Eastern Economy Edition

Second Edition

Power System Analysis

Operation and Control

Abhijit Chakrabarti

Sunita Halder

PHI

Rs. 525.00

POWER SYSTEM ANALYSIS: Operation and Control, 2nd Ed.
Abhijit Chakrabarti and Sunita Halder

ISBN-978-81-203-3585-1

The export rights of this book are vested solely with the publisher.

Fourth Printing (Second Edition) ... ... **October, 2008**

Published by Asoke K. Ghosh, PHI Learning Private Limited, M-97, Connaught Circus, New Delhi-110001 and Printed by Jay Print Pack Private Limited, New Delhi-110015.

CONTENTS

PREFACE

The book, now in its Second Edition, has been thoroughly revised mainly to reorient the chapters and to include new topics as per university syllabi requirements. The chapters included in this edition are power system stability, contingency analysis and power system security, power system compensation, small signal stability, and state estimation and load forecasting. Some of the chapters, namely, power system matrix operations, complex power flows, economic operation, optimal power flow, reactive power control and voltage stability, have been rewritten and new topics have been included in those chapters. The appendices have also been reoriented and a number of solved examples have been included at appropriate places in the text.

We appreciate the patience and support of our spouses and children during the long period we devoted in revising the book.

We hope the Second Edition will be more helpful to the students, practising engineers and researchers. Any constructive criticism of the text is most welcome.

Abhijit Chakrabarti
Sunita Halder

PREFACE TO THE FIRST EDITION

The fundamental aim of this text is to present a number of engineering and economic matters in power system planning operation and control in a comprehensive way. The topics substantiated by a number of illustrations and computer programs describe analytical methods of power system and their operation and control. To understand the text, some acquaintance with the basic concepts in power system as well as advanced calculus methods is needed.

The chapters have been methodically arranged, starting with the basic aspects of power engineering problems. In each chapter, the relevant methods have been dealt with the help of suitable computer-based examples. In a few sections, while dealing with operational problems, optimization methods have been preferred as they can be used without extensive mathematical proofs and are useful in solving practical problems.

The text begins with an introductory discussion on common operating problems and basic aspects of power system operation, including structures of power system, power system representation, and representation of power system elements. Different conventional models are briefly described and analytical treatments are presented to show the modeling concepts of power apparatus like synchronous generator, transformer, transmission lines, motors, etc. Matrix operational methods applicable to power network also get proper attention. Exhaustive analytical treatments are presented for the conventional load flow methods. All the conventional methods of optimization are explained with the help of suitable examples. Some practical and applicational aspects of basic philosophy of ALFC also form part of the discussion. Fundamental aspects of reactive power control and voltage problems in transmission network followed by modern developments in this field including advanced treatments have been detailed. Computerized methods for the analysis of faulted power system have been furnished as well.

The text is self-contained and thorough. It is intended for a one-semester course for postgraduate students as well as a one-year course for senior undergraduate students in electrical engineering. Practising engineers and researchers will also find the book suitable for their use.

The authors acknowledge the constant encouragement they received from the respected Vice-Chancellors Prof. N.R. Banerjea of Bengal Engineering and Science University, Shibpur and Prof. A.N. Basu of Jadavpur University for this project. They also express their gratitude to the respective Deans, Registrars, and Heads of the Departments of both these universities for offering all facilities in course of preparation of the manuscript.

The authors cordially invite any constructive criticism of or comment about the book.

Abhijit Chakrabarti
Sunita Halder

Chapter 1

INTRODUCTION

1.1 STRUCTURE OF A POWER SYSTEM

Electricity is the only form of energy used in the industrial, domestic, commercial, and transportation sectors. It is a coveted form of energy, since it can be generated in bulk and transmitted economically over long distances. Electric power system deals with the generation, transmission and distribution of electric energy associated with the unique feature of control of the flow or demand of energy at desired nodes throughout the power network. Figure 1.1 represents the fundamental structure of a power network where generators produce electric energy, transformers transform this energy into one voltage level from another voltage level, and transmission lines wheel the power from the generating stations to the load centres for the final distribution of electrical energy to different loads. Tie-lines interconnect one system with the neighbouring electric system belonging to the same grid. The circuit breakers isolate a faulty part of the network (the fault being sensed by the relays) while static/rotary compensators may be used for voltage control at load or remote buses. Conventionally, loads are represented in a lumped or composite form.

The best location of a generating station being at a place very close to electrical load centre (i.e., the region where the major energy demand exists), the practical location of the primary conventional energy sources does not necessarily coincide with the urban centres. The location of a power plant is frequently governed by its closeness to the energy resource and transportation facility of the fuel as well as availability of nearest load centre. Environmental aspects are also key factors in determining the site of the plant. Mostly, a generating plant consists of generating units complete with necessary accessories. Control elements like different valves, exciters, regulators etc., also step up transformers, and instrument transformers along with breakers are intended in the station switchyard for the transmission of power and protection of the system. Sources of input to the generating system are conventionally *fossil fuels* (e.g., coal, oil and gas), hydrosource and nuclear fuel. However, non-conventional sources such as wind power, solar energy, tidal power, geothermal power etc. are also being used for *stand-alone systems*.

An electric power system, even a small one, usually constitutes an electric network of vast complexity. The diversity of the system magnitude being great, there is no general rule regarding the structure of the system that applies to any power system. However, any power system could be categorised by a combination of generation, transmission and distribution networks. After generation, transmission

plays a vital role in transporting power from the generating station to load centres. Transmission of power is usually done at HV / EHV / UHV range due to the known fact that it reduces the power loss in the line as well as improves stability. The common transmission voltages across the globe are 33 kV/66 kV/114 kV/132 kV/138 kV/161 kV/220 kV/230 kV/345 kV/400 kV/500 kV in the HV and EHV ranges and 765 kV/800 kV/1100 kV/1500 kV in the UHV ranges in most parts of the world while the generation voltages have commonly been 6 kV/11 kV/12.47 kV/13.2 kV/13.8 kV/ 15 kV/16 kV/22 kV (all are line-to-line voltage).

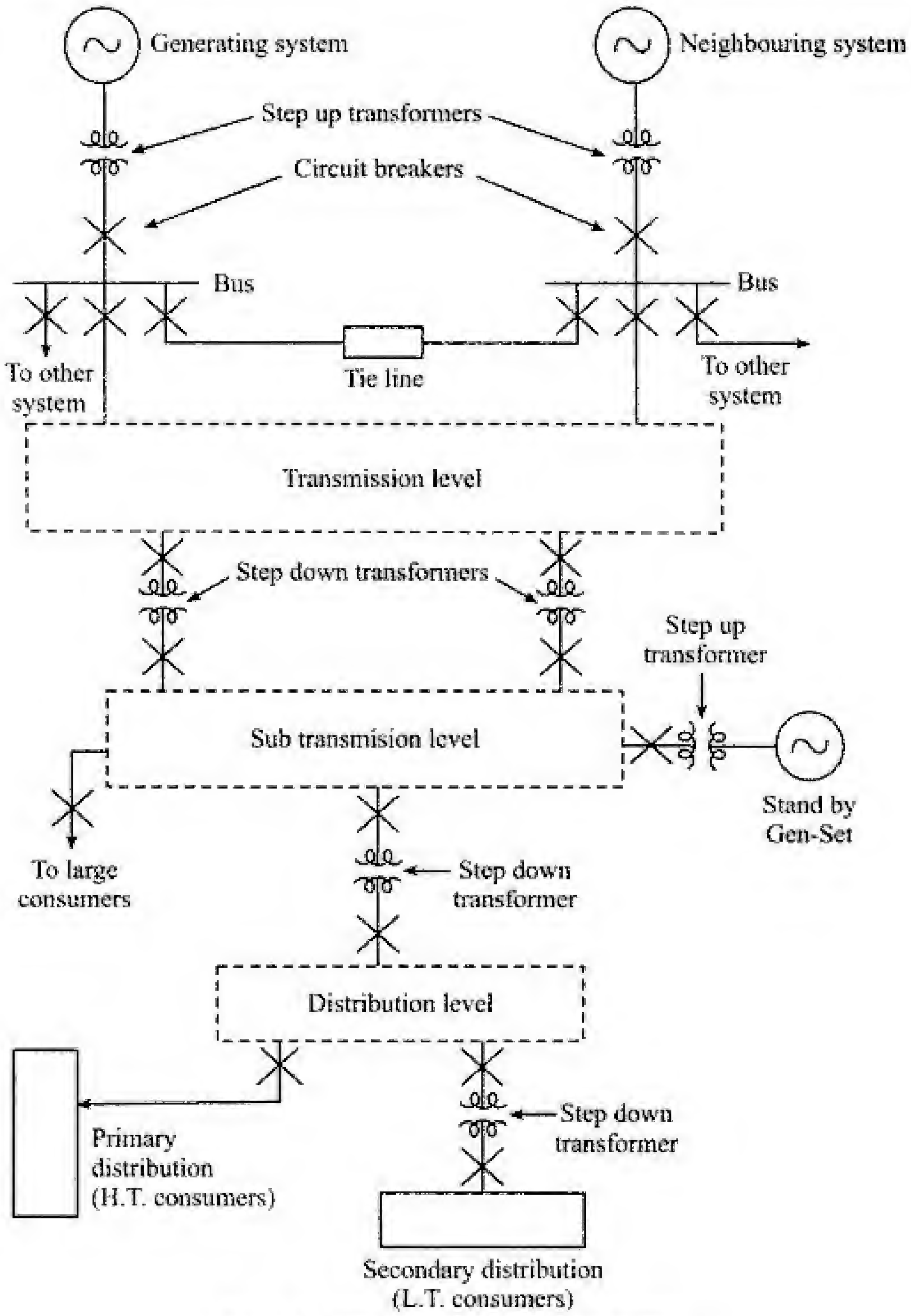


Fig. 1.1 Basic structure of a power system.

In sub-transmission level, the circuits distribute electric power to a number of distribution centres in a certain geographical region at a voltage level that typically varies between 23 kV and 138 kV, the most common grades being 33 kV/66 kV/110 kV/120 kV/132 kV. The sub-transmission circuits may also receive electric power directly from any generator bus. Larger customers are mostly served by sub-transmission level circuits. In small power systems, the sub-transmission level may coincide with the distribution level.

The distribution level consists of the distribution circuits in the overall region of distribution. The larger consumers, i.e. high tension (H.T.) have been termed as *primary distributors* while low tension (L.T.) consumers are the *secondary distributors*. The consumers consuming energy between 3 kV and 23 kV are H.T. consumers while the consumers in the category of 110 V–400/440 V lie in the class of secondary or L.T. consumers. The increasing demand on the electrical energy has led not only to diversification of the generation, transmission and distribution network but also raised the points of proper utilisation and reliability of the electric power. This, in turn, has necessitated the pooling of larger number of power systems into a common grid and consequently insisting for proper scheduling of generation and demand. It also turned out that the incorporation of a large number of systems into a common grid makes the operation of the entire system very sensitive to the operating conditions. Thus in addition to the study of *power system operation*, the knowledge of *power system control* is very much required in order to run the system economically and to maintain a continuous balance between generation and varying load demand. In one way, the problems of dynamic and transient stability, steady state stability, voltage and frequency regulation, power optimisation need to be properly analysed and on the other hand, a methodology of overall system control is to be devised. Digital computers are the most effective tool for the analysis of a power system.

1.2 THE NECESSITY OF CONTROL OF A POWER SYSTEM

Present-day power systems operating as interconnected grid networks have several advantages. First transfer of power between areas is made feasible, enabling the advantage of each generation to be exploited and resulting in improved compensation of load fluctuations with reduced running costs. A reduction in the spare capacity of each of the interconnected system is also possible as a result of mutual assistance between the areas. Power system control is very much required to maintain demand, while the system frequency, voltage level and security are maintained. Overall system control is based on a combination of manual intervention, feedback loops, optimisation techniques and load demand. The requirements for control of frequency and power exchange can be implemented by load frequency control. This control is generally autonomous and each area is responsible for its own steady state power balance. The need for direct action to control network voltage is usually done by the installation of automatic voltage regulating equipment.

The control responsibility is basically divided according to the frequency of intervention of the physical phenomenon involved. The area level decision may also include system voltage control with an optimal scheduling of reactive power flow. Distribution of reactive power generally does not affect the system operating cost significantly, but an optimum allocation may be important for maintaining steady state system stability and voltage levels.

The management and control of a power system is a complex process and it requires proper interaction between many levels. Figure 1.2 indicates the salient elements of the control hierarchy. Manual control is generally slower than digital control. The availability of digital computers has resulted in consideration of digital computer implementation coordinating the control parameters of various levels previously under manual or analog control.

in the control loop and offers the most feasible control output, taking into account the complexity and variety of decisions that have to be taken in view of efficient operation of a modern interconnected power system.

1.2.2 Advantages of Computer Control

The major advantages of computer control of a power system are as follows:

- Highest speed of operation and fastest control action, maximum accuracy and high reliability
- Optimal operation and control
- Fast network state scanning and monitoring
- Scope of implementing adaptive control
- Low maintenance and operating cost

1.2.3 Types of Computer Control System

Basically, the duties of the computer system to control the power system operation are of two types:

(a) Supervisory
(b) Direct

In supervisory type, the computer generates an output to change the set point of the controller. In this case, the computer is just the decision-making tool while the controller is the workhorse in the control system. The controller could be an analog or a digital type.

In direct control, the computer itself acts as controller and executes the decision taken by itself in order to control the process.

Depending on the design of the system, the computer control can be

(i) Off-line
(ii) On-line
(iii) In-line

When the control is off-line, the computer is fed about the data regarding the process through a human operator. The computer is not at all connected to the actual system. The duty of the computer is to process these data and output the results seeing which the operator can recommend a control action.

In on-line systems, the computer is physically connected to the power system through suitable interfacing circuitry and receives the necessary data without any human intervention. The computer processes the input data and outputs the result to an operator who then implements the control action. This is basically the simplest on-line control and is termed as open-loop on-line control. It is also possible to have closed-loop on-line control where the computer requires no manual intervention in implementing the output decision. The computer decision is transmitted to the power system network through necessary interfacing network automatically.

In the in-line type control, the operator collects data from the system and enters them rapidly and directly into the computer through the keyboard.

The digital computer is not only the most valuable tool in power system control but also the most sophisticated. To economise the computer-controlled system, in the lower levels of control, some analog control equipment can still be used. This includes the purpose of instrumentation and metering too. Analog controllers can be used to simulate the power system control aspects for training purposes also.

Before applying direct computer control, it is still possible to use cheaper digital instruments to gather data and provide track up for analog instruments. Digital means may also be adopted for data storage and control of power system elements in the utility sector. Careful planning is needed for the

successful implementation of a computer as a *controller*. Figure 1.3 shows the steps that are to be considered before implementing the decision of using computer controls.

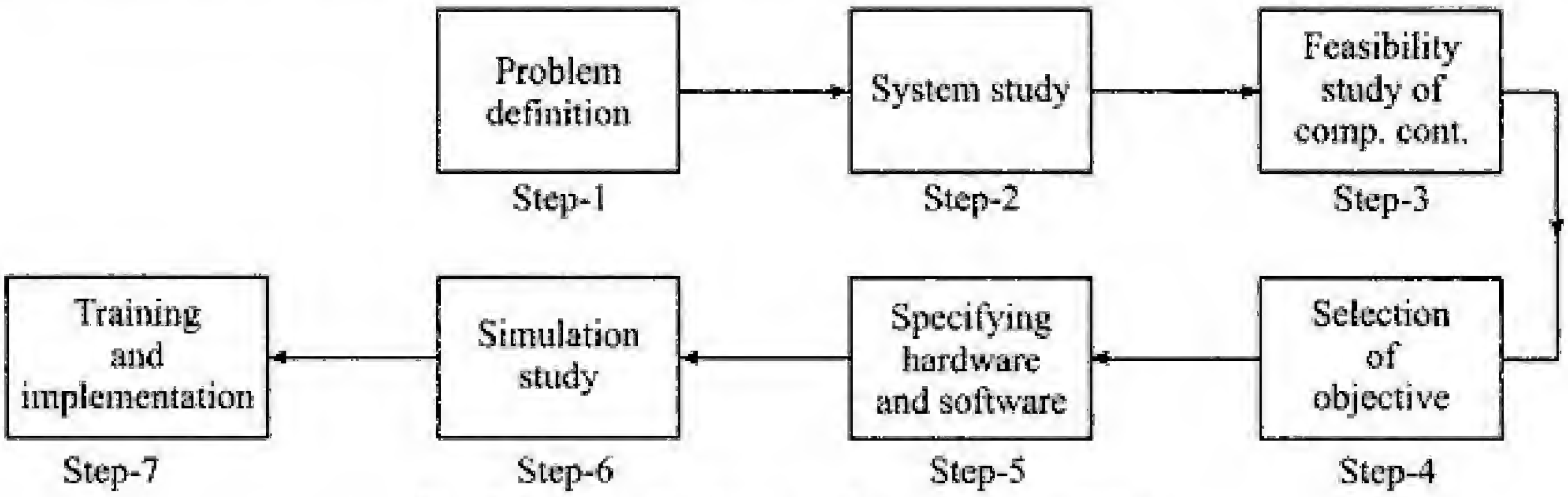


Fig. 1.3 Steps of planning of a computer control.

1.3 POWER SYSTEM REPRESENTATION

As a complete diagram of a practical power system presenting all the three phases (generation, transmission and distribution) is too complicated, it is a normal practice to represent a power system by means of simple systems for each component resulting in single-line diagram, as shown in Fig. 1.4.

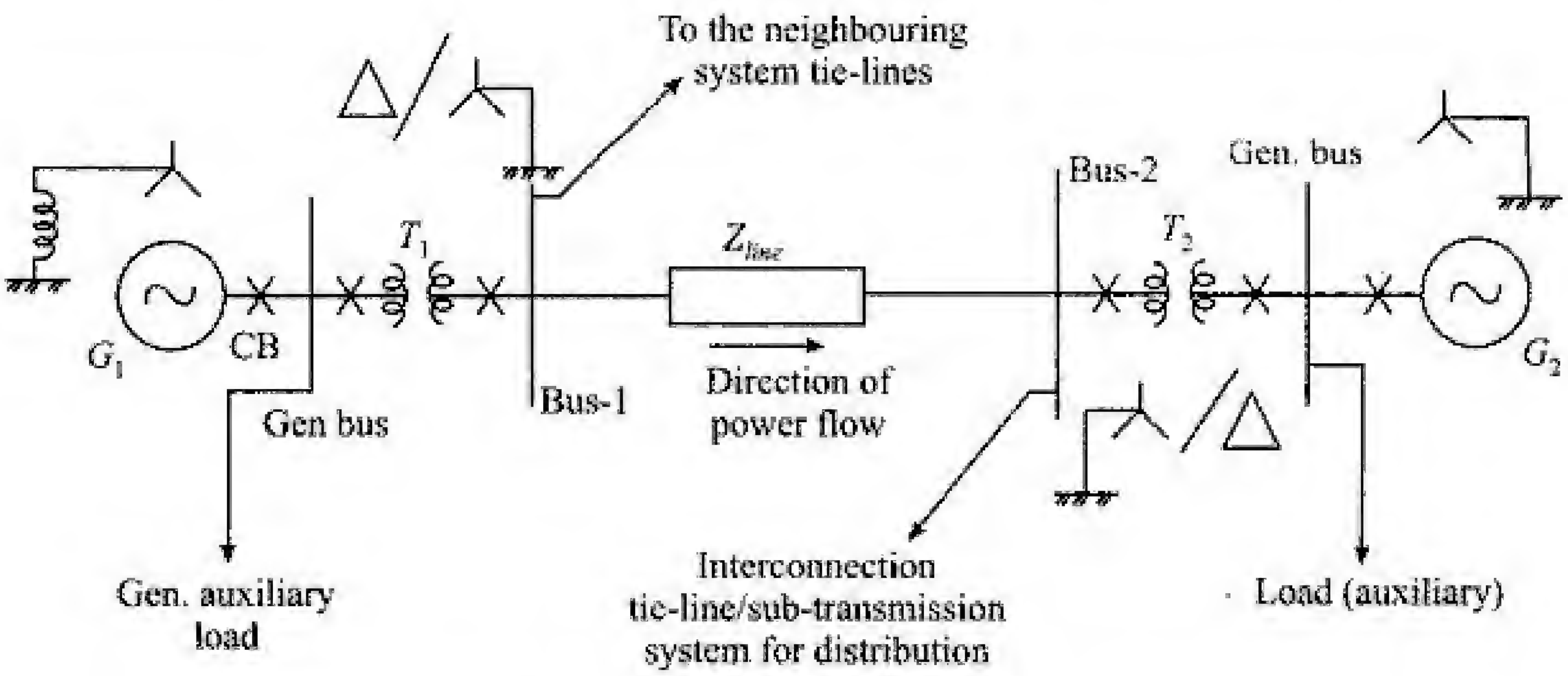


Fig. 1.4 Single-line representation of a simple two-bus system.

Any particular component may or may not be shown in the diagram depending on the information required in a system study, e.g. circuit breakers need not be shown in a load flow analysis diagram but are to be shown for a protection study. Different generator and transformer connections are indicated by proper symbols. Equivalent circuits of power circuit components can be represented in the diagrams.

1.4 POWER SYSTEM AT NORMAL OPERATING STATE

A power system operates in a normal state if the following conditions are satisfied:

(a) There is a perfect balance between power generation and demand; consequently, the load flow equations are satisfied.

(b) The frequency, f, is constant throughout the system.

(c) The bus voltage magnitude $|V_i|$ is within the prescribed limit, i.e.

$$|V_i|_{\min} \le |V_i| \le |V_i|_{\max} \tag{1.1}$$

(This is required as all the power equipment and apparatuses are supposed to be operated at a specified voltage.)

(d) No power system component is to be overloaded.

However, the load is mostly a constantly varying parameter and in order to meet this slow change of the load demand, the normal operating state drifts with time (the load is mostly met by optimal generation scheduling). Change in frequency causes change in the speed of the drives in the consumer's plant. Further, it is necessary to maintain network frequency constant so that the power stations run satisfactorily in parallel, the various motors operating on the system run at the desired speed and other devices function properly. However, the most important reason for keeping frequency of the electrical system constant is that its constancy indicates power balance of the total system.

Overloading of any power system component results in higher temperature of operation and the component is likely to be damaged. System stability, given by the *maximum power* that can be transmitted, also indicates the power system operating at normal state. This *steady state stability limit* (also known as *static transmission capacity*) is given by

$$P_{ij_{\max}} = \frac{|V_i||V_j|}{X_{ij}} \tag{1.2}$$

In an attempt to transmit more power than this limit, synchronism is lost and the transmission system collapses. For short lines (less than 100 km), the thermal limit capability fixes the loading of line whereas for medium or long line, the static transmission capacity becomes the limiting factor. *Voltage stability* is another operating parameter that needs to be considered.

1.5 OPERATING PROBLEMS IN POWER SYSTEMS

An insight into the operation of any electric power system reveals that frequency and voltage are the prime and main indicators of proper system operation. Any disturbance in the system operation causes variation in these two parameters separately or jointly and in cases of severe system disturbances, the frequency and/or voltage variations may be abnormally high indicating the loss of system stability. Frequency variation being the cause of real power mismatch, voltage is the sole indicator of the reactive power imbalances in the system. Common operating problems that are inherent in EHV power lines have been classified and briefly described below. Major areas of study in the relevant area consist of loadability, frequency dynamics, transient stability, power line oscillations and voltage stability problem, in addition to the conventional steady state and transient state power stability.

1.5.1 Loadability of Transmission Lines

Loadability of transmission lines is defined as the *optimum power transfer capability of an EHV line under a specified set of operating criteria*. In an EHV power system, the power transfer capability of a transmission line is substantially affected by nodal power injections and topological changes. It has been generally accepted that the *nodal strength*, i.e. capability of a transmission line, is substantially affected by nodal power injections and topological changes. It has been generally accepted that nodal strength, i.e. the *short circuit capability* (S.C.C.) of the system, is the inverse of the positive sequence equivalent impedance in per unit and it indicates the *robustness* of the power network concerned. This impedance, also consisting of the source reactance, is usually dictated by the series reactance of the line when

analysed in a loss-less frame. The most conventional form of representing the loadability being in terms of *surge impedance loading* (*SIL*), where $SIL = (V^2/Z_0)$ in p.u., the basic expression of power transfer is given by

$$P = \frac{V^2}{X}\sin\delta \tag{1.3}$$

(assuming equal sending and receiving end voltage, the power angle being δ and transfer reactance being X, $X = xL$, x being the reactance per unit length and L, the length of the line). Also,

$$X = xL = \omega\ell L = \omega L\sqrt{\frac{\ell}{c}}\cdot\sqrt{\ell c} = \sqrt{\frac{\ell}{c}}\cdot\omega\sqrt{\ell c}\cdot L$$

or,

$$X = Z_0\beta L = Z_0\theta \approx Z_0\sin\theta \tag{1.4}$$

where Z_0 = surge impedance = $\sqrt{\frac{\ell}{c}}$, ℓ and c being the line inductance and shunt capacitance per unit length, respectively, β = phase constant of the wave of propagation ($= \omega\sqrt{\ell c}$, ω being the angular frequency), θ electrical line length of the line in radian and θ ($= \beta L$) being small, $\sin\theta \approx \theta$.

Substituting equation (1.4) into equation (1.3)

$$P = \frac{V^2}{\sin\theta}\times\frac{1}{Z_0}\sin\delta = (SIL)\frac{\sin\delta}{\sin\theta}$$

or,

$$\frac{P}{SIL} = \frac{\sin\delta}{\sin\theta} \quad [\text{in p.u., } SIL = (V^2/Z_0)] \tag{1.5}$$

Equation (1.5) indicates that the power transfer capability can be represented in terms of *SIL*. Figure 1.5 represents the loadability of a typical EHV single circuit line assuming various line lengths.

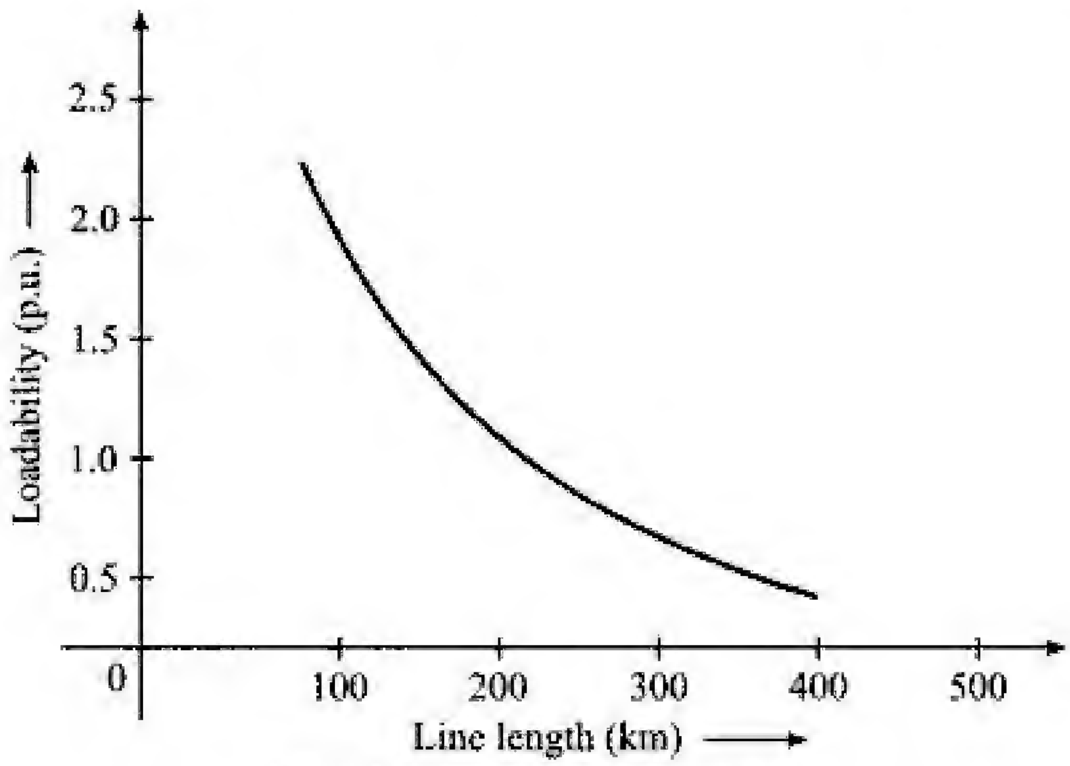


Fig. 1.5 Profile of line loadability.

High source reactance plays a vital role in limiting the line loadability. Loadability can be improved by reducing the reactance of overhead wires and placing series capacitor in line as well as relaxing the voltage drop constraint.

As the machine consists of heavy rotational mass, the speed change cannot be instantaneous and is governed by the following equation,

$$T = J\frac{d\omega}{dt}$$

or,

$$\frac{d\omega}{dt} = \frac{T}{J} = \frac{P_a}{\omega} \times \frac{1}{J}$$

or,

$$\frac{d\omega}{dt} = \frac{1}{J\omega}\left(P_T - P_g\right) \tag{1.8}$$

As, P_a = accelerating power = $P_T - P_g$; P_T being the turbine power output and P_g, the electrical power output. Also,

$$M = J\omega \quad \text{then,} \quad \frac{d\omega}{dt} = \frac{1}{M}\left(P_T - P_g\right)$$

i.e.,

$$\frac{d\omega}{dt} = \frac{1}{\frac{2H}{\omega_{syn}}}\left(P_T - P_g\right)$$

where $M = (2H/\omega_{syn})$, ω_{syn} being the synchronous speed of alternator rotor. Thus,

$$\frac{d(2\pi f)}{dt} = \frac{2\pi f}{2H}\left(P_T - P_g\right)$$

or,

$$\frac{df}{dt} = \frac{f}{2H}\left(P_T - P_g\right) \tag{1.9}$$

In a 50 Hz system from equation (1.9), the initial frequency decay rate for a system to encounter a sudden load demand can then be represented by

$$\frac{df}{dt} = \frac{25}{H}\left(P_T - P_g\right) \tag{1.10}$$

The initial frequency decay rate for different types of loads for a typical radial system for varying attempted overloads has been graphically represented in Fig. 1.7.

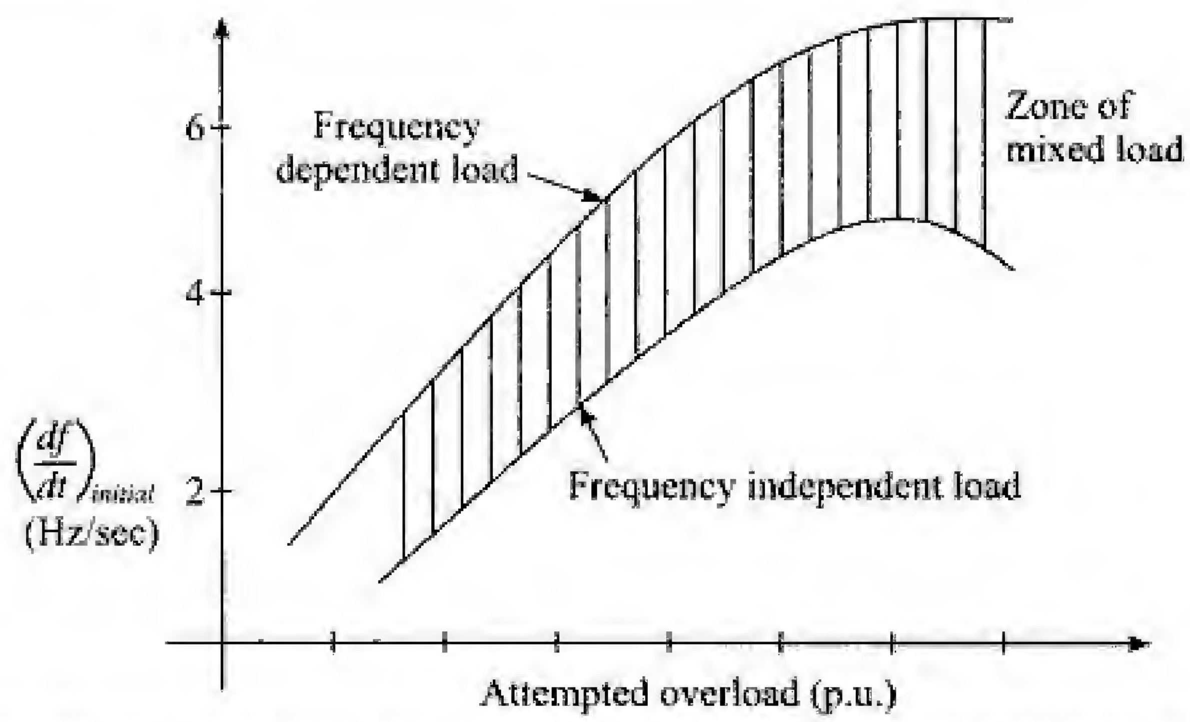


Fig. 1.7 Initial frequency rate for varying attempted overloads of static and frequency-dependent loads.

Low frequency oscillations exist particularly during heavy loading conditions and if not damped properly, they may cause instability. Damping of oscillations can be enhanced by eliminating the non-linear loads in the distribution system. Installation of HVDC line between two regional EHV grids in addition to EHV AC link can eliminate the problem of low frequency oscillations provided the line power flows in AC and DC links are carefully monitored during heavy loading periods.

1.5.6 Reactive Power Limitations and Voltage Control Problems

In an EHV power system, the reactive power flow may be incoherent and the limits of reactive power availability may be restricted, leading to the system voltage collapse in case the line is reactive power constrained. A sudden increase in reactive power demand in a reactive power constrained line is generally due to the contingency in transmission network (e.g., the tripping of a heavily loaded EHV line causing an increase of the load burden of the adjacent line(s) for maintenance of the constant system load). The additional reactive demand caused by the disturbance is generally compensated by the system reactive reserve, if available, allowing the system to settle down at a reduced level of transmission voltage. On the other hand, where the reactive reserve cannot cope with the sudden rise of reactive demand, system voltage instability results. This collapse may occur even though the real power requirements of the system are met and the frequency is stabilised.

In case of a contingency in a transmission system, the *series reactive loss* (Q_{sl}) increases. When the remaining healthy line(s) loading surpasses the *SIL* (surge impedance loading), the rising rate of series reactive losses is substantial showing a steep increase in the rate of series reactive loss against *SIL* loading at its higher magnitudes. This, in turn, depresses the system voltage in the lines. As the *SIL* is directly proportional to the square of the system voltage ($SIL = V^2/Z_0$), it starts to drop as the voltage decreases causing further series reactive loss. In addition to this effect, reactive charging capability, being proportional to the square of the transmission voltage, decreases with decaying voltage causing further deterioration of the system voltage stable state. In extreme cases, all these effects may add to create high magnitude of the line reactive loss for each extra unit of rise in real load. This enormous rise in demand of reactive power invites severe voltage control problems, and in case of weak systems, there may be spontaneous voltage instability.

In a reactive power constrained system, the condition of voltage instability is always governed by the limitation of reactive power availability. The magnitude of the limiting value of the reactive power at any operating condition can be determined analytically and it reveals that the stable voltage state can only be maintained if the system possesses the corresponding limiting value of the reactive power transfer capability.

1.6 SECURITY ANALYSIS AND CONTINGENCY EVALUATION

Under normal operating conditions a power system may face a contingency condition such as outage (complete or partial) of a generating unit or of a line, a sudden increase or decrease of the power demand on the system. A system operator has to analyse the effect of such highly probable contingencies so that the operator may take corrective action in the event of their occurrence. Thus, the analysis of some of the most probable contingencies helps in enhancing system security. The security assessment and its enhancement form an important part of planning and operation of power systems that are continuously expanding.

The main operating states of a power system may be classified as

(a) Normal
(b) Emergency
(c) Restorative

However, later on two more states, "Alert" and "Extremis", were added. For the sake of understanding, only the three-state transition diagram of Fig. 1.10 will be considered here as this diagram provides a good conceptual picture of the overall computer control requirements of a power system.

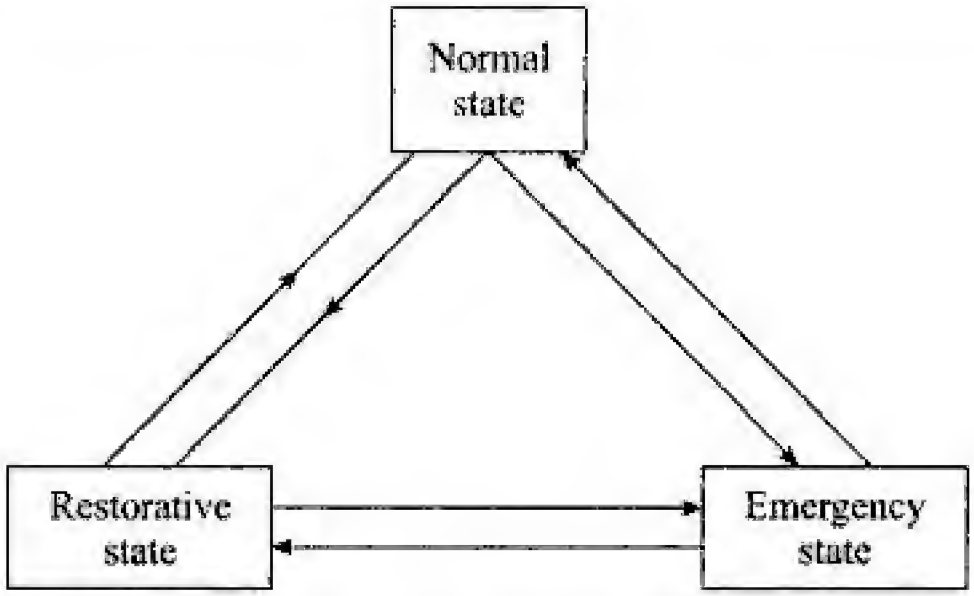


Fig. 1.10 State transition diagram.

Most of the times, the system remains in the normal state as stated earlier. In this state, the load flow equations are satisfied and voltage constancy is maintained, with all operating (or inequality) constraints being satisfied.

When these constraints are not satisfied, the system is said to be in alert state. Contingency evaluation is, therefore, required to find out if the prevailing normal operating condition is secured. The important and probable contingencies to be considered are:

- Outage of a line
- Outage of a generating unit
- Single phase or three-phase fault.

The modern power system control centres (or load dispatch centres) are the places for security monitoring. In these centres, on-line identification of the actual operating condition is undertaken utilising a computer-based technique, known as *state estimation*. The state estimation gives the load dispatcher the best estimate of the complex bus voltage at any instant from the redundant set of telemetered data and breaker status. The security analysis, with the help of the state estimator, then finds out the impact of the contingencies using some fast load flow method such as *Fast Decoupled Load Flow* (FDLF). In this way, the real time data obtained at the energy control centres are examined by the security analyser to find out the security of the system. If the system is found to be insecure, then the system engineer determines the preventive controls to be applied to bring the system back into the secure zone. This may require generation rescheduling and/or a change in the interchange schedule. This normally would deviate the system from the most economic operation, but is quite justified and is very much desirable.

In case the emergency occurs due to cascading events or contingencies, the corrective emergency controls such as *optimum load shedding*, the network rearrangement, starting up of some quick-start units are to be applied to bring the system back into a secured state.

Contingency such as outage of a line, generator or loss of a transformer would reduce the security level. The operating problems as indicated earlier may also lead the system to a state having lesser security. This state is now the alert state where the system remains stable and the operating constraints are satisfied, but an abnormal voltage and frequency condition may arise. This type of state can be tolerated for some time. Preventive controls (for example, start up of standby units and/or compensators) may bring the system from this state to the normal state.

However, when the system is in alert state, some additional contingency may take place such as further loss of unit or line. The contingencies in the distribution or sub-transmission levels may also lead the system to another state with lesser security. This is an emergency state and emergency controls must be implemented to save the system from vulnerable collapse. There may be undue voltage depression and/or overloadings of lines during emergency state. If the emergency controls fail, then the overloaded line must be tripped and the system faces the risk of total shutdown (the extreme state). Load shedding and intentional voltage degradation are the two most effective means of implementing emergency control in order to save the power system. The restorative state involves rescheduling of active and reactive power, re-synchronisation and gradual load pick-up. The system now returns either to a new normal state or to the previous normal state.

In order to avoid the damage to the costly components of the power network, as a first line of defence, protective devices are used at the appropriate places in the system. Functions such as relaying and voltage control are carried out within milliseconds and executed locally throughout the system and no centralised decision-making process is involved. Typically, a relay detects the fault and initiates a circuit breaker tripping to remove the unhealthy part of the network or faulted components from the rest of the system. Another important objective of the emergency control is to perform automatic reorganisation of components. The re-closing of a line must be fast enough. The fast application of emergency controls saves the system from the loss of synchronisation and subsequent islanding.

1.7 AUTOMATIC CONTROL

The necessity of control of a power system being highlighted, it is imperative to mention that the control measures are most effective once the automatic devices are the control elements. Automatic load frequency control (ALFC) and automatic voltage control (AVC) are the two most important aspects that can be implemented to ensure proper system operation.

1.7.1 Automatic Load Frequency Control (ALFC)

In this control circuit, there are two feedback loops, primary and secondary. The purpose of both these loops is to achieve real power balance or load tracking in the system. ALFC loops are designed to maintain power balance by an appropriate adjustment of the turbine torque. By means of the primary loop, a relatively fast but course frequency control is achieved. The secondary ALFC loop works in a slow reset mode to eliminate the remaining small frequency errors. This loop also controls the power interchange between pool members. While the primary loop response is over seconds, the secondary fine adjustments may take about minutes and will stop only after achieving zero frequency error. It may be noted that since the whole group of generators within a given area move coherently, the frequency dynamics is slow, thus characterising them all with the same Δf (frequency error).

In the case of interconnected power systems, tie-lines are erected to interconnect the neighbouring areas. Multi-area dynamics is important to be discussed. All the power commands can be executed in unison among all the generators under control. The secondary ALFC loops in a multi-area system contain control signals, now referred as *area control errors* (ACE), which, in addition to frequency error Δf, also contain the errors in the tie-line powers. These concepts have been discussed in Chapter 8.

1.7.2 Automatic Voltage Control (AVC)

In automatic voltage control systems, bus voltage is measured utilising a potential transformer and is compared to a reference after being rectified and filtered. The resulting error voltage, after amplification, serves as input to an excitation control system where output directly feeds the generator field. A drop in

the terminal voltage causes a boost in the field current. This increases the reactive power output of the machine, thus tending to offset the initial voltage drop.

The AVR loop maintains reactive power balance of a generator by maintaining a constant voltage level. Besides generator buses, shunt capacitors are used to key buses to ensure an overall good voltage profile. By controlling these capacitors and/or reactor banks from an error voltage similar to that of AVR loop, automatic closed-loop voltage control can be achieved. Modern installations use thyristor controls which allow continuous smooth variations of reactive power.

1.7.3 Control Components in Power System

AC Power System is controlled primarily by mechanical means such as circuit breakers, OLTC of transformers and isolators. The solid state devices and communications system are used for monitoring, data-logging and protection. Thus, the primary control on power side suffers from the following disadvantages:

- Comparatively slow operation
- Controls are of On/Off nature and are not regulatory
- Controls cannot be used frequently as the control mechanism has a tendency to wear-out.

Apart from the above, some time the present-day power system suffers from lack of a decision support system for optimal and reliable power system operation. Sophisticated computer systems and faster and reliable communication network between regional and state load dispatch centres (RLDCs and SLDCs) should form the backbone of the decision support system. The data from remote sub-stations need to be gathered at RLDCs/SLDCs for the analysis of grid operating conditions. The analysis can then be used to generate control commands for transmission to SLDCs and remote sub-stations.

In a number of cases, due to the mechanical control mechanism supplemented by lack of proper load dispatch tools, the present-day power system is inflexible and stiff. This makes the system manager, a helpless spectator of various grid problems such as overloading of transmission elements, poor VAR management etc. Lack of control measures to deal with emergent operating conditions often leads to grid disturbances and blackouts. However, with the availability of thyristor valves for power applications, it has become possible to replace the mechanical operations by electronic switches. Though the ON/OFF operation can still be performed by mechanical closing/opening of circuit breaker, it is now possible to change the basic characteristic of the network by electronic devices to achieve the requisite flexibility.

The availability of faster control is a necessity but not sufficient for making the AC system flexible. One should first address the objectives to be achieved by the FACTS (Flexible AC Transmission System). Some of the objectives can be as follows:

- Regulate power flow on AC lines with a view to either avoid overloading or to minimise power loss
- To operate the system at a safe power-angle for same power delivered
- To enhance the power transfer capability of the system by introducing improved dynamic characteristics
- System islanding under extreme conditions
- Strategies to save the system/islands from total collapse.

After the objectives have been identified, the following strategies need to be decided.

(i) Planning and operational system strategies:
  - System analysis and planning
  - Loss optimisation
  - System security

(ii) FACTS controllers strategy
(iii) Inter-utility communication strategy.

The details of the FACTS project for a region can be worked out based on the following:

- Installation of series capacitors on certain sections/lines
- Installation of *static VAR compensators* (SVCs) at strategic locations
- Installation of phase-shifters, if required
- Low-frequency oscillation dampers, if required
- Communication network
- FACTS controller with on-line data monitoring
- Computer software for grid analysis.

The above items not only require huge investments but also coordination among the various utilities. A systematic approach is to be adopted and the investments are to be phased out over a period of time. The following phases are important aspects in FACT planning.

Phase I: System Security

In the first phase, emphasis should be laid on prevention of faults spreading into the system and creating grid instability. This phase can be termed as *system security* phase.

In this phase a few pilot project(s) can be taken up for the installation of switched series capacitors in certain selected locations. There shall not be any necessity of any elaborate FACTS controller at the stage. The control actions can be derived from terminal sub-stations.

Phase II: Strong Interconnections

In the second phase emphasis should be on strong inter-utility interconnections free from low-frequency oscillations. This shall involve:

- Extension of switched series capacitors to many other sections
- Installation of SVCs at grid points
- Phase shifter, if required
- Development of FACTS controller
- Communication means.

Phase III: Optimal Operation

Optimal grid operation can be the watch-ward of the third and last phase. In this, loss optimisation can be carried out through the FACTS controller. Many other advanced control means can be used for optimal system operation, viz. phase-shifters, SSR (sub-synchronous resonance) dampers, dynamic loads, etc.

The evolution of FACTS has to be progressive with time, not only because of the huge resource requirements but also because of the fast development on technological front. FACTS is likely to become more and more economically viable with the passage of time.

1.8 USE OF COMPUTERS AND FUTURE TRENDS

The use of digital computers for solving a load flow problem was first made in 1956. Subsequently, other studies such as fault level analysis, transient stability analysis, economic load dispatch etc. were carried out off-line using digital computers. In more recent times, however, it has been realised that digital computer can also be greatly helpful as on-line monitoring and controlling agents for the modern large-scale integrated power systems. As mentioned earlier, a compelling reason for this is the critical

dependence of the reliable operation of the large interconnected power system on a proper setting of the operating conditions. Yet another reason is the dramatic improvement in the capabilities of the computing system associated with a steep fall in their costs. Due to the availability of modern, fast and cost-effective computers, it has been possible to achieve a greater flexibility, accuracy, speed and economy in real time control and monitoring of power systems. Micro, mini and mainframe computers are increasingly being used for both off-line computation and on-line monitoring and control of power systems, both inside and outside energy control centres.

The addition of system security function has initiated a significant change in the scope of control centres. This addition involves major changes in real time data requirements and the sophistication in data and information processing. The functions can be implemented in a completely automatic manner using *supervisory control and data acquisition* (SCADA) systems. It involves data collection (the data involved are active and reactive powers flowing through the lines and transformers, voltage and frequencies at various bus bars, status of breakers and switches etc.) and display of the desired data as well as data processing for network state estimation, automatic generation control etc. The output commands are used to alter generation, open or close circuit breakers, switch on or off reactive power control elements and so on. In addition to automatic generation control (AGC) and automatic voltage control (AVC), the other denigrated works of the computer control are economic dispatch, security monitoring, security analysis, off-line short circuit calculations and state estimation.

1.9 APPLICATION OF COMPUTER METHODS IN POWER SYSTEM ANALYSIS

1.9.1 Gaussian Elimination Method

Gaussian elimination method is an elementary elimination method that reduces a set of equations to an equivalent upper triangular matrix, which can be solved by backward substitution. Although this method is applicable to N no. of equations, it is illustrated here for three simultaneous equations. Let the equations be

$$Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 = I_1 \tag{1.11a}$$

$$Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 = I_2 \tag{1.11b}$$

$$Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 = I_3 \tag{1.11c}$$

In a matrix form

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \tag{1.12}$$

Dividing both sides of equation (1.11a) by Y_{11} we get

$$V_1 + Y'_{12}V_2 + Y'_{13}V_3 = I'_1 \tag{1.13}$$

where $$Y'_{12} = \frac{Y_{12}}{Y_{11}},\ Y'_{13} = \frac{Y_{13}}{Y_{11}} \text{ and } I'_1 = \frac{I_1}{Y_{11}}$$

Let us now eliminate V_1 from equation (1.11b) [by multiplying equation (1.11a) by $\left(-\frac{Y_{21}}{Y_{11}}\right)$ and then adding it to equation (1.11b)]. Then we have

$$Y'_{22}V_2 + Y'_{23}V_3 = I'_2 \tag{1.14a}$$

where $$Y'_{22} = -\frac{Y_{21}}{Y_{11}} \times Y_{12} + Y_{22}; \; Y'_{23} = -\frac{Y_{21}}{Y_{11}} \times Y_{13} + Y_{23}$$

and $$I'_2 = -\frac{Y_{21}}{Y_{11}} \times I_1 + I_2$$

Dividing both sides of equation (1.14a) by Y'_{22} we get

$$V_2 + Y''_{23}V_3 = I''_2 \tag{1.14b}$$

where $$Y''_{23} = \frac{Y'_{23}}{Y'_{22}} \quad \text{and} \quad I''_2 = \frac{I'_2}{Y'_{22}}$$

Similarly, if we eliminate V_1 from equation (1.11c) [by multiplying equation (1.11a) by $\left(-\frac{Y_{31}}{Y_{11}}\right)$ and then adding it to equation (1.11c)], we get

$$Y'_{32}V_2 + Y'_{33}V_3 = I'_3 \tag{1.14c}$$

Let us eliminate V_2 from equation (1.14a) [by multiplying equation (1.14a) by $\left(-\frac{Y'_{32}}{Y'_{22}}\right)$ and then adding it to equation (1.14c)] we get

$$Y''_{33}V_3 = I''_3 \tag{1.15a}$$

Dividing both side of equation (1.15a) by Y''_{33} we get

$$V_3 = I'''_3 \tag{1.15b}$$

where $$I'''_3 = \frac{I''_3}{Y''_{33}}$$

Equations (1.13), (1.14b) and (1.15b) can be written in a matrix form as follows:

$$\begin{bmatrix} 1 & Y'_{12} & Y'_{13} \\ 0 & 1 & Y''_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I'_1 \\ I''_2 \\ I'''_3 \end{bmatrix} \tag{1.16}$$

Thus, equation (1.12) has been converted into an equation containing an upper triangular matrix. From the above equation, we can easily get the solution for V_1, V_2 and V_3 by backward substitution.

From equation (1.15b), we have

$$V_3 = I'''_3$$

Substituting V_3 in equation (1.14b), we get

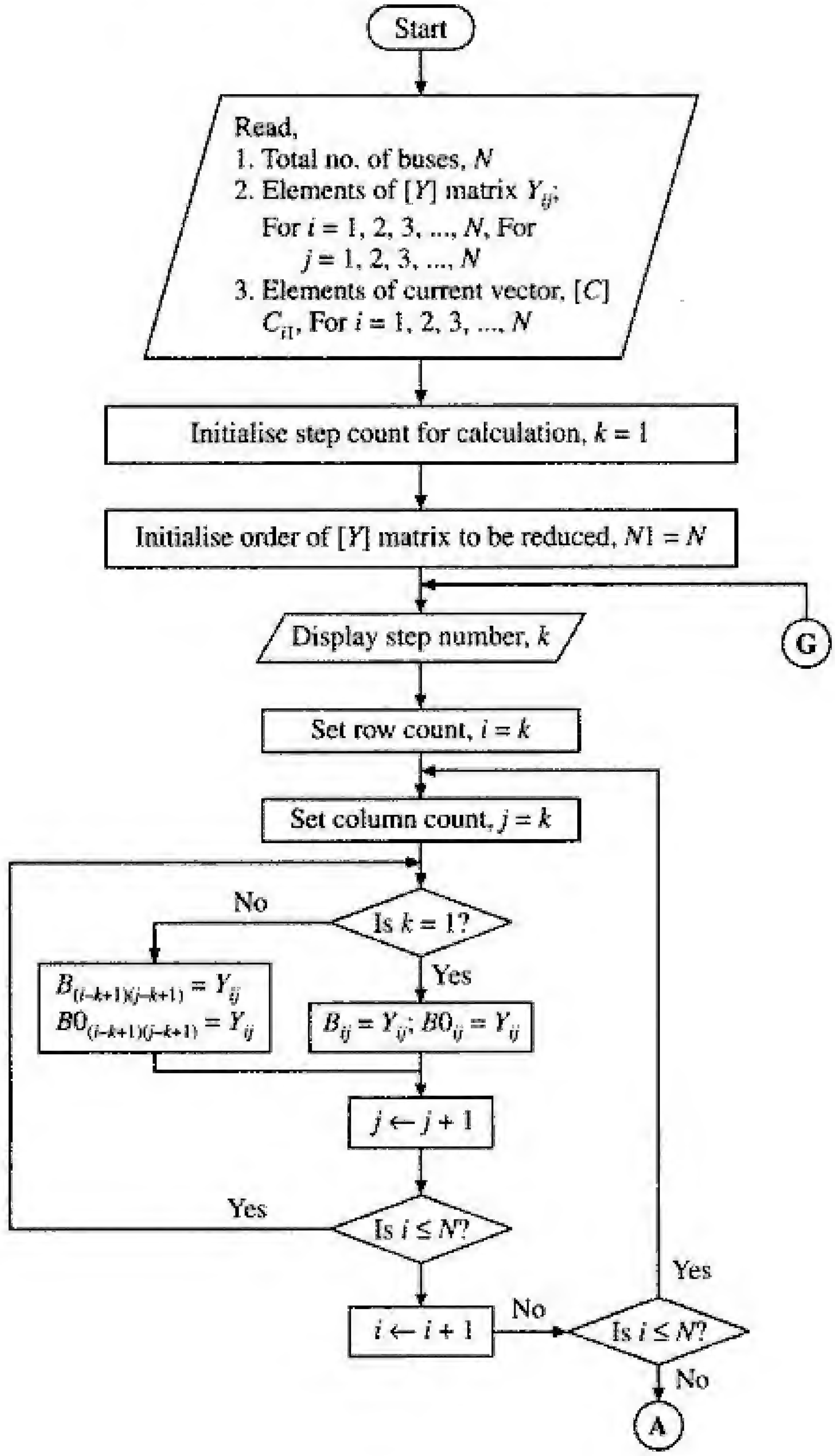
$$V_2 = I''_2 - Y''_{23}V_3$$

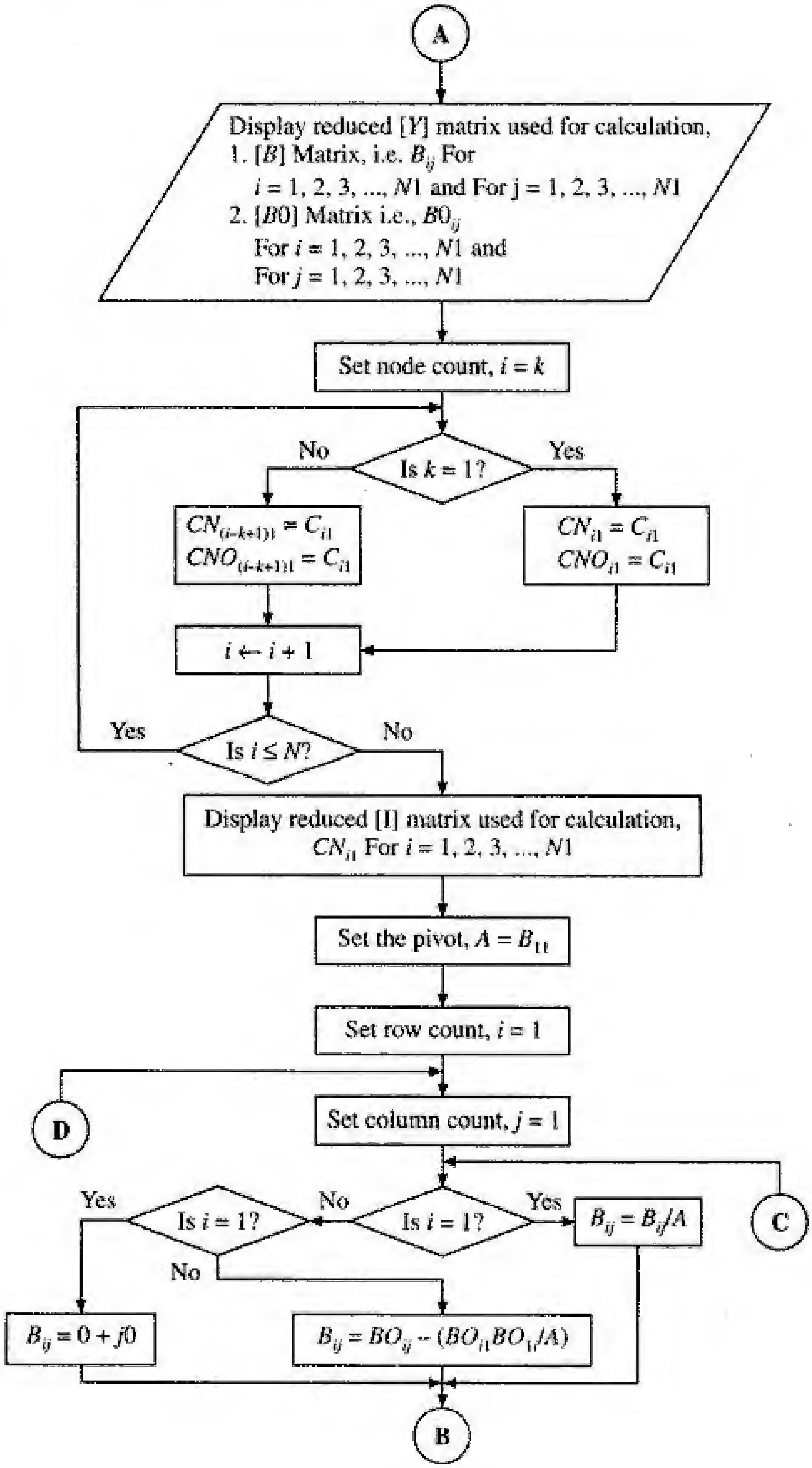
Substituting V_2 and V_3 in equation (1.13), we get

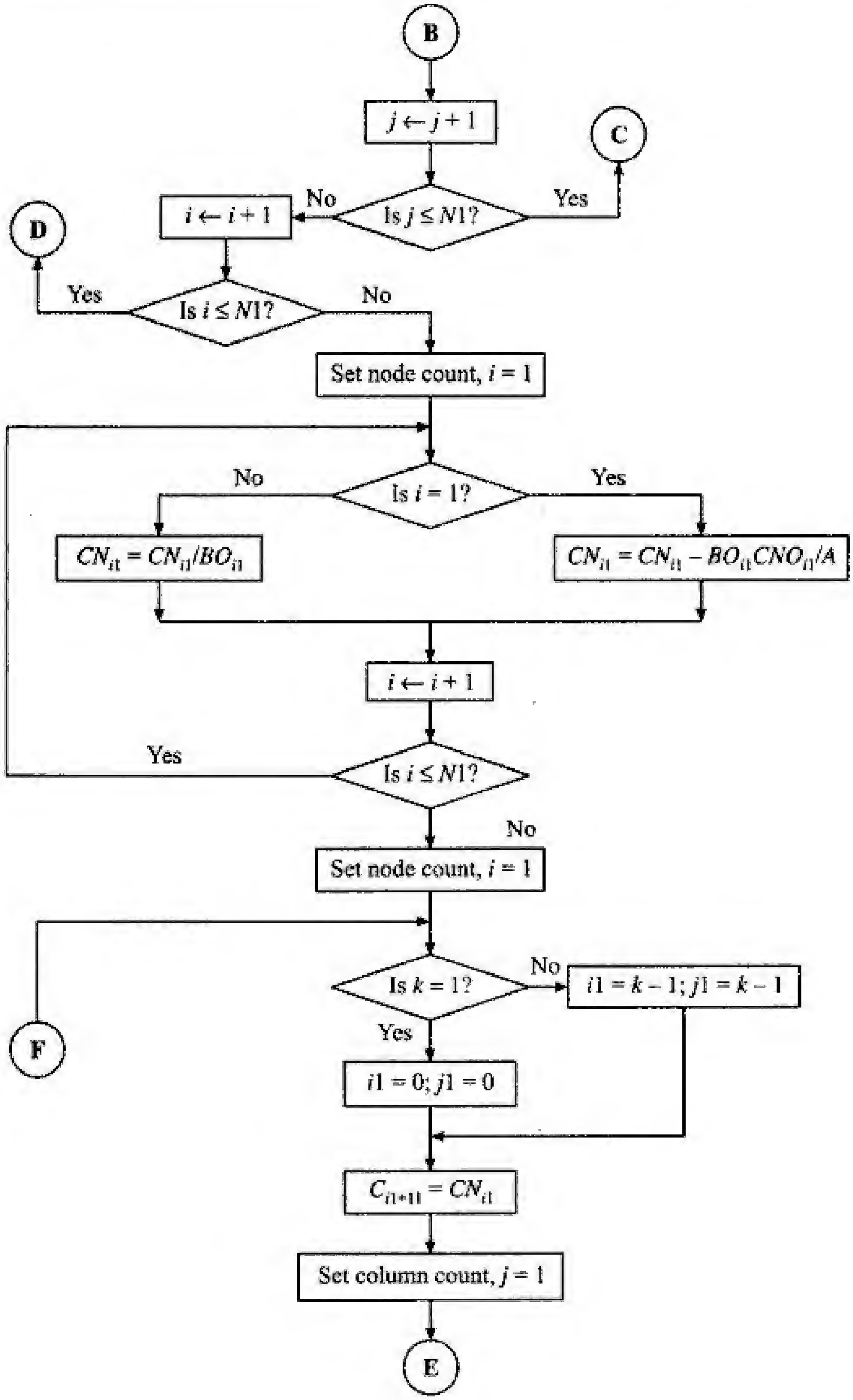
$$V_1 = I'_1 - (Y'_{12}V_2 + Y'_{13}V_3)$$

Hence we can get the final solution of V_1, V_2 and V_3.

The detailed flowchart of the method is given in Fig. 1.11.




A
Display reduced [Y] matrix used for calculation,
1. [B] Matrix, i.e. B_{ij} For
$i = 1, 2, 3, \ldots, N1$ and For $j = 1, 2, 3, \ldots, N1$
2. [B0] Matrix i.e., $B0_{ij}$
For $i = 1, 2, 3, \ldots, N1$ and
For $j = 1, 2, 3, \ldots, N1$
Set node count, $i = k$
Is $k = 1$?
No
Yes
$CN_{(i-k+1)1} = C_{i1}$
$CNO_{(i-k+1)1} = C_{i1}$
$CN_{i1} = C_{i1}$
$CNO_{i1} = C_{i1}$
$i \leftarrow i + 1$
Is $i \leq N$?
Yes
No
Display reduced [I] matrix used for calculation,
CN_{i1} For $i = 1, 2, 3, \ldots, N1$
Set the pivot, $A = B_{11}$
Set row count, $i = 1$
Set column count, $j = 1$
D
Is $i = 1$?
Yes
No
Is $i = 1$?
Yes
No
$B_{ij} = B_{ij}/A$
C
$B_{ij} = 0 + j0$
$B_{ij} = BO_{ij} - (BO_{i1}BO_{1j}/A)$
B


B
j ← j + 1
C
No
Is j ≤ N1?
Yes
i ← i + 1
D
Yes
Is i ≤ N1?
No
Set node count, i = 1
No
Is i = 1?
Yes
CN_i1 = CN_i1/BO_i1
CN_i1 = CN_i1 – BO_i1CNO_i1/A
i ← i + 1
Yes
Is i ≤ N1?
No
Set node count, i = 1
Is k = 1?
No
i1 = k – 1; j1 = k – 1
Yes
F
i1 = 0; j1 = 0
C_i1+11 = CN_i1
Set column count, j = 1
E

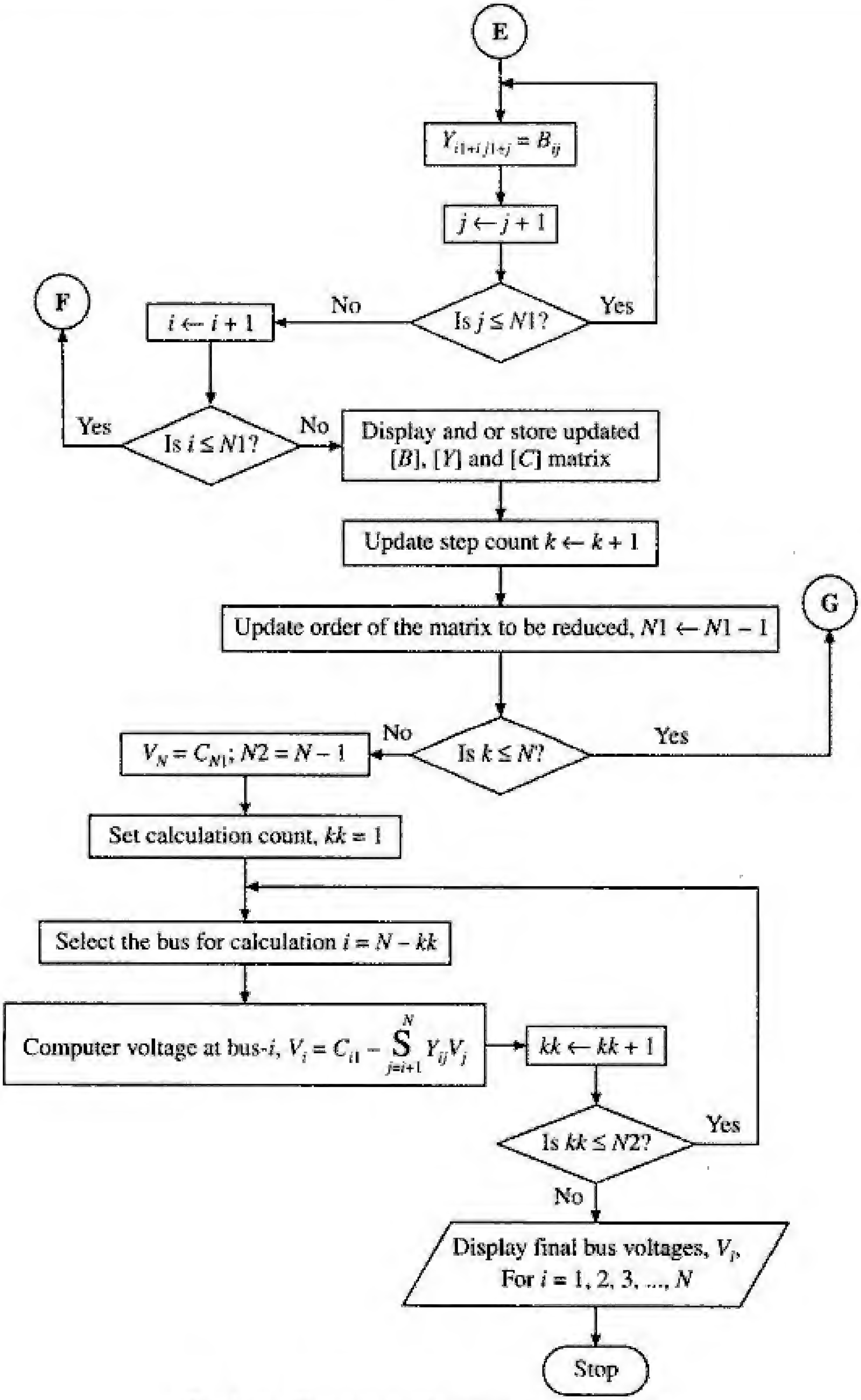


Fig. 1.11 Flowchart of Gauss elimination method.

Example 1.1: Use Gaussian elimination method to solve the nodal equations given below and find out final bus voltages of a three bus power system as shown in Fig. E1.1.

$$\begin{bmatrix} -j10.952 & j5 & j5 \\ j5 & -j10.952 & j5 \\ j5 & j5 & -j10 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1\angle -90^\circ \\ 0.857\angle -120^\circ \\ 0 \end{bmatrix}$$

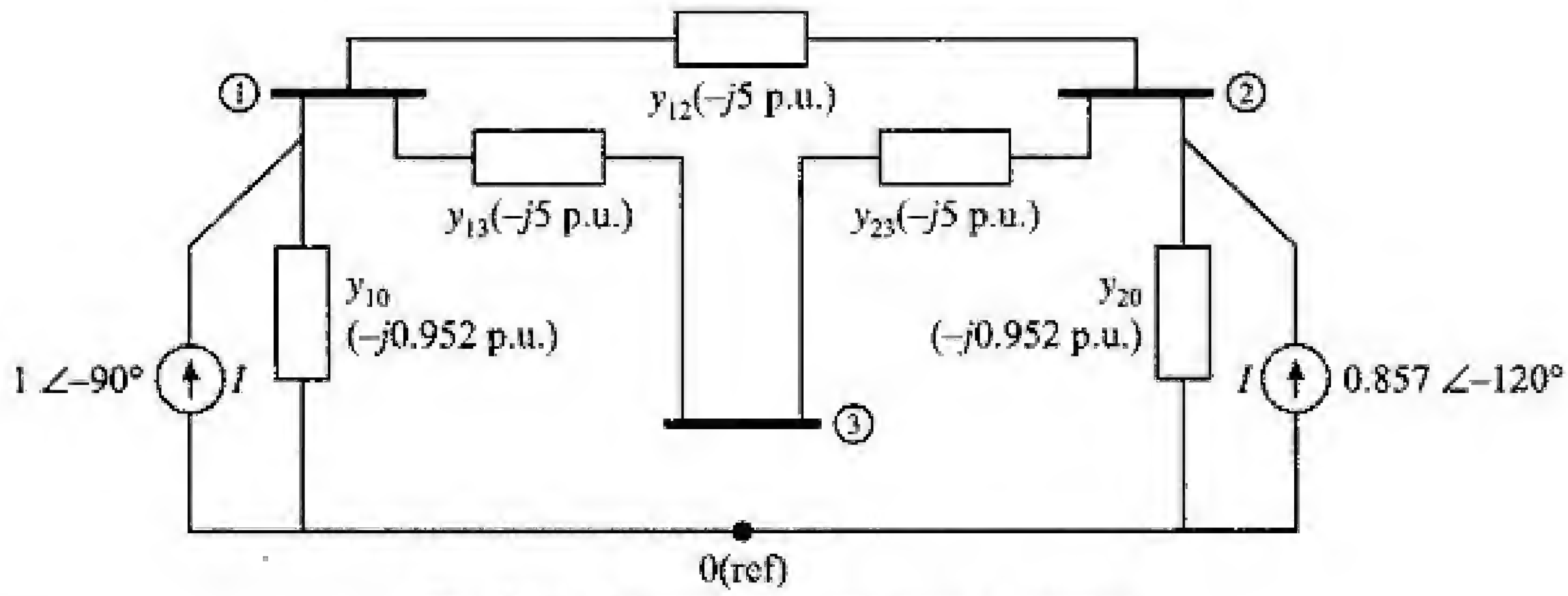


Fig. E1.1 A three bus power system network.

Solution: The three equations are:

$$-j10.952V_1 + j5V_2 + j5V_3 = -j \tag{1}$$

$$j5V_1 - j10.952V_2 + j5V_3 = -0.4285 - j0.742184 \tag{2}$$

$$j5V_1 + j5V_2 - j10V_3 = 0 \tag{3}$$

From equation (1)

$$V_1 - 0.456538V_2 - 0.456538V_3 = 0.091308 \tag{4}$$

Let us eliminate V_1 from the above equation (2)

$$Y'_{22} = -\frac{Y_{21}}{Y_{11}} \times Y_{12} + Y_{22} = -\frac{j5}{-j10.952} \times j5 - j10.952 = -j8.669312$$

$$Y'_{23} = -\frac{Y_{21}}{Y_{11}} \times Y_{13} + Y_{23} = -\frac{j5}{-j10.952} \times j5 + j5 = -j7.282688$$

and
$$I'_2 = -\frac{Y_{21}}{Y_{11}} \times I_1 + I_2$$

$$= -\frac{j5}{-j10.952} \times (-j) + (-0.4285 - j0.742184) = -0.4285 - j1.198722$$

∴
$$-j8.669312V_2 + j7.282688V_3 = -0.4285 - j0.285646 \tag{5}$$

or
$$V_2 - 0.840054V_3 = 0.138272 - j0.049427 \tag{6}$$

Let us eliminate V_1 from the equation (3)

$$Y'_{32} = -\frac{Y_{31}}{Y_{11}} \times Y_{12} + Y_{32} = -\frac{j5}{-j10.952} \times j5 + j5 = j7.282688$$

$$Y'_{33} = -\frac{Y_{31}}{Y_{11}} \times Y_{13} + Y_{33} = -\frac{j5}{-j10.952} \times j5 - j10 = -j7.717312$$

and $$I'_3 = -\frac{Y_{31}}{Y_{11}} \times I_1 + I_3 = -\frac{j5}{-j10.952} \times (-j) + 0 = -j0.456538$$

$\therefore$ $$j7.282688V_2 - j7.717312V_3 = j0.456538 \quad (7)$$

Let us now eliminate V_3 from equation (7).

where $$Y''_{33} = -\frac{Y'_{32}}{Y'_{22}} \times Y'_{23} + Y'_{33} = -\frac{j7.282688}{-j8.669312} \times j7.282688 - j7.717312$$

$$= -j1.599462$$

$$I''_3 = -\frac{Y'_{32}}{Y'_{22}} \times I'_2 + I'_3 = -\frac{j7.282688}{-j8.669312} \times (-0.4285 - j1.198722) - j0.456538$$

$$= (-0.359963 - j1.463528)$$

$\therefore$ $$-j1.599462V_3 = -0.359963 - j1.463528$$

or, $$V_3 = (0.915013 - j0.225053) \quad (8)$$

Therefore, equations (4), (6) and (8) are required to be solved to get V_1, V_2 and V_3.

From equation (6),

$$V_2 = 0.138272 - j0.049427 + 0.840054V_3 = (0.906932 - j0.238484)$$

From equation (4),

$$V_1 = 0.091308 + 0.456538V_2 + 0.456538V_3 = (0.923094 - j0.211622)$$

All the voltages, currents and admittances are expressed in p.u.; the angles are expressed in degree.

Execution of computer program to solve bus voltages by Gauss elimination method for Example 1.1

[Y_{Bus}] matrix of the system: NMYBUS1.DAT

```
3
(0.0, -10.952) Y11      (0.0, 5.0) Y12          (0.0, 5.0) Y13
(0.0, 5.0) Y21          (0.0, -10.952) Y22      (0.0, 5.0) Y23
(0.0, 5.0) Y31          (0.0,5.0) Y32           (0.0, -10.0) Y33
```

Bus currents: NMCUR1.DAT

```
(0.0, -1.0) C1
(-0.4285, -0.742184) C2
(0.0, 0.0) C3
```

Final bus voltages (Output of NMGAUSS.FOR): NMVOLT1.DAT

```
Step  =      1

REDUCED [Y] MATRIX USED FOR SUCCESSIVE ELIMINATION

[B]

B(   1,   1 ) = (    .000000, -10.952000 )
B(   1,   2 ) = (    .000000,   5.000000 )
```

```
B(  1,   3 ) = (    .000000,   5.000000 )
B(  2,   1 ) = (    .000000,   5.000000 )
B(  2,   2 ) = (    .000000,-10.952000 )
B(  2,   3 ) = (    .000000,   5.000000 )
B(  3,   1 ) = (    .000000,   5.000000 )
B(  3,   2 ) = (    .000000,   5.000000 )
B(  3,   3 ) = (    .000000, -10.000000 )
```

[BO]

```
BO(  1,   1 ) = (    .000000, -10.952000 )
BO(  1,   2 ) = (    .000000,   5.000000 )
BO(  1,   3 ) = (    .000000,   5.000000 )
BO(  2,   1 ) = (    .000000,   5.000000 )
BO(  2,   2 ) = (    .000000, -10.952000 )
BO(  2,   3 ) = (    .000000,   5.000000 )
BO(  3,   1 ) = (    .000000,   5.000000 )
BO(  3,   2 ) = (    .000000,   5.000000 )
BO(  3,   3 ) = (    .000000, -10.000000 )
```

REDUCED [C] MATRIX USED FOR SUCCESSIVE ELIMINATION

[CN]

```
CN(  1 ) = (    .000000, -1.000000 )
CN(  2 ) = (  -.428500, -.742184 )
CN(  3 ) = (    .000000, .000000 )
```

UPDATED [B] MATRIX USED FOR SUCCESSIVE ELIMINATION

```
B(  1,   1 ) = (  1.000000,    .000000 )
B(  1,   2 ) = (  -.456538,    .000000 )
B(  1,   3 ) = (  -.456538,    .000000 )
B(  2,   1 ) = (   .000000,    .000000 )
B(  2,   2 ) = (   .000000, -8.669312 )
B(  2,   3 ) = (   .000000,  7.282688 )
B(  3,   1 ) = (   .000000,    .000000 )
B(  3,   2 ) = (   .000000,  7.282688 )
B(  3,   3 ) = (   .000000, -7.717312 )
```

UPDATED [Y] MATRIX

```
Y(  1,   1 ) = (  1.000000,    .000000 )
Y(  1,   2 ) = (  -.456538,    .000000 )
Y(  1,   3 ) = (  -.456538,    .000000 )
Y(  2,   1 ) = (   .000000,    .000000 )
Y(  2,   2 ) = (   .000000, -8.669312 )
```

```
Y(  2,  3 ) = (   .000000,  7.282688 )
Y(  3,  1 ) = (   .000000,   .000000 )
Y(  3,  2 ) = (   .000000,  7.282688 )
Y(  3,  3 ) = (   .000000, -7.717312 )

UPDATED [C] MATRIX

C(  1 ) = (   .091308,    .000000 )
C(  2 ) = (  -.428500, -1.198722 )
C(  3 ) = (   .000000,   -.456538 )

Step  =     2

REDUCED [Y] MATRIX USED FOR SUCCESSIVE ELIMINATION

[B]

B(  1,  1 ) = (   .000000, -8.669312 )
B(  1,  2 ) = (   .000000,  7.282688 )
B(  2,  1 ) = (   .000000,  7.282688 )
B(  2,  2 ) = (   .000000, -7.717312 )

[BO]

BO(  1,  1 ) = (   .000000, -8.669312 )
BO(  1,  2 ) = (   .000000,  7.282688 )
BO(  2,  1 ) = (   .000000,  7.282688 )
BO(  2,  2 ) = (   .000000, -7.717312 )

REDUCED [C] MATRIX USED FOR SUCCESSIVE ELIMINATION

[CN]

CN(  1 ) = (  -.428500, -1.198722 )
CN(  2 ) = (   .000000,   -.456538 )

UPDATED [B] MATRIX USED FOR SUCCESSIVE ELIMINATION

B(  1,  1 ) = (  1.000000,    .000000 )
B(  1,  2 ) = (  -.840054,    .000000 )
B(  2,  1 ) = (   .000000,    .000000 )
B(  2,  2 ) = (   .000000, -1.599462 )

UPDATED [Y] MATRIX

Y(  1,  1 ) = (  1.000000,    .000000 )
```

```
Y(  1,  2 ) = (  -.456538,    .000000 )
Y(  1,  3 ) = (  -.456538,    .000000 )
Y(  2,  1 ) = (   .000000,    .000000 )
Y(  2,  2 ) = (  1.000000,    .000000 )
Y(  2,  3 ) = (  -.840054,    .000000 )
Y(  3,  1 ) = (   .000000,    .000000 )
Y(  3,  2 ) = (   .000000,    .000000 )
Y(  3,  3 ) = (   .000000, -1.599462 )

UPDATED [C] MATRIX

C(  1 ) = (   .091308,    .000000 )
C(  2 ) = (   .138272,   -.049427 )
C(  3 ) = (  -.359963, -1.463528 )

Step  =      3

REDUCED [Y] MATRIX USED FOR SUCCESSIVE ELIMINATION

[B]

B(  1,  1 ) = (   .000000, -1.599462 )

[BO]

BO(  1,  1 ) = (   .000000, -1.599462 )

REDUCED [C] MATRIX USED FOR SUCCESSIVE ELIMINATION

[CN]

CN(  1 ) = (  -.359963, -1.463528 )

UPDATED [B] MATRIX USED FOR SUCCESSIVE ELIMINATION

B(  1,  1 ) = (  1.000000,    .000000 )

UPDATED [Y] MATRIX

Y(  1,  1 ) = (  1.000000,    .000000 )
Y(  1,  2 ) = (  -.456538,    .000000 )
Y(  1,  3 ) = (  -.456538,    .000000 )
Y(  2,  1 ) = (   .000000,    .000000 )
Y(  2,  2 ) = (  1.000000,    .000000 )
Y(  2,  3 ) = (  -.840054,    .000000 )
Y(  3,  1 ) = (   .000000,    .000000 )
```

```
Y(  3,  2 ) = (   .000000,   .000000 )
Y(  3,  3 ) = (  1.000000,   .000000 )

UPDATED [C] MATRIX

C(  1 ) = (   .091308,   .000000 )
C(  2 ) = (   .138272,  -.049427 )
C(  3 ) = (   .915013,  -.225053 )

     Bus-code            VOLTAGE
        1       (   .923094,   -.211622) V1
        2       (   .906932,   -.238484) V2
        3       (   .915013,   -.225053) V3
```

Example 1.2: Solve the following simultaneous equation by gauss elimination method:

$$\overbrace{\begin{bmatrix} \begin{pmatrix}3.448276\\-j8.540689\end{pmatrix} & \begin{pmatrix}-1.724138\\+j4.310345\end{pmatrix} & \begin{pmatrix}-1.724138\\+j4.310345\end{pmatrix} \\ \begin{pmatrix}-1.724138\\+j4.310345\end{pmatrix} & \begin{pmatrix}3.448276\\-j8.540689\end{pmatrix} & \begin{pmatrix}-1.724138\\+j4.310345\end{pmatrix} \\ \begin{pmatrix}-1.724138\\+j4.310345\end{pmatrix} & \begin{pmatrix}-1.724138\\+j4.310345\end{pmatrix} & \begin{pmatrix}3.448276\\-j8.540689\end{pmatrix} \end{bmatrix}}^{[B]} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \overbrace{\begin{bmatrix} 0 \\ 1\angle 120^\circ \\ 0.5\angle -90^\circ \end{bmatrix}}^{[A]}$$

Execution of computer program to solve the given simultaneous equations by Gauss elimination method for Example 1.2.

Application of the given input data to the computer program of the Gauss elimination method reveals the following:

Final solution (Output of NMGAUSS.FOR): NMVOLT2.DAT

```
                    V
( -5.717692,   2.106743) V1
( -5.673174,   2.049917) V2
( -5.684209,   2.093266) V3
```

1.9.2 Kron's Method of Network Reduction

In power system studies it is often required to handle large systems, and for computational purposes it may be required to reduce the dimension of the network by eliminating certain buses.

Let $$[I_{Bus}] = [Y_{Bus}][V_{Bus}] \tag{1.17}$$

We will assume that the elements of $[I_{Bus}]$ and $[V_{Bus}]$ are ordered in such a way that buses to be retained would appear first and denoted by subscript R, while the buses to be eliminated are denoted by E.

$$\therefore \quad [I_{Bus}] = \begin{bmatrix} I_{Bus-E} \\ I_{Bus-R} \end{bmatrix}; [V_{Bus}] = \begin{bmatrix} V_{Bus-E} \\ V_{Bus-R} \end{bmatrix}$$

Since $[Y_{Bus}] = \begin{bmatrix} Y_{mm} & Y_{mn} \\ Y_{nm} & Y_{nn} \end{bmatrix}$; where bus numbers 1, 2, 3, ..., m are load buses and $(m + 1)$ to n are generator buses.

We can write,

$$\begin{bmatrix} I_m \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{mm} & Y_{mn} \\ Y_{nm} & Y_{nn} \end{bmatrix} \begin{bmatrix} V_m \\ E_n \end{bmatrix}$$

where $[I_m]$ and $[I_n]$ are load bus and generator bus currents and $[V_m]$ and $[E_n]$ are load bus and generator bus voltages.

If there are no sources at load buses,

$$[I_m] = [I_{Bus-E}] = 0$$

Thus we can write

$$0 = Y_{mm}V_m + Y_{mn}E_n \tag{1.18}$$

and

$$I_n = Y_{nm}V_m + Y_{nn}E_n \tag{1.19}$$

From equation (1.18), we have

$$V_m = -Y_{mm}^{-1}Y_{mn}E_n \tag{1.20}$$

Substituting into equation (1.19), we get

$$[I_n] = [I_{Bus-R}] = (Y_{nn} - Y_{mn}^T Y_{mm}^{-1} Y_{mn})E_n = Y_{Bus(reduced)}E_n$$

$$\therefore \qquad Y_{Bus(reduced)} = (Y_{nn} - Y_{mn}^T Y_{mm}^{-1} Y_{mn}) \tag{1.21}$$

In power systems, instead of taking inverse of Y_{mm}, we can perform the elimination bus by bus.

Let us suppose we have a bus admittance matrix*

$$[Y_{Bus}] = \begin{bmatrix} Y_{11} & \cdots & Y_{1k} & \cdots & Y_{1n} \\ Y_{2k} & \cdots & Y_{2k} & \cdots & Y_{2n} \\ \vdots & & \vdots & & \vdots \\ Y_{k1} & & Y_{kk} & & Y_{kn} \\ \vdots & & \vdots & & \vdots \\ Y_{n1} & \cdots & Y_{nk} & \cdots & Y_{nn} \end{bmatrix}$$

Our aim is to eliminate bus k; we will reorient $[Y_{Bus}]$ in such a way so that bus k elements take position in the last row and column.

$$\therefore \qquad [Y_{Bus}] = \left[\begin{array}{ccc|c} Y_{11} & \cdots & Y_{1n} & Y_{1k} \\ Y_{21} & \cdots & Y_{2n} & Y_{2k} \\ \vdots & & \vdots & \vdots \\ Y_{n1} & \cdots & Y_{nn} & Y_{nk} \\ \hline Y_{k1} & \cdots & Y_{kn} & Y_{kk} \end{array}\right]$$

After elimination of k-th row, i.e. k-th node, new elements would be obtained using equation (1.21).

$$Y_{11(new)} = Y_{11(old)} - \frac{Y_{1k(old)}Y_{k1(old)}}{Y_{kk}},$$

*Formation of bus admittance matrix has been discussed in Chapter 3.

$$Y_{12(new)} = Y_{12(old)} - \frac{Y_{1k(old)}Y_{k2(old)}}{Y_{kk}}$$

and so on.

In general we can write for any element in the reduced $[Y_{Bus}]$ matrix, using Kron's formula,

$$Y_{ij(new)} = Y_{ij(old)} - \frac{Y_{ik(old)}Y_{kj(old)}}{Y_{kk}} \tag{1.22}$$

where node k is to be eliminated.

The flowchart of the Kron's method of node reduction is shown in Fig. 1.12.

Example 1.3: Using Kron's method of node elimination, eliminate node 3 and corresponding voltage V_3 from nodal equation matrix of three bus system given in Example 1.1. Draw the reduced network.

Solution: We are required to eliminate node-3. Using Kron's, we can write

$$Y_{11(new)} = Y_{11(old)} - \frac{Y_{13(old)}Y_{31(old)}}{Y_{33}} = -j10.952 - \frac{j5 \times j5}{-j10} = -j8.452$$

$$Y_{12(new)} = Y_{12(old)} - \frac{Y_{13(old)}Y_{32(old)}}{Y_{33}} = j5 - \frac{j5 \times j5}{-j10} = j7.5 = Y_{21(new)}$$

$$Y_{22(new)} = Y_{22(old)} - \frac{Y_{23(old)}Y_{32(old)}}{Y_{33}} = -j10.952 - \frac{j5 \times j5}{-j10} = -j8.452$$

Execution of computer program of Kron's method of node elimination for Example 1.3

$[Y_{Bus}]$ matrix of the system: NMYBUS1.DAT

Given in Example 1.1.

Bus currents: NMCUR1.DAT

Given in Example 1.1.

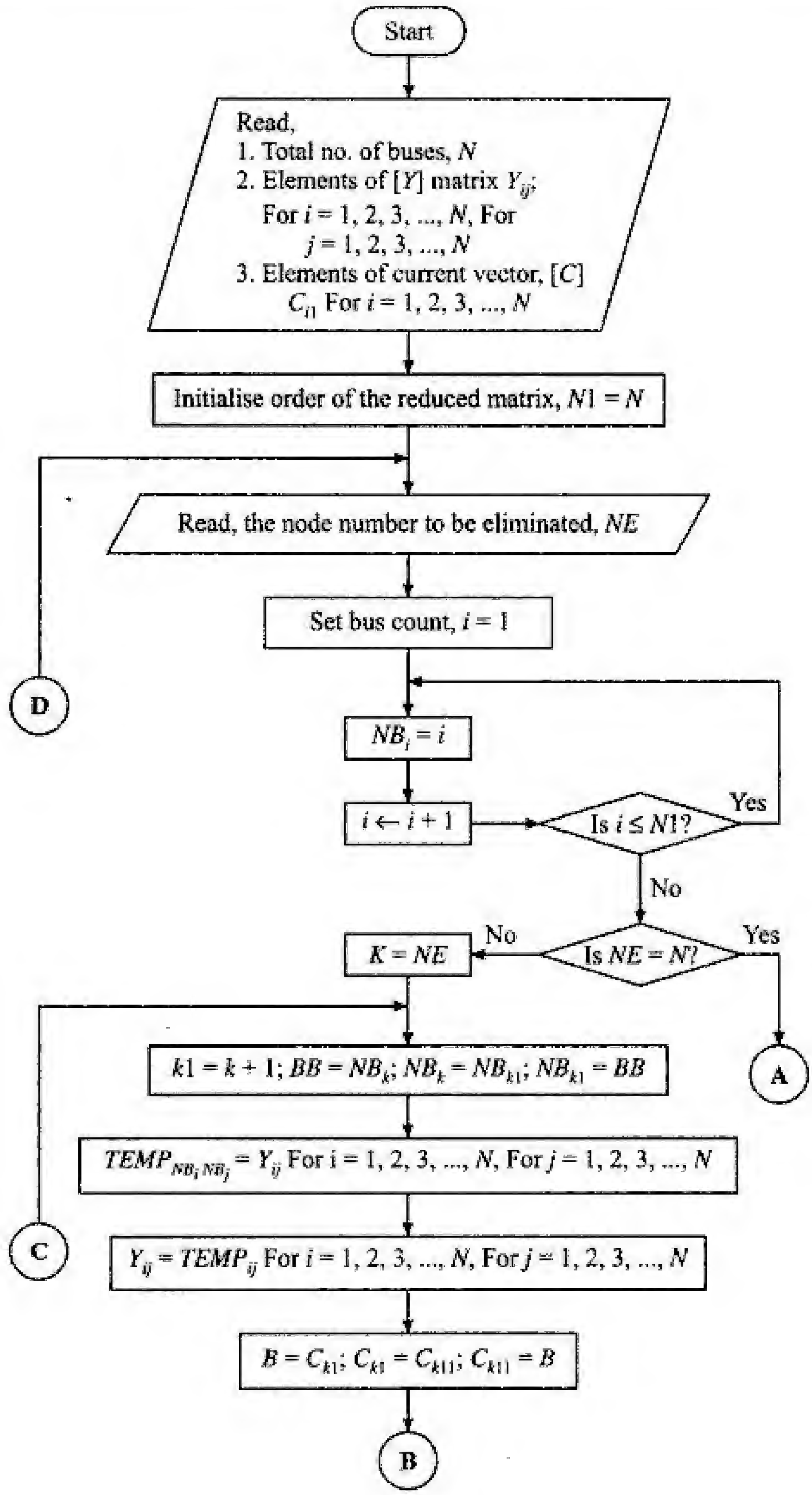
Final $[Y_{Bus}]$ and bus currents matrices (Output of NMKRON.FOR): NMKRON1.DAT

```
UPDATED [Y] MATRIX

Y(  1,   1 ) = (   .000000, -8.452000 ) Y11
Y(  1,   2 ) = (   .000000,  7.500000 ) Y12
Y(  2,   1 ) = (   .000000,  7.500000 ) Y21
Y(  2,   2 ) = (   .000000, -8.452000 ) Y22

UPDATED [C] VECTOR

C(  1 ) = (   .000000, -1.000000 ) C1
C(  2 ) = (  -.428500, -.742184 ) C2
```


Start
Read,
1. Total no. of buses, N
2. Elements of $[Y]$ matrix Y_{ij};
For $i = 1, 2, 3, ..., N$, For
$j = 1, 2, 3, ..., N$
3. Elements of current vector, $[C]$
C_{i1} For $i = 1, 2, 3, ..., N$
Initialise order of the reduced matrix, $N1 = N$
Read, the node number to be eliminated, NE
Set bus count, $i = 1$
D
$NB_i = i$
$i \leftarrow i + 1$
Is $i \leq N1$?
Yes
No
Is $NE = N$?
No
Yes
$K = NE$
A
$k1 = k + 1$; $BB = NB_k$; $NB_k = NB_{k1}$; $NB_{k1} = BB$
$TEMP_{NB_i NB_j} = Y_{ij}$ For i = 1, 2, 3, ..., N, For $j = 1, 2, 3, ..., N$
C
$Y_{ij} = TEMP_{ij}$ For $i = 1, 2, 3, ..., N$, For $j = 1, 2, 3, ..., N$
$B = C_{k1}$; $C_{k1} = C_{k11}$; $C_{k11} = B$
B

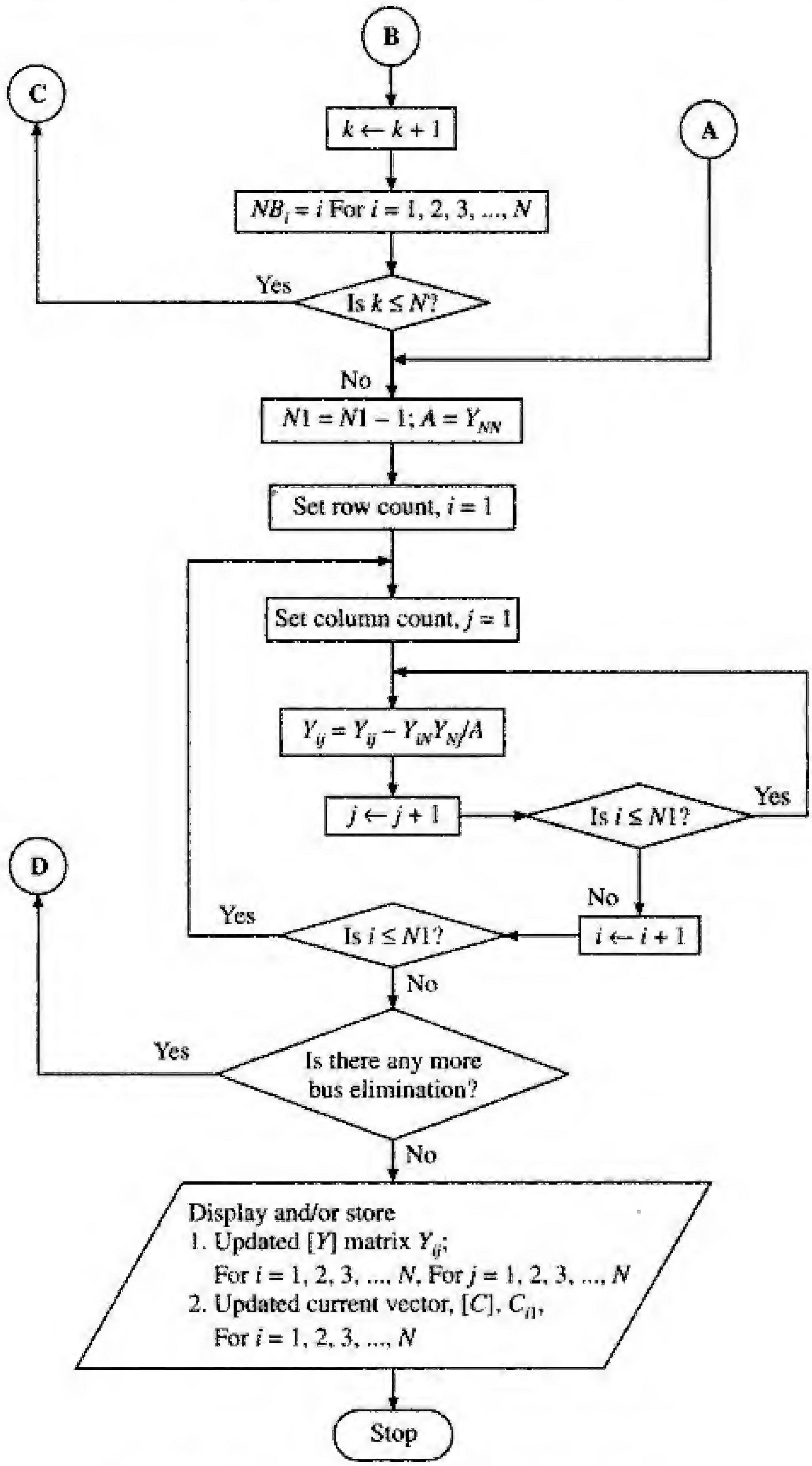


Fig. 1.12 Flowchart of Kron's method of node elimination.

The reduced network is shown in Fig. E1.2.

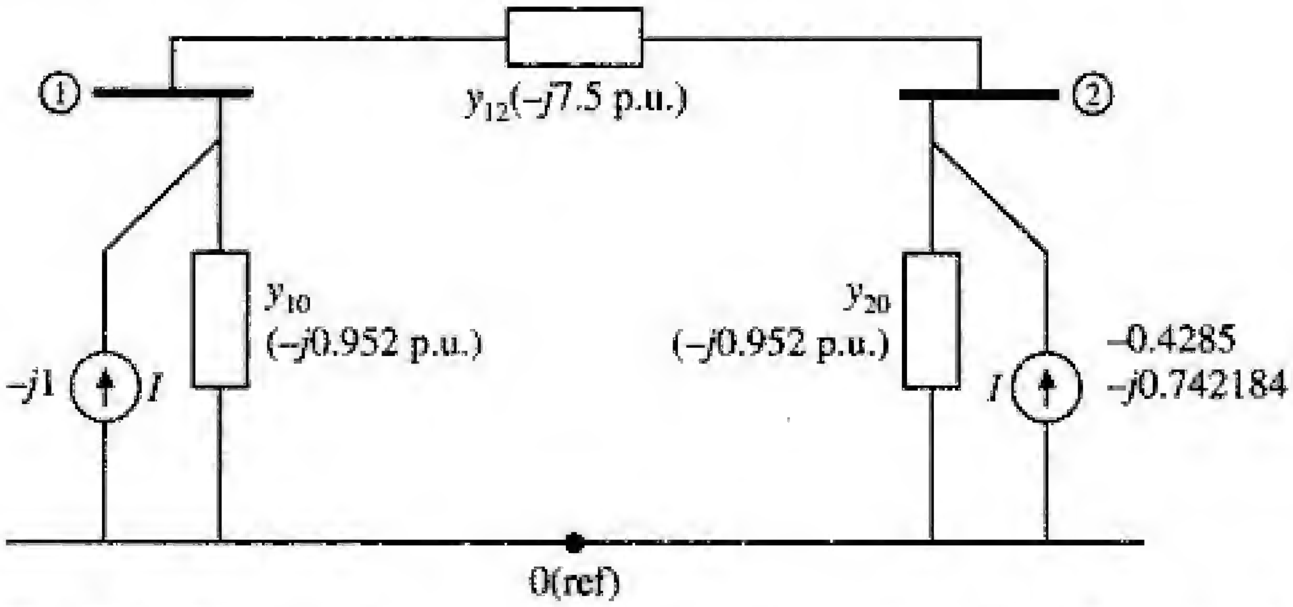


Fig. E1.2 Reduced two bus system after elimination of node ③ of Example 1.3.

Example 1.4: Eliminate V_1 of Example 1.2 using Kron's method of node elimination.

Solution: Execution of computer program of Kron's method of node elimination for Example 1.4

Final [*B*] and bus currents matrices (Output of NMKRON.FOR): NMKRON2.DAT

```
UPDATED [B] MATRIX

B(  1,   1 ) = (  2.586272, -6.365355 )
B(  1,   2 ) = ( -2.586142,  6.485679 )
B(  2,   1 ) = ( -2.586142,  6.485679 )
B(  2,   2 ) = (  2.586272, -6.365355 )

UPDATED [B] VECTOR

B(  1 ) = (  -.500000,  -.866025 )
B(  2 ) = (   .000000,  -.500000 )
```

1.9.3 Solution of Differential Equation

Euler's method

Euler's method is a method to solve first order differential equation. Let us consider the differential equation as

$$\frac{dx}{dt} = f(x, t) \tag{1.23}$$

Figure 1.13 illustrates the geometrical method of solution by Euler's method.

At $x = x_0$, $t = t_0$, it can be assumed that the curve representing the true solution by its tangent having a slope

$$\left.\frac{dx}{dt}\right|_{x=x_0} = f(x_0, t_0) \tag{1.24}$$

$$\therefore \qquad \Delta x = \left.\frac{dx}{dt}\right|_{x=x_0} \Delta t \tag{1.25}$$

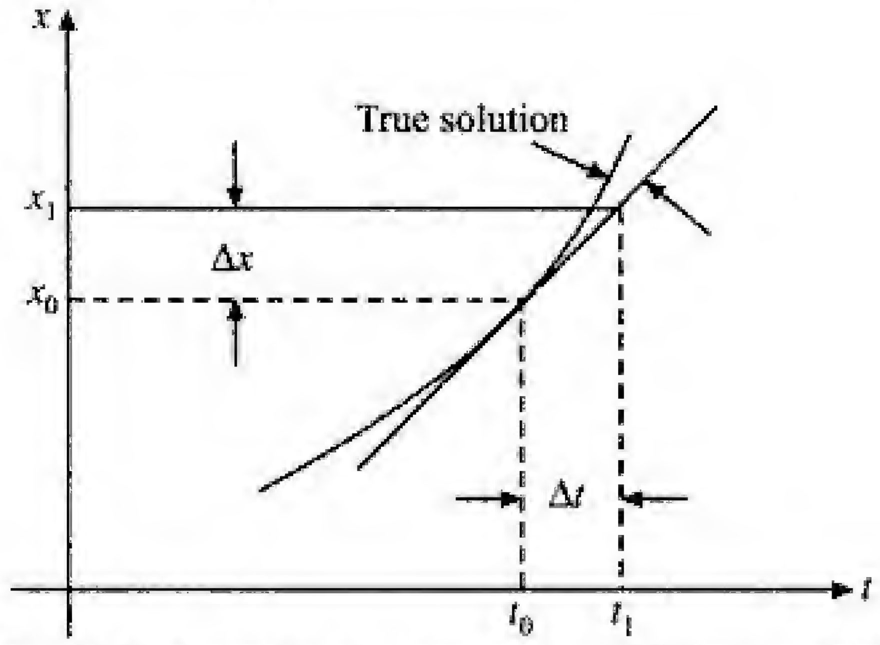


Fig. 1.13 Geometrical representation of Euler's method.

The value of x at $t = t_1 = t_0 + \Delta t$ is given by

$$x_1 = x_0 + \Delta x = x_0 + \left.\frac{dx}{dt}\right|_{x=x_0} \Delta t \tag{1.26}$$

The Euler's method is equivalent to using the first two terms of Taylor series expansion for x around point (x_0, t_0) as follows:

$$x_1 = x_0 + \Delta t(\dot{x}_0) + \frac{\Delta t^2}{2!}(\ddot{x}_0) + \frac{\Delta t^3}{3!}(\dddot{x}_0) + \cdots \tag{1.27}$$

After using the Euler's method to determine $x(=x_1)$ at $t = t_1$, we can take another short step Δt and determine $x(=x_1)$ at $t = t_2 = t_0 + 2\Delta t$ as follows:

$$x_2 = x_1 + \left.\frac{dx}{dt}\right|_{x=x_1} \Delta t \tag{1.28}$$

By applying this technique successively, values of x can be computed for different values of t.

Only the first derivative of x is used in this method, therefore it is called *first order* method. For more accuracy, Δt is required to be very small.

Example 1.5: Given $\frac{dy}{dx} = 2x^2 + 5xy + 3y$ with $x_0 = 0$; $y_0 = 1$

Using the Euler's method, find the solution correct upto six decimal points using step size, $\Delta x = 0.1$ and upto $x = 0.5$.

Solution: Given $\frac{dy}{dx} = f(x, y) = 2x^2 + 5xy + 3y$ and $x_0 = 0$; $y_0 = 1$, $\Delta x = 0.1$

Let us use equation (1.26) for the solution.

Now $$\left.\frac{dy}{dx}\right|_{y=y_0} = f(x_0, y_0) = f(0, 1) = (2x_0^2 + 5x_0y_0 + 3y_0)$$

$$= (2 \times 0 + 5 \times 0 \times 1 + 3 \times 1) = 3$$

$\therefore$ $$y_1 = y_0 + \Delta y = y_0 + \left(\left.\frac{dy}{dx}\right|_{y=y_0}\right)\Delta x = 1 + 3 \times 0.1 = 1.3$$

$$x_1 = x_0 + \Delta x = 0.1$$

Now $\left.\dfrac{dy}{dx}\right|_{y=y_0} = f(x_0, y_0) = f(0.1) = (2x_0^2 + 5x_0y_0 + 3y_0)$

$$= (2 \times 0 + 5 \times 0 \times 1 + 3 \times 1) = 3$$

$\therefore$ $$y_1^p = y_0 + \left.\frac{dy}{dx}\right|_{y=y_0} \Delta x = 1 + 3 \times 0.1 = 1.3$$

$$\left.\frac{dy}{dx}\right|_{y=y_1^p} = f(x_0, y_1^p) = f(0, 1.3) = 3.9$$

$$y_1^c = y_0 + \frac{1}{2}\left(\left.\frac{dy}{dx}\right|_{y=y_0} + \left.\frac{dy}{dx}\right|_{y=y_1^p}\right)\Delta x = 1 + \frac{1}{2}(3 + 3.9) \times 0.1 = 1.345$$

$\therefore$ $$y_1 = y_1^c = 1.345$$

$$x_1 = x_0 + \Delta x = 0.1$$

Similarly, to get y_2,

$$\left.\frac{dy}{dx}\right|_{y=y_1} = f(x_1, y_1) = f(0.1, 1.345) = 4.7275$$

$\therefore$ $$y_2^p = y_1 + \left.\frac{dy}{dx}\right|_{y=y_1} \Delta x = 1.345 + 4.7275 \times 0.1 = 1.81775$$

$$\left.\frac{dy}{dx}\right|_{y=y_2^p} = f(x_1, y_2^p) = f(0.1, 1.81775) = 6.382125$$

$$y_2^c = y_1 + \frac{1}{2}\left(\left.\frac{dy}{dx}\right|_{y=y_1} + \left.\frac{dy}{dx}\right|_{y=y_2^p}\right)\Delta x = 1.345 + \frac{1}{2}(4.7275 + 1.81775) \times 0.1$$

$$= 1.900481$$

$\therefore$ $$y_2 = y_2^c = 1.900481$$

In a similar way, the other solutions can be obtained.

Execution of computer program of Modified Euler's method for Example 1.6

Final solution (Output of EULER.FOR): MEULERS.DAT

x	y
1.000000E-01	1.345000
2.000000E-01	1.900481
3.000000E-01	2.822312
4.000000E-01	4.400162
5.000000E-01	7.190264

Runge–Kutta method

The Runge–Kutta (R–K) method approximates the Taylor series solution. Only the first order derivatives are used in this method. The effect of higher order derivatives is included by considering the first order derivatives several times. There are R–K methods of different orders.

Second order Runge–Kutta method: Consider the differential equation given in equation (1.23). The second order R–K formula for $x(=x_1)$ at $t = t_1$ is

$$x_1 = x_0 + \Delta x = x_0 + \frac{1}{2}(d_1 + d_2) \tag{1.30}$$

where, $d_1 = f(x_0, t_0)\Delta t,\ d_2 = f(x_0 + d_1, t_0 + \Delta t)\Delta t$

This method is equivalent to using first and second derivative terms of Taylor series. The error is in order of Δt^3.

In general, the value of x for $(n + 1)^{th}$ step is given by

$$x_{n+1} = x_n + \frac{1}{2}(d_1 + d_2) \tag{1.31}$$

where, $d_1 = f(x_n, t_n)\Delta t,\ d_2 = f(x_n + d_1, t_n + \Delta t)\Delta t$

Fourth order Runge-Kutta method: The general formula for the value of x for $(n + 1)^{th}$ step is given by

$$x_{n+1} = x_n + \frac{1}{6}(d_1 + 2d_2 + 2d_3 + d_4) \tag{1.32}$$

where, $d_1 = f(x_n, t_n)\Delta t,\ d_2 = f\left(x_n + \frac{d_1}{2}, t_n + \frac{\Delta t}{2}\right)\Delta t$

$$d_3 = f\left(x_n + \frac{d_2}{2}, t_n + \frac{\Delta t}{2}\right)\Delta t,\ d_4 = f(x_n + d_3, t_n + \Delta t)\Delta t$$

The physical interpretation of the above solution is as follows:

d_1 = slope at the beginning of time step $t = \Delta t$,

d_2 = first approximation to slope at midstep of $t = \Delta t$,

d_1 = second approximation to slope at midstep of $t = \Delta t$,

d_1 = slope at the end of time step $t = \Delta t$,

$$\Delta x = \frac{1}{6}(d_1 + 2d_2 + 2d_3 + d_4) \tag{1.33}$$

Thus, Δx is the incremental value of x given by the weighted average of estimates based on slopes at the beginning, midpoint and at the end of time step. This method is equivalent to considering up to fourth derivative terms in the Taylor series expansion and it has an error in order of Δt^5.

Equation (1.32) may also be written as

$$x_{n+1} = x_n + \frac{\Delta t}{6}(d_1 + 2d_2 + 2d_3 + d_4) \tag{1.34}$$

where, $d_1 = f(x_n, t_n),\ d_2 = f\left(x_n + \frac{\Delta t}{2}d_1, t_n + \frac{\Delta t}{2}\right)$

$$d_3 = f\left(x_n + \frac{\Delta t}{2}d_2, t_n + \frac{\Delta t}{2}\right),\ d_4 = f(x_n + d_3\Delta t, t_n + \Delta t)$$

The flowchart is given in Fig. 1.14.

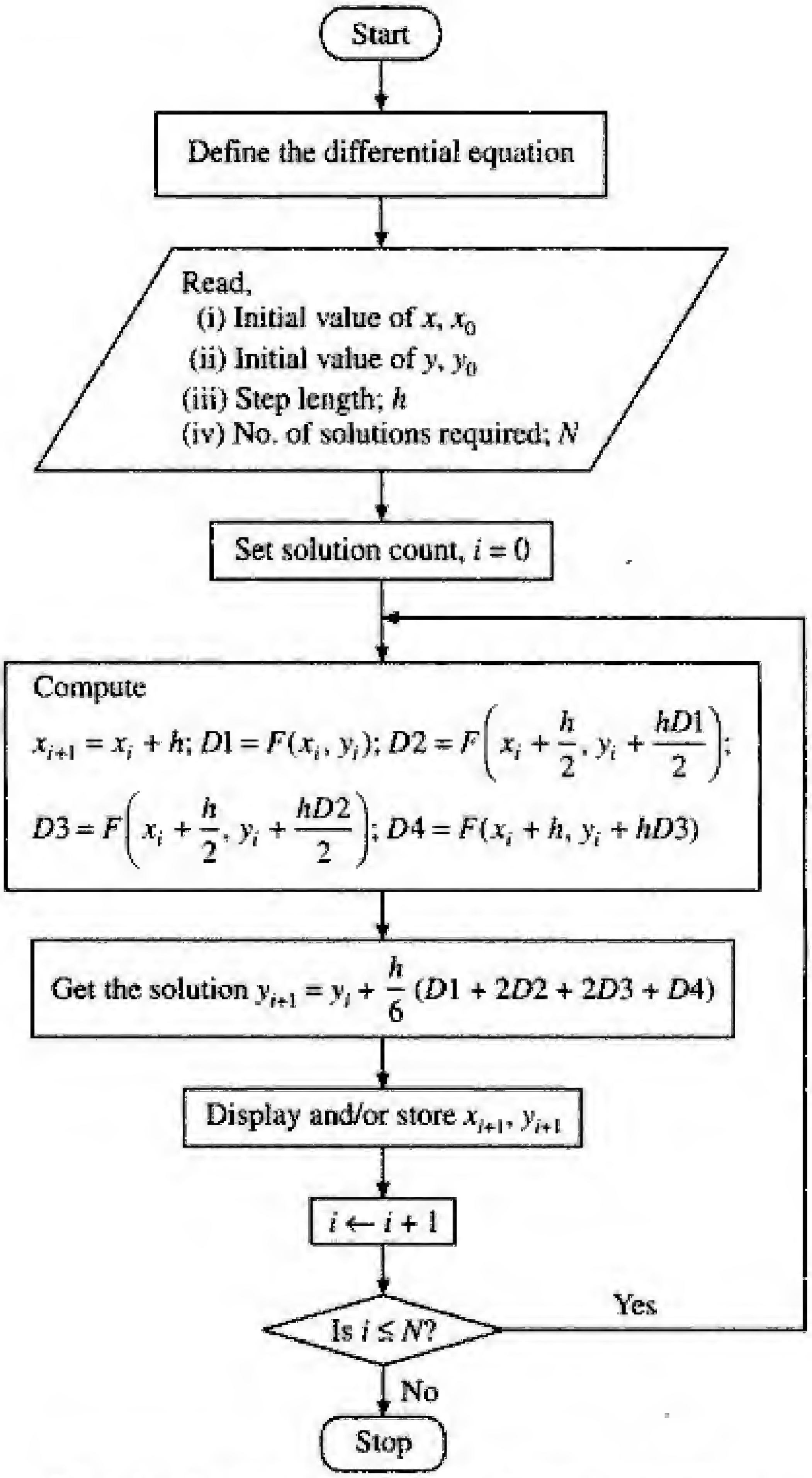


Fig. 1.14 Flowchart of fourth order Runge–Kutta method.

Example 1.7: Given $\frac{dy}{dx} = 2x^2 + 5xy + 3y$ with $x_0 = 0$; $y_0 = 1$

Using Runge–Kutta method, find the solution correct upto six decimal points using step size, $h = 0.1$ and upto $x = 0.5$.

Solution: Given $\quad \frac{dy}{dx} = f(x, y) = 2x^2 + 5xy + 3y$

and $\quad x_0 = 0$; $y_0 = 1$, $\Delta x = 0.1$

Let us rewrite equation (1.34) for solution.

$$x_{n+1} = x_n + \frac{\Delta t}{6}(d_1 + 2d_2 + 2d_3 + d_4)$$

where, $d_1 = f(x_n, t_n)$, $d_2 = f\left(x_n + \frac{\Delta t}{2}d_1, t_n + \frac{\Delta t}{2}\right)$

$$d_3 = f\left(x_n + \frac{\Delta t}{2}d_2, t_n + \frac{\Delta t}{2}\right), d_4 = f(x_n + d_3\Delta t, t_n + \Delta t)$$

Here, $d_1 = f(y_0, x_0) = f(1, 0) = (2x_0^2 + 5x_0y_0 + 3y_0)$

$$= (2 \times 0 + 5 \times 0 \times 1 + 3 \times 1) = 3$$

$$d_2 = f\left(y_0 + \frac{\Delta x}{2}d_1, x_0 + \frac{\Delta x}{2}\right) = f(1.15, 0.05) = 3.7425$$

$$d_3 = f\left(y_0 + \frac{\Delta x}{2}d_2, x_0 + \frac{\Delta x}{2}\right) = f(1.187125, 0.05) = 3.863156$$

$$d_4 = f(y_0 + d_3\Delta x, x_0 + \Delta x) = f(1.386316, 0.1) = 4.872105$$

$$\therefore \quad y_1 = y_0 + \frac{\Delta x}{6}(d_1 + 2d_2 + 2d_3 + d_4) = 1 + 0.384724 = 1.384724$$

$$x_1 = x_0 + \Delta x = 0.1$$

Similarly to get y_2,

$$d_1 = f(y_1, x_1) = f(1.384724, 0.1) = 4.866533$$

$$d_2 = f\left(y_1 + \frac{\Delta x}{2}d_1, x_1 + \frac{\Delta x}{2}\right) = f(1.628051, 0.15) = 6.150189$$

$$d_3 = f\left(y_1 + \frac{\Delta x}{2}d_2, x_1 + \frac{\Delta x}{2}\right) = f(1.692233, 0.15) = 6.390874$$

$$d_4 = f(y_0 + d_3\Delta x, x_0 + \Delta x) = f(2.023812, 0.2) = 8.175244$$

$$\therefore \quad y_2 = y_1 + \frac{\Delta x}{6}(d_1 + 2d_2 + 2d_3 + d_4) = 1.384724 + 0.635398 = 2.020122$$

In a similar way, the other solutions can be obtained.

Execution of computer program of Runge–Kutta method for Example 1.7

Final solution (Output of NMRK.FOR): NMRKOUT1.DAT

x	y
1.000000E-01	1.384724
2.000000E-01	2.020122
3.000000E-01	3.105102
4.000000E-01	5.023332
5.000000E-01	8.542507

Example 1.8: Given $\frac{dy}{dx} = (x + 1)(y + 1)$ with $x_0 = 1$; $y_0 = 5$

Using Runge–Kutta method, find the solution correct upto six decimal points using step size, h = 0.05 and upto $x = 1.5$.

Solution: Execution of computer program of Runge–Kutta method for Example 1.8

Final solution (Output of NMRK.FOR): NMRKOUT2.DAT

x	y
1.050000	5.639319
1.100000	6.365149
1.150000	7.190780
1.200000	8.131765
1.250000	9.206338
1.300000	10.435910
1.350000	11.845690
1.400000	13.465380
1.450000	15.330070
1.500000	17.481270

1.9.4 Eigenvalues and its Properties

Eigenvalues

The eigenvalues of a matrix are given by the values of the scalar parameter λ for which there exist non-trivial solutions (i.e. other than $X = 0$) to the equation

$$AX = \lambda X \tag{1.35}$$

where A is a $n \times n$ matrix (real for a physical system such as a power system)

X is a $n \times 1$ vector.

To find eigenvalues, equation (1.35) may be written as

$$(A - \lambda I)X = 0 \tag{1.36}$$

For a non-trivial solution

$$det(A - \lambda I)X = 0 \tag{1.37}$$

The expansion of the determinant gives the *characteristic equation*, and the n number of solutions of equation (1.37), i.e. $\lambda = \lambda_1, \lambda_2, \lambda_3, ..., \lambda_n$ are the eigenvalues of A. The eigenvalues may be real or complex. If A is real, complex eigenvalues always occur in conjugate pairs. Similar matrices have similar eigenvalues. A matrix and its transpose have same eigenvalues.

Example 1.9: Compute eigenvalues of matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

Solution: $$det(A - \lambda I)X = 0$$

or, $$det(A - \lambda I)X = 0$$

or, $$\left| \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

or, $$\begin{vmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix} = 0$$

or, $$1 - 2\lambda + \lambda^2 - 4 = 0$$

or, $$\lambda^2 - 2\lambda - 3 = 0$$

or, $$(\lambda - 3)(\lambda + 1) = 0$$

$\therefore$ $$\lambda = 3, -1$$

Eigenvectors

For any eigenvalue λ_i, the n column vector ξ_i which satisfies equation (1.35) is called *right eigenvector* of A associated with the eigenvalue λ_i. Therefore, we have

$$A\xi_i = \lambda_i \xi_i \quad \text{for} \quad i = 1, 2, 3, \ldots, n \tag{1.38}$$

where
$$\xi_i = \begin{bmatrix} \xi_{1i} \\ \xi_{2i} \\ \vdots \\ \xi_{ni} \end{bmatrix} \tag{1.39}$$

Equation (1.36) is homogeneous, $k\xi_i$ (where k is a scalar) is also a solution. Thus, eigenvectors are determined within a scalar multiplier. Similarly, the n row vector η_i which satisfies

$$\eta_i A = \lambda_i \eta_i \quad \text{for } i = 1, 2, 3, \ldots, n \tag{1.40}$$

is called *left eigenvector* associated with eigenvalue λ_i.
The left and right eigenvector of two different eigenvalues are orthogonal, i.e. if $\lambda_i \neq \lambda_j$ then

$$\eta_j \xi_i = 0 \tag{1.41}$$

In case the eigenvector corresponds to the same eigenvalue,

$$\eta_i \xi_i = C_i \tag{1.42}$$

where C_i is a non-zero constant.

As stated above, eigenvectors are computed within scalar multiplier. It is a common practice to normalize these vectors so that

$$\eta_i \xi_i = 1 \tag{1.43}$$

Modal matrices

To demonstrate the eigenproperties of matrix A, let us define the following matrices first:

$$\xi = [\xi_1\ \xi_2 \cdots \xi_n],$$
$$\eta = [\eta_1\ \eta_2 \cdots \eta_n],$$

λ = diagonal matrix, with eigenvalues $\lambda_1\ \lambda_2 \ldots \lambda_n$ as diagonal elements. All the matrices are $n \times n$ matrices.
In terms of the above matrices, equations (1.38) and (1.40) can be written as

$$A\xi = \xi\lambda \tag{1.44}$$

$$\eta\xi = I \tag{1.45}$$

and
$$\eta = \xi^{-1} \tag{1.46}$$

From equation (1.44)

$$\xi^{-1} A \xi = \lambda \tag{1.47}$$

Eigenvalue and stability

The stability of a system can be determined by eigenvalues as follows:

1. A real eigenvalue corresponds to non-oscillatory mode. A negative real eigenvalue indicates the decaying mode. The larger its magnitude, the faster the decay. A positive real eigenvalue represents *aperiodic instability*.
2. Complex eigenvalues occur in conjugate pairs and each pair corresponds to an oscillatory mode.

The real part of the eigenvalue gives the damping and the imaginary part frequency of oscillation. A negative real part represents a damped oscillation whereas a positive real part represents oscillation of increasing magnitude.

Example 1.10: Comment on the stability for systems having following state matrix.

System-1:
$$[A] = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$$

System-2:
$$[A] = \begin{bmatrix} -0.4 & -0.25 & 0 & 0.12 \\ 0 & 0 & 314 & 0 \\ -0.17 & -0.15 & 0 & 0 \\ -710 & -10 & 0 & -5 \end{bmatrix}$$

System-3:
$$[A] = \begin{bmatrix} -0.4 & -0.25 & 0 & 0.12 \\ 0 & 0 & 314 & 0 \\ -0.17 & -0.15 & 0 & 0 \\ -710 & 10 & 0 & -5 \end{bmatrix}$$

Solution:

System 1: Eigenvalues are $\lambda_{1,2} = \pm j1.7321$

So, the eigenvalues have no real part. Hence, the system is oscillatory but stable.

System 2: Eigenvalues are $\lambda_{1,2} = -2.6794 \pm j9.0895$

and $\lambda_{3,4} = -0.0206 \pm j6.6544$

So, the eigenvalues have negative real part. Hence, the system is stable with damped oscillation.

System 3: Eigenvalues are $\lambda_{1,2} = -2.7846 \pm j9.0280$

and $\lambda_{3,4} = 0.0846 \pm j6.7806$

So, one pair of eigenvalues have positive real part. Hence, the system is unstable with increasing oscillation.

Concept of eigenvalue sensitivity

Let us determine the sensitivity of eigenvalues to the element of state matrix A. From equation (1.38), we know

$$A\xi_i = \lambda_i \xi_i$$

Differentiation of the above equation with respect to any kj-th element of matrix A (i.e. a_{kj}) yields,

$$\frac{dA}{da_{kj}}\xi_i + A\frac{d\xi_i}{da_{kj}} = \frac{d\lambda_i}{da_{kj}}\xi_i + \lambda_i\frac{d\xi_i}{da_{kj}}$$

Premultiplying both sides by η_i we get

$$\eta_i\frac{\partial A}{\partial a_{kj}}\xi_i + \eta_i A\frac{\partial \xi_i}{\partial a_{kj}} = \eta_i\frac{\partial \lambda_i}{\partial a_{kj}}\xi_i + \eta_i\lambda_i\frac{\partial \xi_i}{\partial a_{kj}}$$

or,
$$\eta_i\frac{\partial A}{\partial a_{kj}}\xi_i + \eta_i\frac{\partial \xi_i}{\partial a_{kj}}(A - \lambda_i I) = \eta_i\frac{\partial \lambda_i}{\partial a_{kj}}\xi_i$$

or, $$\eta_i \frac{\partial A}{\partial a_{kj}} \xi_i = \frac{\partial \lambda_i}{\partial a_{kj}} \qquad [\because \quad (A - \lambda_i I) = 0 \text{ and } \eta_i \xi_i = 1] \tag{1.48}$$

All the elements of $\frac{\partial A}{\partial a_{kj}}$ are zero, except for the element in the k-th row and j-th column which is equal to one. Therefore,

$$\frac{\partial \lambda_i}{\partial a_{kj}} = \eta_{ik} \xi_{ji} \tag{1.49}$$

Thus, the sensitivity of the eigenvalue λ_i to the element a_{kj} of the state matrix is equal to the product of the left eigenvector element η_{ik} and the right eigenvector element ξ_{ji}.

Participation factor

The problem in using the left and right eigenvectors individually for identifying the relationship between states and modes is, the elements of eigenvectors are dependent on units and scaling associated with state variables. Therefore, a matrix called *participation matrix* is developed, which combines left and right eigenvectors as follows:

$$P = [P_1 \; P_2 \; P_3 \; \dots \; P_n] \tag{1.50}$$

where $$P_i = \begin{bmatrix} P_{1i} \\ P_{2i} \\ \vdots \\ P_{ni} \end{bmatrix} = \begin{bmatrix} \xi_{1i}\eta_{i1} \\ \xi_{2i}\eta_{i2} \\ \\ \xi_{ni}\eta_{in} \end{bmatrix} \tag{1.51}$$

Here, ξ_{ki} = the element of k-th row and i-th column of modal matrix ξ
= k-th entry of right eigenvector ξ_i
η_{ik} = the element of i-th row and k-th column of modal matrix η
= k-th entry of left eigenvector η_i

The element $P_{ki} = \xi_{nk}\eta_{ik}$ is termed *participation factor*. It is a measure of relative participation of the k-th state variable in i-th mode and vice-versa. The effect of multiplying left and right eigenvectors is, P_{ki} is dimensionless.

In view of eigenvector normalization [equation (1.43)], the sum of participation factors associated with any mode $\left(\sum_{i=1}^{n} P_{ki}\right)$ or with any state variable $\left(\sum_{k=1}^{n} P_{ki}\right)$ is equal to one. From equation (1.49), it can be seen that the participation factor P_{ki} is actually equal to the sensitivity of the eigenvalue λ_i to the diagonal element a_{kk} of the state matrix A.

$$P_{ki} = \frac{\partial \lambda_i}{\partial a_{kk}} \tag{1.52}$$

The participation factor is generally an indicator of relative participations of the respective states in the corresponding modes.

1.9.5 Triangular Factorization

Triangular factorization is a well-known numerical method to solve a set of simultaneous algebraic equations. This method reduces a set of equations to two equivalent upper triangular and lower triangular

matrices, which can be solved by backward and forward substitution. Although this method is applicable to N no. of equations, here it is illustrated for three simultaneous equations (1.11a)–(1.11c) as follows:

$$Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 = I_1 \tag{1.11a}$$

$$Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 = I_2 \tag{1.11b}$$

$$Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 = I_3 \tag{1.11c}$$

In a matrix form

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \tag{1.12}$$

or,

$$\begin{bmatrix} \ell_{11} & 0 & 0 \\ \ell_{21} & \ell_{22} & 0 \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \tag{1.53}$$

where $\begin{bmatrix} \ell_{11} & 0 & 0 \\ \ell_{21} & \ell_{22} & 0 \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix}$ is a lower triangular matrix and $\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$ is an upper triangular matrix.

Now, equation (1.53) can be written as

$$\begin{bmatrix} \ell_{11} & 0 & 0 \\ \ell_{21} & \ell_{22} & 0 \\ \ell_{31} & \ell_{32} & \ell_{33} \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \tag{1.54}$$

where

$$\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} \tag{1.55}$$

From equation (1.54), we can write,

$$\ell_{11}Z_1 = I_1 \tag{1.56a}$$

$$\ell_{21}Z_1 + \ell_{22}Z_2 = I_2 \tag{1.56b}$$

$$\ell_{31}Z_1 + \ell_{32}Z_2 + \ell_{33}Z_3 = I_3 \tag{1.56c}$$

Therefore, equation (1.56a–1.56c) can be solved by forward substitution as follows:

$$Z_1 = \frac{I_1}{\ell_{11}} \tag{1.57a}$$

$$Z_2 = \frac{1}{\ell_{22}}(I_2 - \ell_{21}Z_1) \tag{1.57b}$$

$$Z_3 = \frac{1}{\ell_{33}}(I_3 - \ell_{31}Z_1 - \ell_{32}Z_2) \tag{1.57c}$$

From equation (1.55), we can write,

$$V_1 + u_{12}V_2 + u_{13}V_3 = Z_1 \tag{1.58a}$$

$$V_2 + u_{23}V_3 = Z_2 \tag{1.58b}$$

$$V_3 = Z_3 \tag{1.58c}$$

Therefore, equation (1.58a–1.58c) can be solved by backward substitution as follows:

$$V_3 = Z_3 \tag{1.59a}$$

$$V_2 = Z_2 - u_{23}V_3 \tag{1.59b}$$

$$V_1 = Z_1 - u_{12}V_2 - u_{13}V_3 \tag{1.59c}$$

The elements of lower and upper triangular matrices can be calculated as follows:
From equation (1.53), we can write

$$\begin{bmatrix} \ell_{11} & \ell_{11}u_{12} & \ell_{11}u_{13} \\ \ell_{21} & (\ell_{21}u_{12} + \ell_{22}) & (\ell_{21}u_{13} + \ell_{22}u_{23}) \\ \ell_{31} & (\ell_{31}u_{12} + \ell_{32}) & (\ell_{31}u_{13} + \ell_{32}u_{23} + \ell_{33}) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \tag{1.60}$$

Comparing equation (1.60) with equation (1.12), we can write,

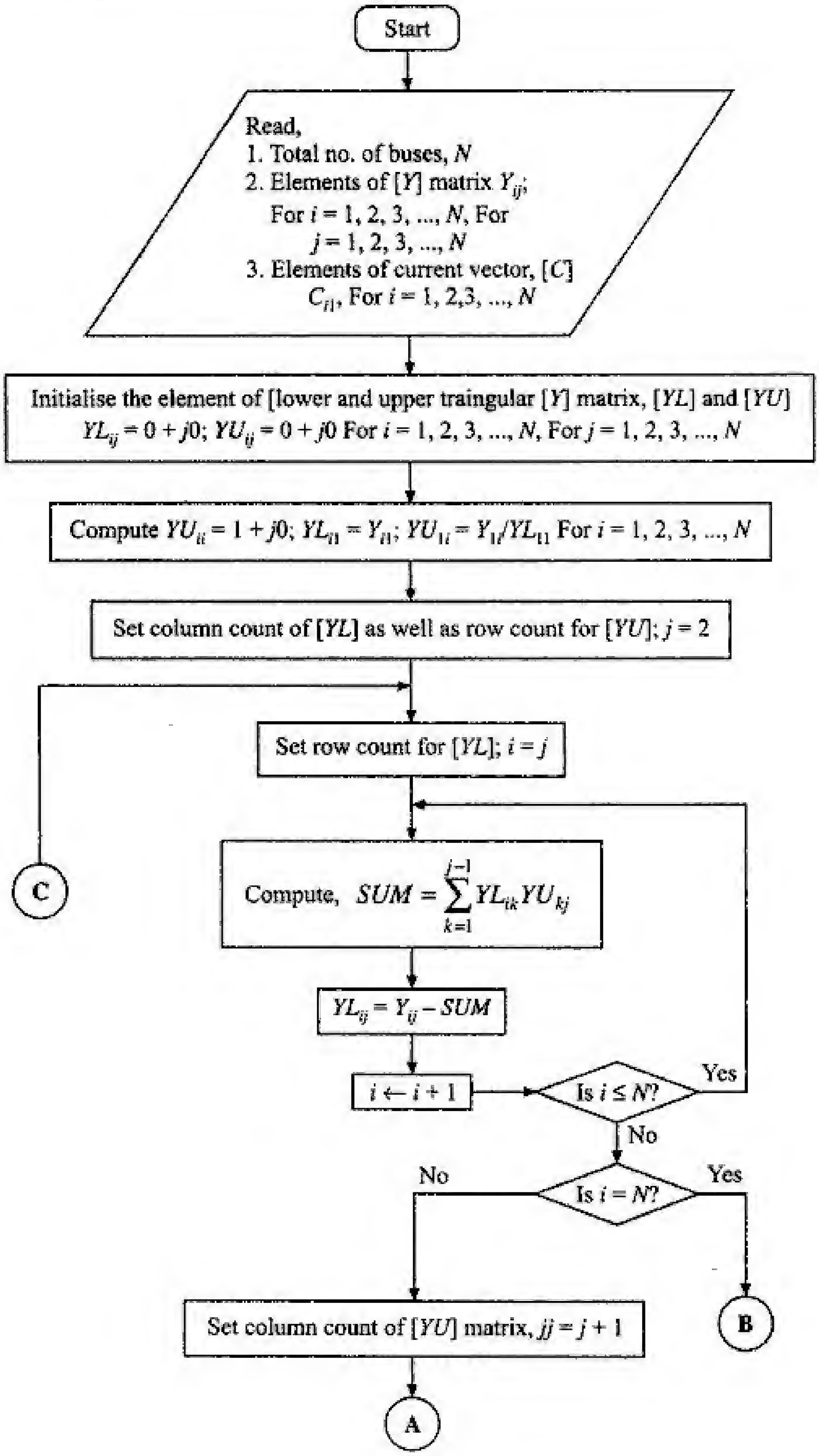
$$\left.\begin{aligned} \ell_{11} &= Y_{11} \\ \ell_{12} &= \ell_{13} = 0 \\ \ell_{21} &= Y_{21} \\ \ell_{22} &= Y_{22} - \ell_{21}u_{12} \\ \ell_{23} &= 0 \\ \ell_{31} &= Y_{31} \\ \ell_{32} &= Y_{32} - \ell_{31}u_{12} \\ \ell_{33} &= Y_{33} - \ell_{31}u_{13} - \ell_{32}u_{23} \end{aligned}\right\} \tag{1.61}$$

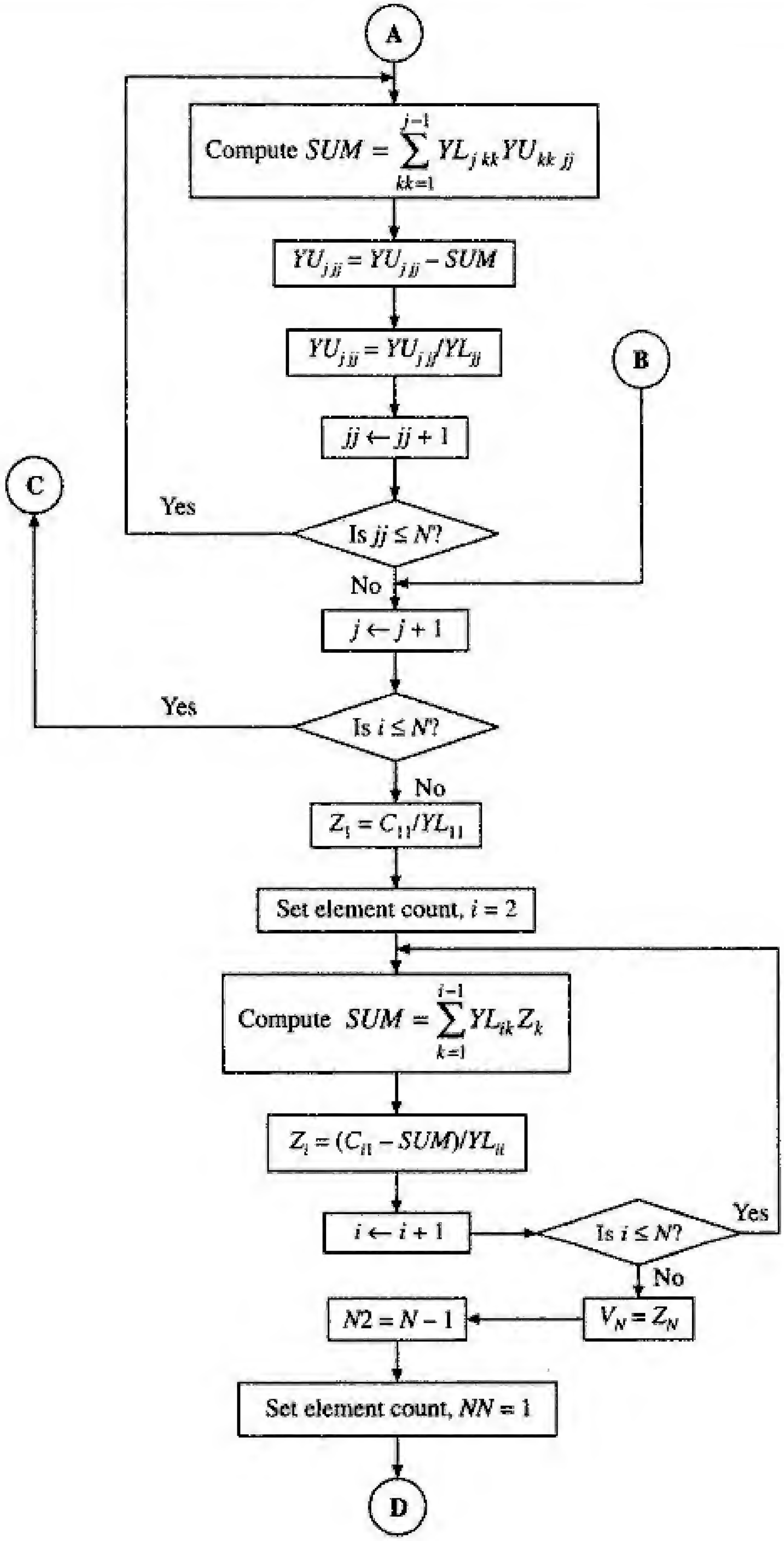
and

$$\left.\begin{aligned} u_{11} &= 1 \\ u_{12} &= \frac{Y_{12}}{\ell_{11}} \\ u_{13} &= \frac{Y_{13}}{\ell_{11}} \\ u_{21} &= 0 \\ u_{22} &= 1 \\ u_{23} &= \frac{1}{\ell_{22}}(Y_{23} - \ell_{21}u_{13}) \\ u_{31} &= u_{32} = 0 \\ u_{33} &= 1 \end{aligned}\right\} \tag{1.62}$$

Hence the lower and upper triangular matrices can be formed using equations (1.61) and (1.62), and then by forward and backward substitution we can get the final solution of V_1, V_2 and V_3.

The detailed flowchart of the method is given in Fig. 1.15.


Start
Read,
1. Total no. of buses, N
2. Elements of $[Y]$ matrix Y_{ij};
For $i = 1, 2, 3, ..., N$, For
$j = 1, 2, 3, ..., N$
3. Elements of current vector, $[C]$
C_{i1}, For $i = 1, 2, 3, ..., N$
Initialise the element of [lower and upper traingular $[Y]$ matrix, $[YL]$ and $[YU]$
$YL_{ij} = 0 + j0$; $YU_{ij} = 0 + j0$ For $i = 1, 2, 3, ..., N$, For $j = 1, 2, 3, ..., N$
Compute $YU_{ii} = 1 + j0$; $YL_{i1} = Y_{i1}$; $YU_{1i} = Y_{1i}/YL_{11}$ For $i = 1, 2, 3, ..., N$
Set column count of $[YL]$ as well as row count for $[YU]$; $j = 2$
Set row count for $[YL]$; $i = j$
C
Compute, $SUM = \sum_{k=1}^{j-1} YL_{ik} YU_{kj}$
$YL_{ij} = Y_{ij} - SUM$
$i \leftarrow i + 1$
Is $i \le N$?
Yes
No
Is $i = N$?
No
Yes
Set column count of $[YU]$ matrix, $jj = j + 1$
B
A


A
Compute $SUM = \sum_{kk=1}^{j-1} YL_{j\,kk} YU_{kk\,jj}$
$YU_{j\,jj} = YU_{j\,jj} - SUM$
$YU_{j\,jj} = YU_{j\,jj}/YL_{jj}$
B
$jj \leftarrow jj + 1$
C
Yes
Is $jj \le N$?
No
$j \leftarrow j + 1$
Yes
Is $i \le N$?
No
$Z_1 = C_{11}/YL_{11}$
Set element count, $i = 2$
Compute $SUM = \sum_{k=1}^{i-1} YL_{ik} Z_k$
$Z_i = (C_{i1} - SUM)/YL_{ii}$
$i \leftarrow i + 1$
Is $i \le N$?
Yes
No
$N2 = N - 1$
$V_N = Z_N$
Set element count, $NN = 1$
D

$$u_{12} = \frac{Y_{12}}{\ell_{11}} = \frac{j5}{(-j10.952)} = -0.456538;$$

$$u_{13} = \frac{Y_{13}}{\ell_{11}} = \frac{j5}{(-j10.952)} = -0.456538$$

Again from equation (1.61), we can write

$$\ell_{22} = Y_{22} - \ell_{21}u_{12} = -j10.952 - \{j5 \times (-0.456538)\} = -j8.66931;$$

$$\ell_{32} = Y_{32} - \ell_{31}u_{12} = j5 - \{j5 \times (-0.456538)\} = j7.28268$$

From equation (1.62),

$$u_{23} = \frac{1}{\ell_{22}}(Y_{23} - \ell_{21}u_{13}) = \frac{1}{-j8.66931}\{j5 - j5 \times (-0.456538)\} = -0.840054$$

From equation (1.61),

$$\ell_{33} = Y_{33} - \ell_{31}u_{13} - \ell_{32}u_{23}$$
$$= -j10 - \{j5 \times (-0.456538)\} - \{j7.28268 \times (-0.840054)\} = -j1.599466$$

From equations (1.57a–1.57c), we get

$$Z_1 = \frac{I_1}{\ell_{11}} = \frac{-j}{(-j10.952)} = 0.091308$$

$$Z_2 = \frac{1}{\ell_{22}}(I_2 - \ell_{21}Z_1) = \frac{1}{-j8.66931}\{(-0.4285 - j0.742184) - (j5 \times 0.091308)\}$$
$$= (0.138272 - j0.049427)$$

$$Z_3 = \frac{1}{\ell_{33}}(I_3 - \ell_{31}Z_1 - \ell_{32}Z_2)$$

$$= \frac{1}{-j1.599466}\{0 - (j5 \times 0.091308) - (j7.28268) \times (0.138272 - j0.049427)\}$$
$$= (0.915012 - j0.225051)$$

Therefore, using equations (1.59a–1.59c) we get the solution by backward substitution as follows:

$$V_3 = Z_3 = (0.915012 - j0.225051)$$
$$V_2 = Z_2 - u_{23}V_3$$
$$= (0.138272 - j0.049427) - (-0.840054) \times (0.915012 - j0.225051)$$
$$= (0.906931 - j0.238482)$$
$$V_1 = Z_1 - u_{12}V_2 - u_{13}V_3$$
$$= 0.091308 - (-0.456538) \times (0.906931 - j0.238482)$$
$$- (-0.456538) \times (0.915012 - j0.225051) = (0.923094 - j0.211620)$$

All the voltages, currents and admittances are expressed in p.u.; the angles are expressed in degree.

Execution of computer program to solve bus voltages by triangular factorization for Example 1.10

[Y_{Bus}] matrix of the system: NMYBUS1.DAT

Given in Example 1.1.

Bus currents: NMCUR1.DAT

Given in Example 1.1.

Final bus voltages (Output of TRANFACT.FOR): NMTVOLT1.DAT

```
LOWER TRIANGULAR MATRIX

  YL(  1,   1 ) = (   .000000, -10.952000 )
  YL(  1,   2 ) = (   .000000,    .000000 )
  YL(  1,   3 ) = (   .000000,    .000000 )
  YL(  2,   1 ) = (   .000000,   5.000000 )
  YL(  2,   2 ) = (   .000000,  -8.669312 )
  YL(  2,   3 ) = (   .000000,    .000000 )
  YL(  3,   1 ) = (   .000000,   5.000000 )
  YL(  3,   2 ) = (   .000000,   7.282688 )
  YL(  3,   3 ) = (   .000000,  -1.599463 )

INTERMEDIATE VOLTAGE MATRIX

              Bus-code                 VOLTAGE
                 1             (    .091308,     .000000)
                 2             (    .138272,   -.049427)
                 3             (    .915013,   -.225053)

UPPER TRIANGULAR MATRIX

  YU(  1,   1 ) = (  1.000000,    .000000 )
  YU(  1,   2 ) = (  -.456538,    .000000 )
  YU(  1,   3 ) = (  -.456538,    .000000 )
  YU(  2,   1 ) = (   .000000,    .000000 )
  YU(  2,   2 ) = (  1.000000,    .000000 )
  YU(  2,   3 ) = (  -.840054,    .000000 )
  YU(  3,   1 ) = (   .000000,    .000000 )
  YU(  3,   2 ) = (   .000000,    .000000 )
  YU(  3,   3 ) = (  1.000000,    .000000 )

VOLTAGE MATRIX

              Bus-code                 VOLTAGE
                 1            (    .923094,   -.211622) V1
                 2            (    .906932,   -.238483) V2
                 3            (    .915013,   -.225053) V3
```

Example 1.12: Solve the simultaneous equations given in Example 1.2 using triangular factorization.

Solution: Execution of computer program to solve for the variables by triangular factorization for Example 1.12

Final solution (Output of TRANFACT.FOR): NMTVOLT2.DAT

```
                           V
              ( -5.717704,   2.106747)
              ( -5.673186,   2.049922)
              ( -5.684221,   2.093270)
```

EXERCISES

1. Draw a block diagram of a Hierarchical Control Structure.
2. What are the advantages of computer control in power system? What are the types of computer control?
3. Draw the single line diagram of a two bus power system. What is the usual range of transmission voltage in India?
4. What are the 'states' in a power system? What do you mean by 'normal operating state'?
5. What do you mean by 'loadability' of transmission line? Derive an expression for it.
6. Find the expression for the frequency decay rate of a turbo-alternator following an attempted overload.
7. Write short notes on
   (i) Security analysis and contingency evaluation.
   (ii) FACT system.
   (iii) Gaussian elimination.
   (iv) Kron's method of network reduction.
8. Briefly describe Euler and Modified Euler methods for solving first order differential equations.
9. Illustrate the use of R–K method.
10. How do you ascertain the stability of a system using eigenvalue analysis?
11. What is eigenvalue sensitivity? Explain.
12. Illustrate the concept of triangular factorization to solve a set of simultaneous equation.

Chapter 2

MODELLING OF POWER SYSTEM COMPONENTS

2.1 INTRODUCTION

In order to implement *computer control* of a power system, it is imperative to gain a clear understanding of the representation of the power system components. Component modelling thus becomes very important. Studies of electrical energy systems are based on the simulation of actual phenomena using models behaving exactly in the identical way as the elements in the physical system. In research, it is necessary to have models permitting precise and detailed simulation. The different parameters must be accessible and the models are required to follow the physical process as closely and as faithfully as possible. Then it is required to solve mathematical equations governing these phenomena. Modelling of active elements, e.g. generator, transformer etc. is relatively difficult while that of passive elements, e.g. transmission line, relay, inductive VAR compensator etc. is easier. Passive circuit elements are mostly modelled by their parameters in the equivalent circuits while the active power system components are modelled by their operation in steady, transient and sub-transient state.

The models used in the power system give precise results in a certain field of hypotheses corresponding to their use. Here, the concept of representation of the physical reality of the phenomena disappears and only the relationship between data and results exists. Their limited use leads to simpler models than the preceding ones and necessitates fewer data processing requirements. This means that they can be more easily integrated into large simulation packages. In these models the process representation is based on the fundamental physical laws. Though the model is simplified, its method of representation takes into account the principle of non-linearity inherent in the physical phenomena involved. The models can be structured in modules to simplify subsequent upgrading and correction of the network. To a greater or lesser extent, the system variables require time in order to respond to any change in their operation. Modelling should take care of the change and system equations are to be written to designate the state of the operation of the element. However, writing of these equations obviously requires assumptions and hence no clear definitive model exists for most of the active elements. Proper model is to be selected by the programmer that suits the requirements of the problem.

The modelling of a synchronous generator needs utmost care as it is the heart of the power system. It may be observed that its modelling is the most difficult task due to its "stiffness" to the changes in the operating conditions external to the machine. On the other hand, there is transmission

network that responds almost immediately to the configurational change and loading alteration. The time constants associated with the network are insignificant in comparison to those of the synchronous machine. The rotary swing further complicates the modelling. The present text will give adequate stress on an alternator modelling such that the basic building blocks for computer-aided analysis of the operation of the power system can be developed at this stage.

2.2 MODELLING OF SYNCHRONOUS GENERATOR (ALTERNATOR)

In modelling of the synchronous generator, the most appropriate frame of reference is one that is attached to the rotor. This frame rotates at the same speed of the rotor. The major axis of this frame is known as *direct axis* (the rotor polar axis) or simply the d-axis and the second axis is 90° (elect.) apart from this polar axis and is known as the *quadrature axis* (the inter polar axis) or the q-axis.

In this text, the synchronous generator has been modelled in five different modes. Each mode is associated with some assumptions and the programmer is to select the particular model depending on the requirements of the following assumptions:

(i) The rotor speed of the alternator does not vary more than the prescribed limit
(ii) Rotational power loss due to windage and friction are neglected
(iii) Mechanical power input is constant.

Model '0'

From the basic concepts on electrical machines, it is well known that a group of synchronous machines or a part of the power system may be represented by a *single equivalent synchronous machine*. Similarly, an *infinite bus*, representing a part of the system having zero impedance and infinite rotational inertia, may be similarly modelled using the operating *state equations* while the machine voltage is assumed to be constant behind *d-axis transient reactance* (X_1'). In this chapter, the salient pole synchronous machine is only considered, as the cylindrical rotor machine model may be regarded as a special case of a salient machine model with $X_d = X_q$; X_d and X_q are the *direct axis* and *quadrature axis* synchronous reactances, respectively.

To model a salient pole generator in transient state, two transient voltages are to be assumed (E_a' and E_q') representing the flux linkage in the rotor winding. The transient operation is associated with addition of transient reactance and voltage to the steady state model (Fig. 2.1). The phasor diagram

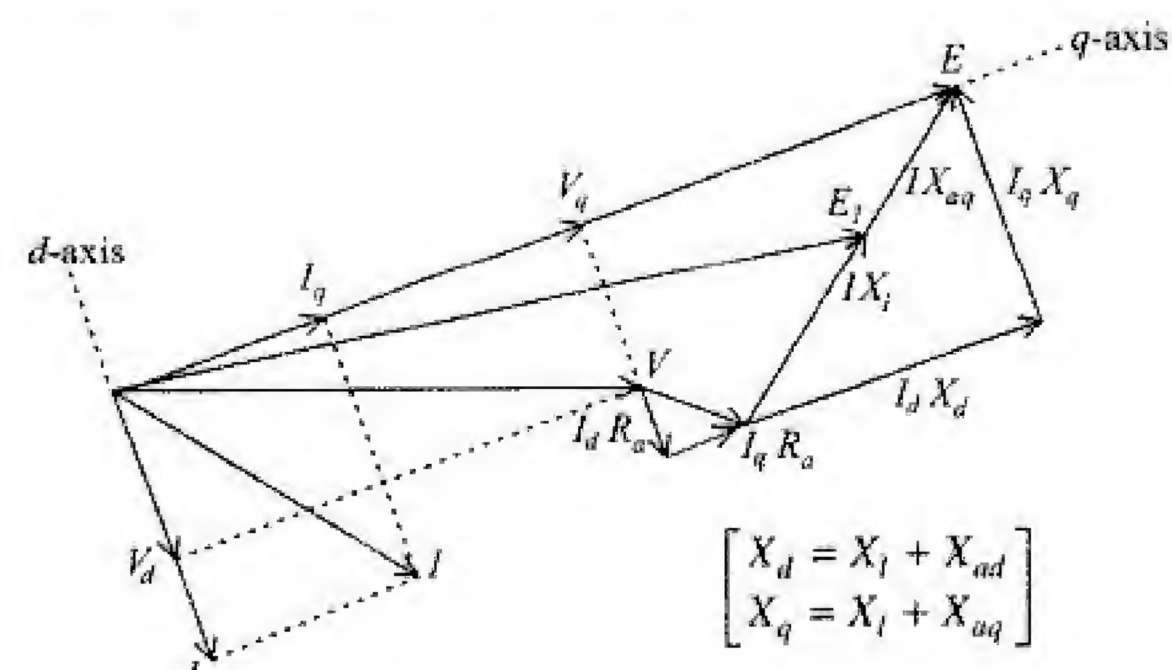


Fig. 2.1 Phasor diagram of steady state operation of the salient pole alternator. Suffix d stands for direct axis quantities and suffix q for quadrature axis quantities. Suffix l indicates leakage quantity; V_d and I_d are numerically negative.

of the transient condition in the machine has been shown in Fig. 2.2, where the induced voltage E has been considered the sum of the two voltages E_d and E_q unlike to that in the steady state model when $E = E_q$ and $E_d = 0$. The transient voltage in this model can be shown to exist behind the transient reactances X'_d and X'_q. The equations representing this model are thus

$$E'_d = V_d + I_d R_a + I_q X'_q \tag{2.1}$$

and

$$E'_q = V_q + I_q R_a - I_d X'_d \tag{2.2}$$

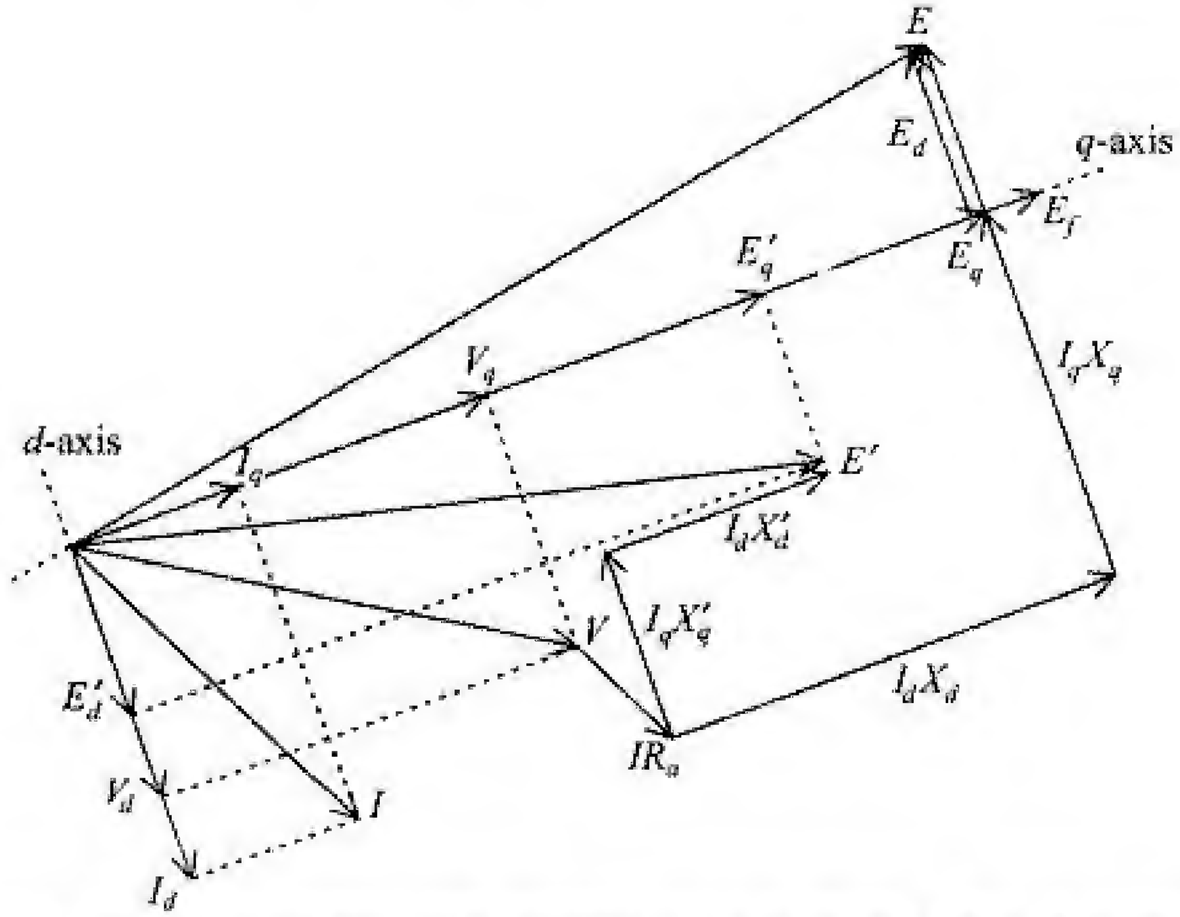


Fig. 2.2 Phasor diagram of the transient state operation of the salient alternator. E' is the transient voltage (projections at d-axis are numerically negative).

[Here, E and V represent induced and terminal voltages while I is the machine current and X' is the transient reactance of the salient pole alternator. Suffixes d and q are used to designate the direct axis and quadrature axis components of the variables. E'_d, V_d and I_d are numerically negative.]

Model 1

Here, the model of the machine has been assumed to have the magnitude of constant voltage behind the d-axis transient reactance only; q-axis transient flux linkage has been assumed to be so small that it has been neglected. However, the mechanical system equations have been considered in this model. Hence, the modelling has been done utilising the equations (2.1) and (2.2) in addition to rotor swing equations given by equations (2.3) and (2.4).

$$\frac{d\omega}{dt} = \frac{1}{M}\left(P_m - P_e - D\frac{d\delta}{dt}\right) \tag{2.3}$$

and

$$\frac{d\omega}{dt} = \omega - 2\pi f_0 \tag{2.4}$$

where, $M = \dfrac{H}{\pi f_0}$

[M = angular momentum
H = inertia constant
f_0 = base frequency
ω = angular frequency
P_m = turbine shaft power
P_e = generator electrical power output
D = damping coefficient
δ = rotor angle]

Model 2

The drawback of Model-0 and Model-1 is that the electrical dynamics have not been considered. Model-2 includes the machine operation with time varying equations assuming d-axis transient effects only. The equations representing this model are given by equations (2.1), (2.2), (2.3) and (2.4) in addition to equation (2.5) that represents the governing differential equation to allow the rotor flux linkage to change with time. From the phasor diagram of Fig. 2.2,

$$\frac{dE_q'}{dt} = \frac{(E_f - E_q)}{T_d'} = \frac{E_f + (X_d - X_d')I_d - E_q'}{T_d'} \tag{2.5}$$

where, T_d' is the *direct axis transient time constant* and E_f is the applied field voltage. I_d is numerically negative.

Model 3

In this model, the transient effects in both the d and q-axes have been considered. The governing equations are represented by equations (2.1) to (2.6). Equation (2.5) considered the flux linkage changing with time for the q-axis while equation (2.6) describes the same for the d-axis. From phasor diagram of Fig. 2.2, equation (2.6) can be formed as

$$\frac{dE_d'}{dt} = -\frac{E_d}{T_q'} = \frac{-(X_q - X_q')I_q - E_d'}{T_q'} \tag{2.6}$$

Here T_d' is the *quadrature axis transient time constant.*

Model 4

Sub-transient state of operation has not yet been considered in any of the models discussed so far. Due to the presence of a *damper winding*, sub-transient state of operation needs attention. Similar to the transient modelling, in this case also, two sub-transient new voltages (E_d'' and E_q'') have been assumed. Figure 2.3 represents the phasor diagram of the alternator during sub-transient state of operation. The governing equations can be written as

$$E_d'' = V_d + I_dR_a + I_qX_q'' \tag{2.7}$$

$$E_q'' = V_q + I_dR_a + I_dX_d'' \tag{2.8}$$

$$\frac{dE_q''}{dt} = [E_q' + (X_d' - X_d'')I_d - E_q'']/T_d'' \tag{2.9}$$

$$\frac{dE_d''}{dt} = [E_d' + (X_q' - X_q'')I_d - E_q'']/T_d'' \tag{2.10}$$

In the above equation, T_d'' and T_q'' are considered to be *sub-transient d-axis* and *q-axis time constants*. This model is characterised by equations (2.5) to (2.10) in addition to equations (2.3) and (2.4). Groups of synchronous machines or parts of the system may be represented by a single synchronous machine model. An infinite bus bar, representing a large *stiff* system, may similarly be modelled as a single machine (Model-0).

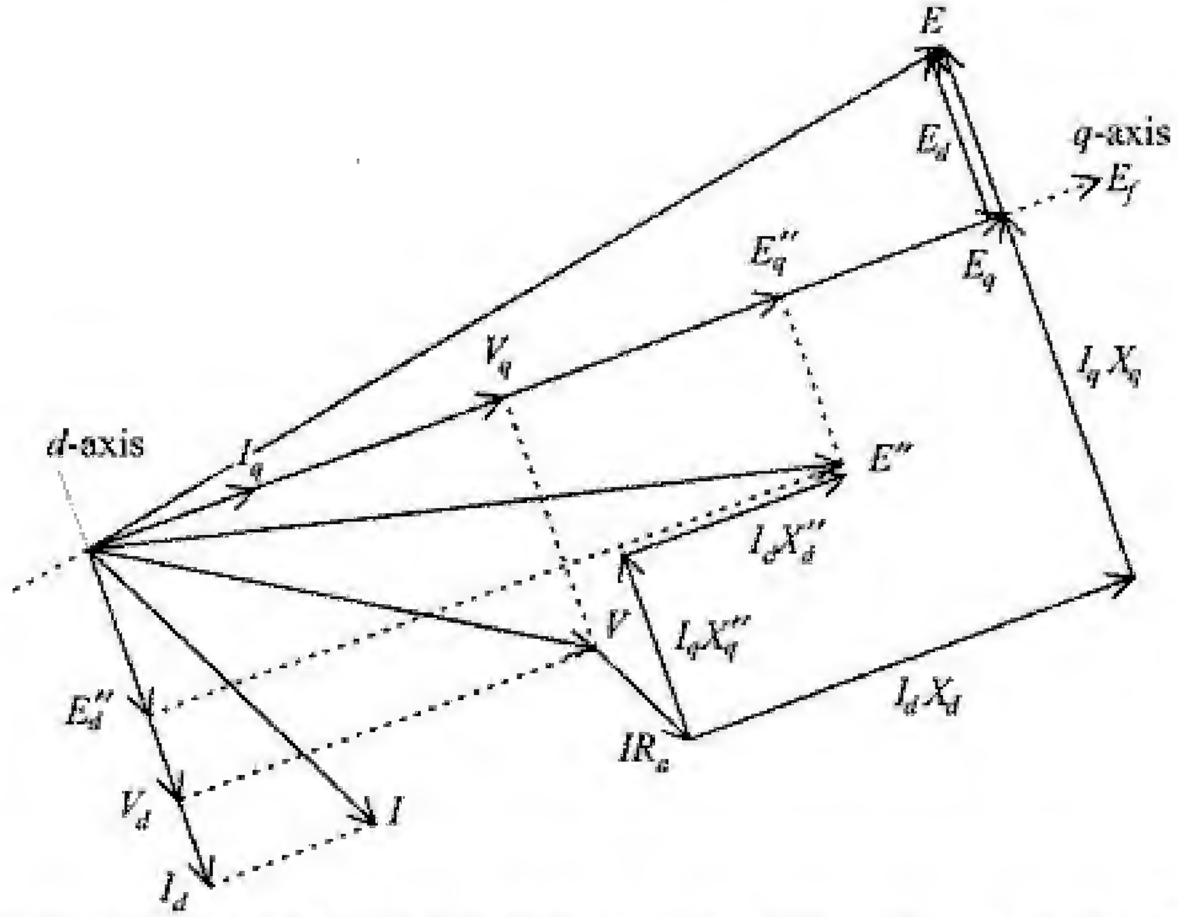


Fig. 2.3 Phasor diagram of the sub-transient state operation of the salient alternator. E' is the transient voltage (d-axis projections are numerically negative).

2.3 MODELLING OF A SYNCHRONOUS GENERATOR IN A NETWORK

The synchronous machine equations have been framed with a reference rotating with its own rotor. The real and imaginary components of the voltages in a network reference frame (Fig. 2.4) can thus be formed as

$$\begin{bmatrix} V_r \\ V_{im} \end{bmatrix} = \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} V_q \\ V_d \end{bmatrix}$$

or,

$$\begin{bmatrix} V_q \\ V_d \end{bmatrix} = \begin{bmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} V_r \\ V_{im} \end{bmatrix} \tag{2.11}$$

Here V_r and V_{im} represent components of voltage V in real and imaginary axis.

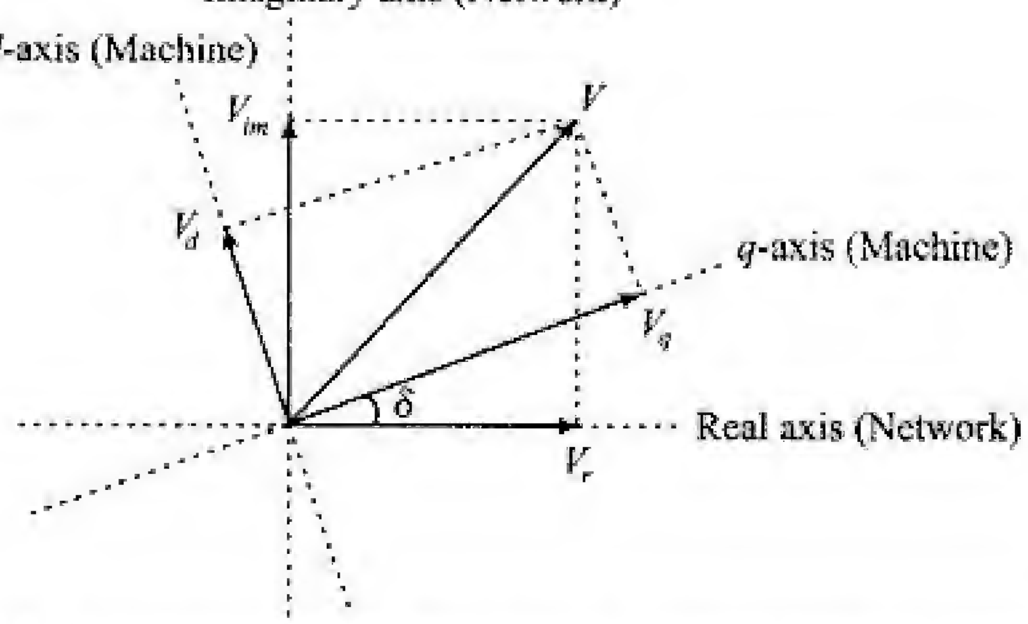


Fig. 2.4 Co-relation between alternation and network frame of reference.

It may be noted here that the two reference frames and the relationship between components of the reference frames (equation 2.11) are commonly discussed in the literature. It may also be noted that a given phasor V has been distributed into two very different forms of components depending on the angle δ of the machine reference frame. It may be observed that the vector V can also be represented in the form of equation (2.12).

$$V = (V_q + jV_d)e^{j\delta} \tag{2.12}$$

where V_q and V_d are purely real quantities. Assuming the positive sequence voltages and currents with the amplitude and phases, the general relation between these variables may be written for the network as

$$[I] = [Y][V] \tag{2.13}$$

[In case of representation of the variables of the machine, the expressed quantities in d–q reference frame must be converted into a common reference frame by axis transformations.]

The power equations for a salient pole alternator can be modelled by any one of the models. The power equations in the steady state and transient state are given by

$$P = \frac{|E||V|}{X_d}\sin\delta + \frac{|V|^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta \tag{2.14}$$

and

$$P_g = \frac{|E'||V|}{X'_d}\sin\delta + \frac{|V|^2}{2}\left(\frac{1}{X'_q} - \frac{1}{X'_d}\right)\sin 2\delta \tag{2.15}$$

when, $\delta = \angle E - \angle V = \angle E' - \angle V$.

2.4 MODELLING OF GENERATOR COMPONENTS

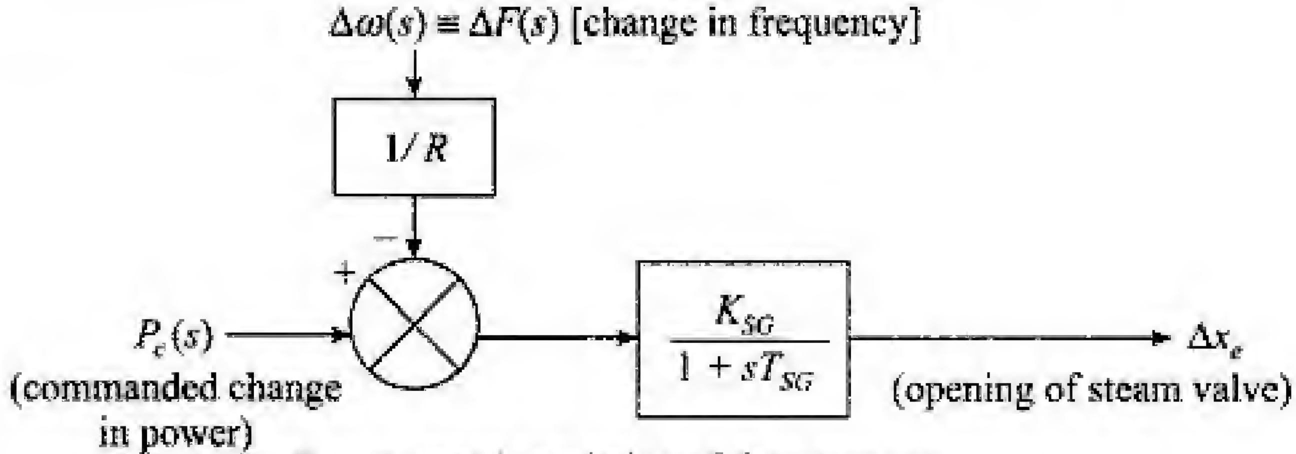
The modelling of the generator remains incomplete if the role of AGC/LFC (*automatic generation control/load frequency control*) and *excitation control* are not included. Just as the AVR (*automatic voltage regulator*) achieves reactive power balance by maintaining a constant voltage, the load frequency control achieves real power balance by maintaining a constant frequency. *Governor Modelling* and *Tturbine Modelling* are thus very much important in implementing AGC.

2.4.1 Governor Modelling

If the load increases, the speed of the synchronous generator reduces slightly. The governor of any thermal unit reacts to this speed variation and permits the entry of some more steam from the boiler to the turbine which, in turn, increases the speed. The increased steam flow reduces the boiler pressure, which reinstates the increase of an adequate fuel, air and water flow to release the steam pressure. Fortunately, the large *thermal inertia* of most boiler systems enables the load frequency performance of the turbine, generator and load to be decoupled from that of the boiler, so that, for short duration of load change, the boiler pressure may be regarded as constant. The generator mainly determines the short-term response of the system to the load fluctuations.

Many forms of the governor system have been devised all of which include, in some way or the other, the variation of the turbine-generator shaft speed as the basis on which the change of position of the turbine working fluid control valve actuates. Typical speed droop characteristics for most governors range between 5 and 10%. The latest trend in the turbine governor design is to provide an electronic controller. A block diagram representation of the speed governor system is shown in Fig. 2.5.

The speed governing system of hydroturbine is more complicated. An additional feedback loop provides temporary droop compensation to *prevent* instability. This is necessitated by the large inertia of the *penstock gate*, which regulates the rate of water input to the turbine.



R = speed regulation of the governor
K_{SG} = gain of the speed governor
T_{SG} = time constant of the speed governor

Fig. 2.5 Block diagram representation of the speed governor system.

Here,
$$\Delta x_e = \frac{K_{SG}}{1+sT_{SG}}\left(\Delta P_c - \frac{1}{R}\Delta\omega\right) \tag{2.16}$$

Equation (2.16) plays an important role in modelling the governor operation. Let us consider a simple example. Assuming an increment $\Delta P_c = 1.0$ at $t = 0$, for a speed governing system under test (i.e. operating on open loop resulting $\Delta\omega = 0$), the increment in steam valve opening Δx_e is obtained from equation (2.16) using the Laplace transform of ΔP_c:

$$\Delta x_e = \frac{K_{SG}}{s\left(1+sT_{SG}\right)}, \text{ using the Laplace transform of } \Delta P_c$$

$$= \frac{K_{SG}/T_{SG}}{s\left(s+\dfrac{1}{T_{SG}}\right)} \tag{2.17}$$

Mathematical manipulation yields,

$$\Delta x_e = \frac{K_{SG}}{s} - \frac{K_{SG}}{s+\dfrac{1}{T_{SG}}} \tag{2.18}$$

which on inverse Laplace transform yields

$$\Delta x_e(t) = K_{SG}\left(1-e^{-t/T_{SG}}\right) \text{ for } t \geq 0 \tag{2.19}$$

The response curve has been plotted in Fig. 2.6. Thus, the governor action has been modelled utilising the concept of transfer functions.

2.4.2 Turbine Modelling

Turbine dynamics are of prime importance as they also affect the overall response of the generating plant to load changes. The actual dynamics ofcourse greatly depends on the type of turbine used. A non-reheat type of steam turbine has been shown in Fig. 2.7.

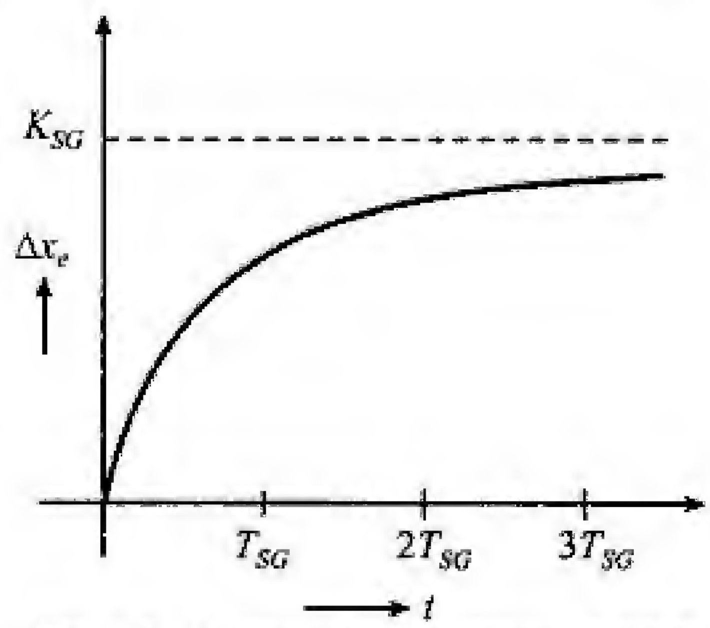


Fig. 2.6 Speed governor response curve.

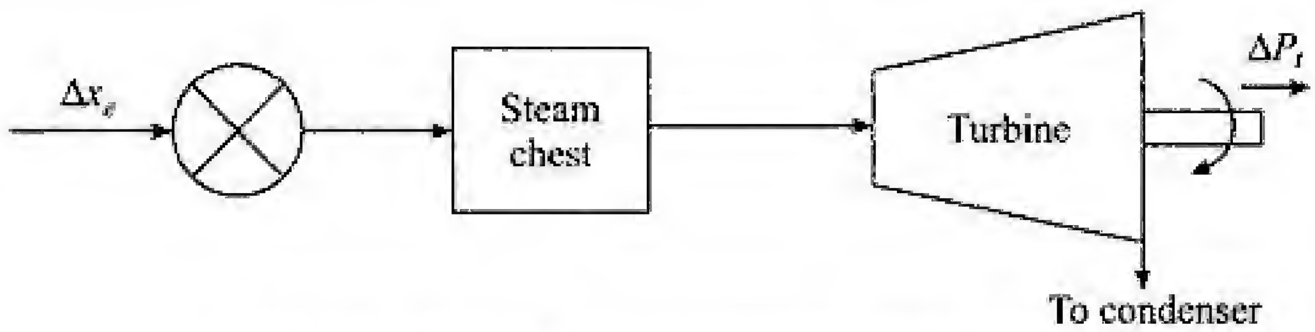


Fig. 2.7 Block diagram of non-reheat type turbine.

After passing the control valve, the high pressure steam enters the turbine via the steam-chest that introduces the delay T_T (in the order of 0.2 s to 0.5 s) in the steam flow resulting in the transfer function

$$G_T = \frac{\Delta P_t(s)}{\Delta x_e(s)} = \frac{1}{1+sT_T} \tag{2.20}$$

The turbine governor block diagram has been shown in Fig. 2.8.

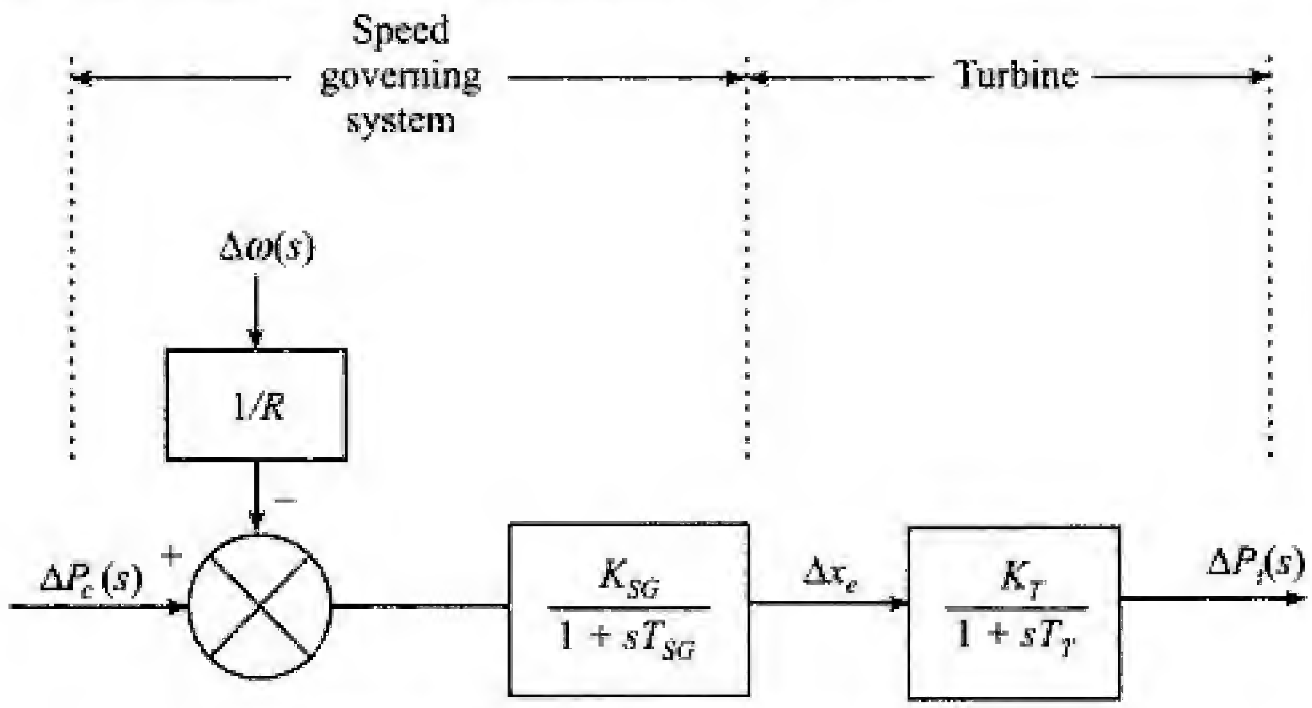


Fig. 2.8 Turbine governor block diagram.

Assuming the command increment to be ΔP_C, at steady state,

$$\Delta P_M = K_{SG} K_T \Delta P_C \tag{2.21}$$

It insists to choose a scale factor so that $\Delta P_t = \Delta P_C$. This is equivalent to picking $K_{SG}K_T = 10$. This gives the model as shown in Fig. 2.9.

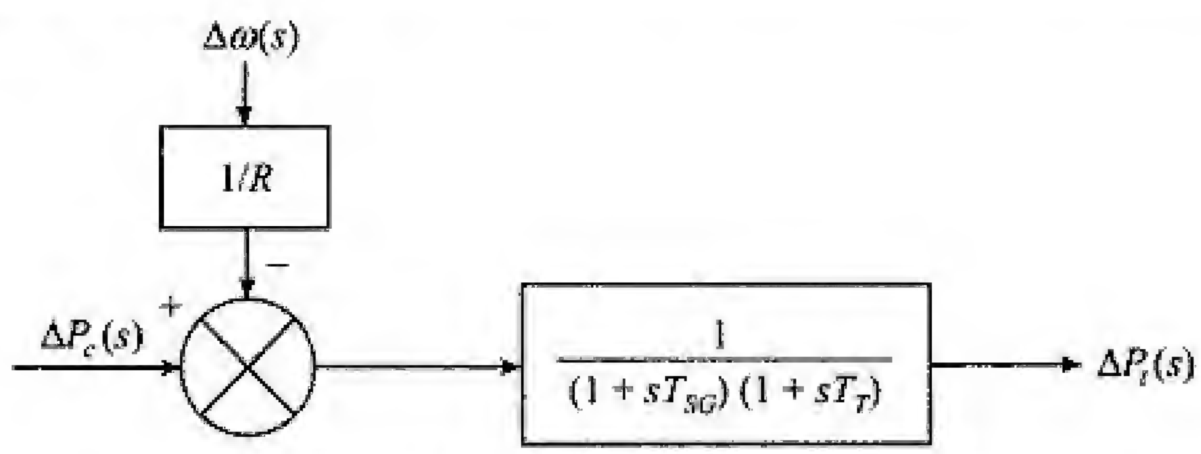


Fig. 2.9 Block diagram for turbine governor modelling.

This model can also be modified to account for reheat cycle steam turbine (Fig. 2.10). This is more efficient and is used for modern-day large sets. The overall transfer function of the reheat type unit is given by

$$G = \frac{\Delta P_t(s)}{\Delta x_e(s)} = \frac{1 + 0.5sT_{RH}}{1 + sT_{RH}} \tag{2.22}$$

where, T_{RH} is the time constant of the reheater having typical values in the range of 5–10 s.

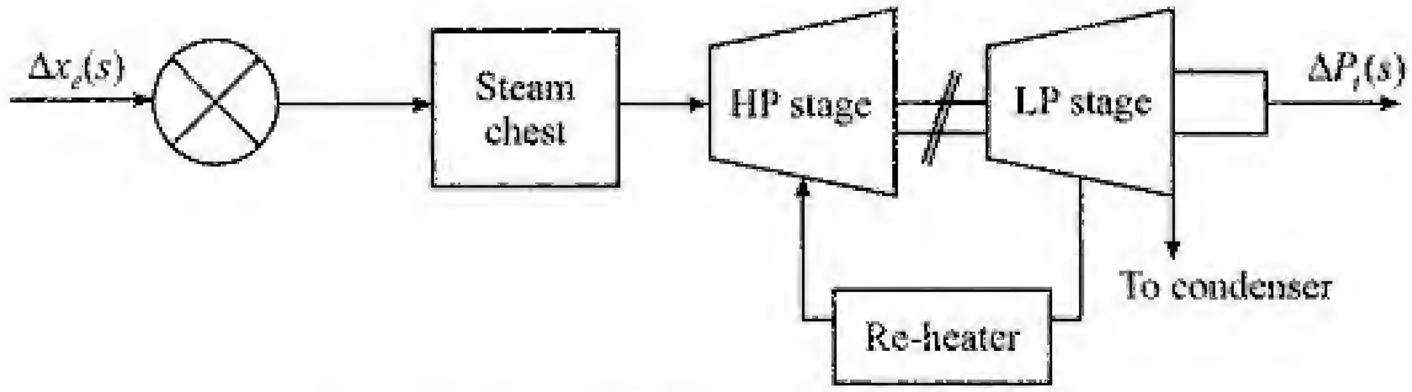


Fig. 2.10 Block diagram of reheat steam turbine.

The hydro turbine design varies with the water head (Fig. 2.11).

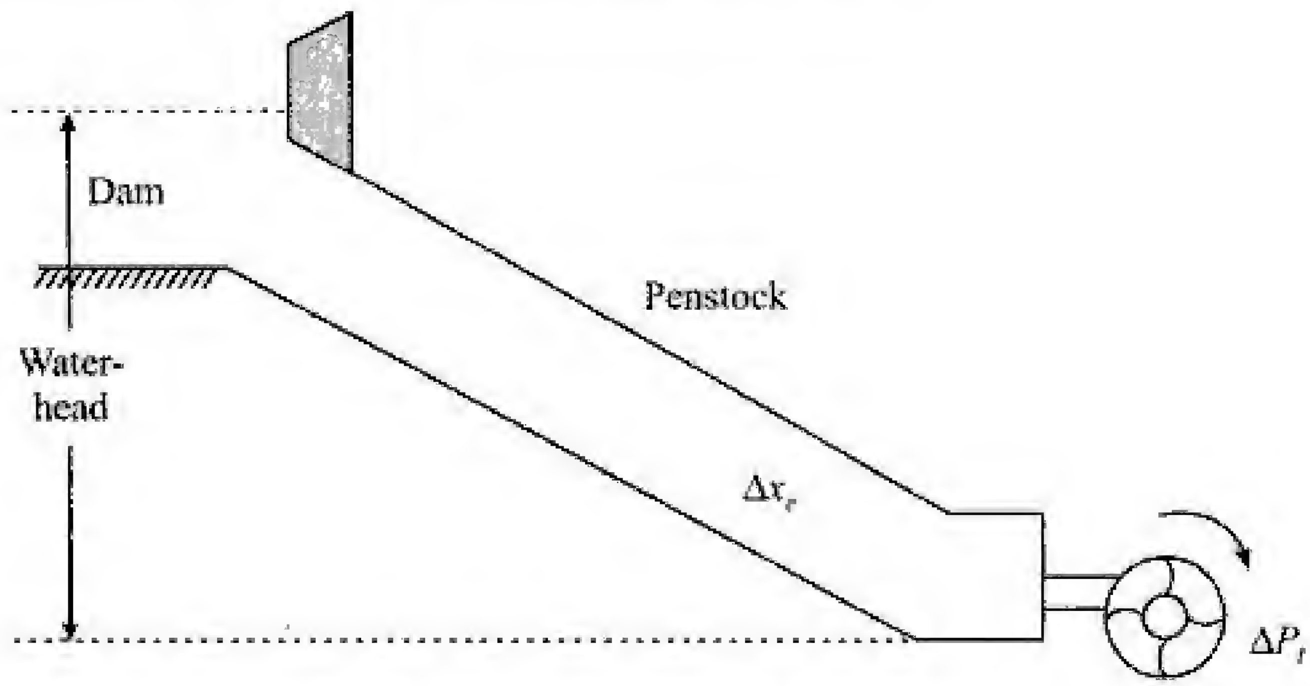


Fig. 2.11 Block diagram of hydroturbine.

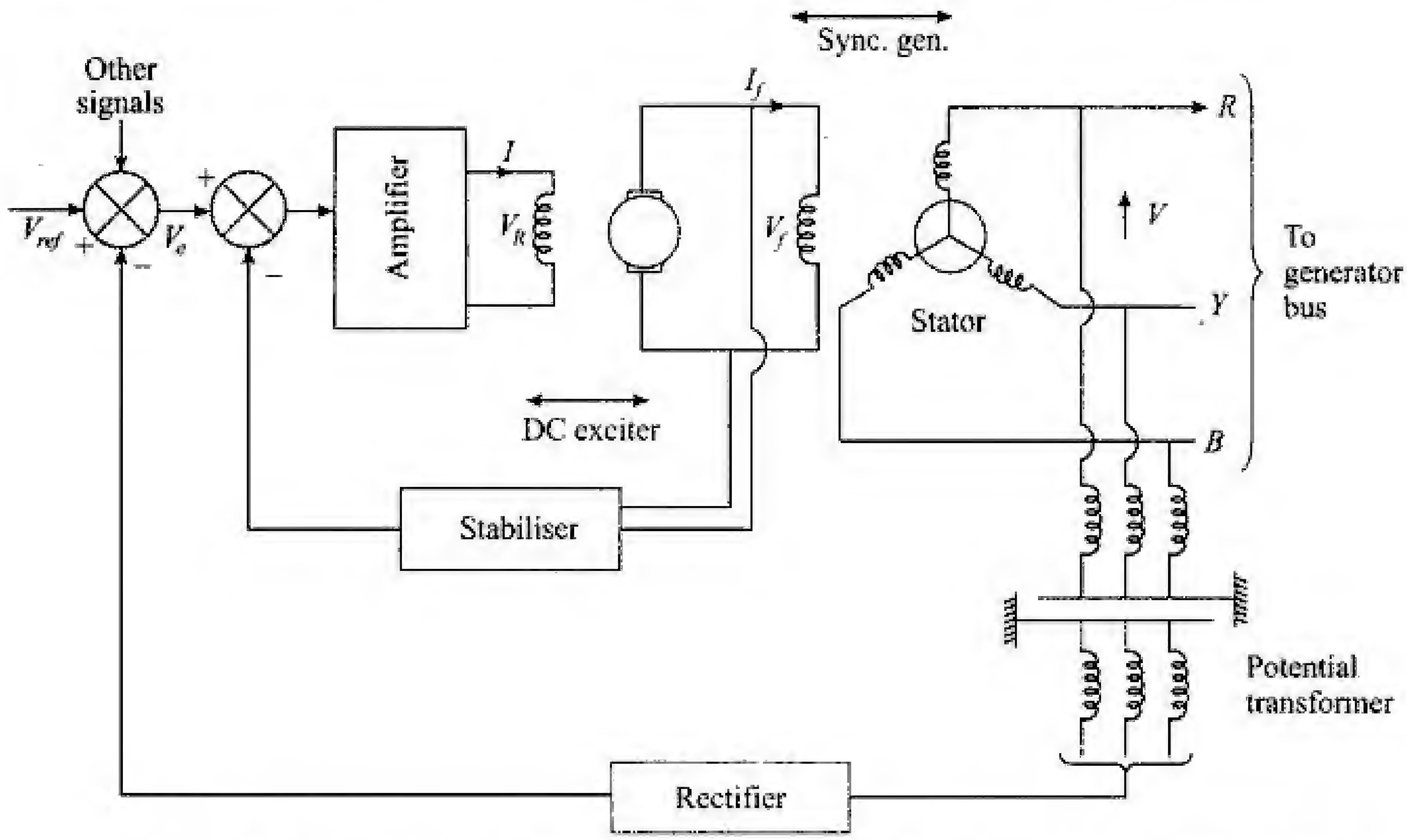
The overall transfer function is then

$$G = \frac{1 - 2sT_p}{1 + sT_p} \tag{2.22a}$$

where, T_p is the time (1 – 5s) it takes for the water to pass through the penstock.

2.4.3 Modelling of Exciter

Figure 2.12 represents the conventional excitation system of an alternator while Fig. 2.13 its block diagram with respective transfer functions.



[V_{ref} = Reference control input. V_e = Error voltage. V_R = Field voltage at the exciter.
V_f = Field voltage at the rotor of the sync. generator. V = Terminal voltage of the alternator.]

Fig. 2.12 Alternator excitation system.

In the block diagram of Fig. 2.13, T_R, the *time constant of the rectifier* is very small and may be neglected. The amplifier gain K_{Amp} is usually high (between 25 and 400). Amplifier time constant (T_{Amp}) is in the range of 0.02–0.4 sec. A stabiliser has also been shown to stabilise the gain of the exciter. K_{st}, the stabiliser gain, is 0.02 to 0.1 while stabiliser time constant T_{st} is in the range of 0.35 to 2.5 sec.

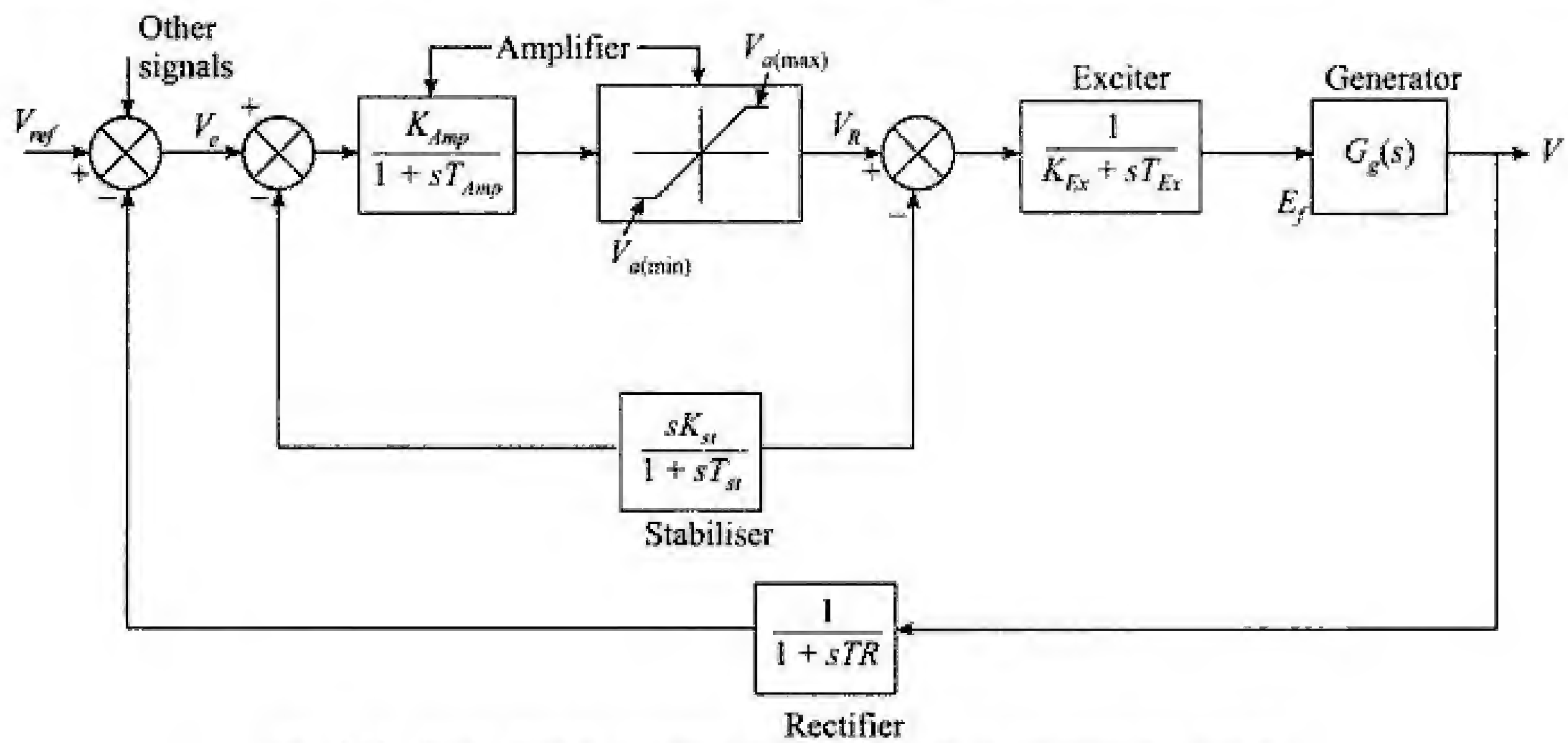


Fig. 2.13 Block diagram showing transfer function of the excitation system.

Some simplifications lead to a simplified block diagram as shown in Fig. 2.14. Here,

$$V = \frac{K_{Amp} K_g \sigma}{K_{ex} + K_{Amp} K_g \sigma} V_{ref};$$

when, σ is a factor associated with the transfer function of the synchronous generator when loaded.

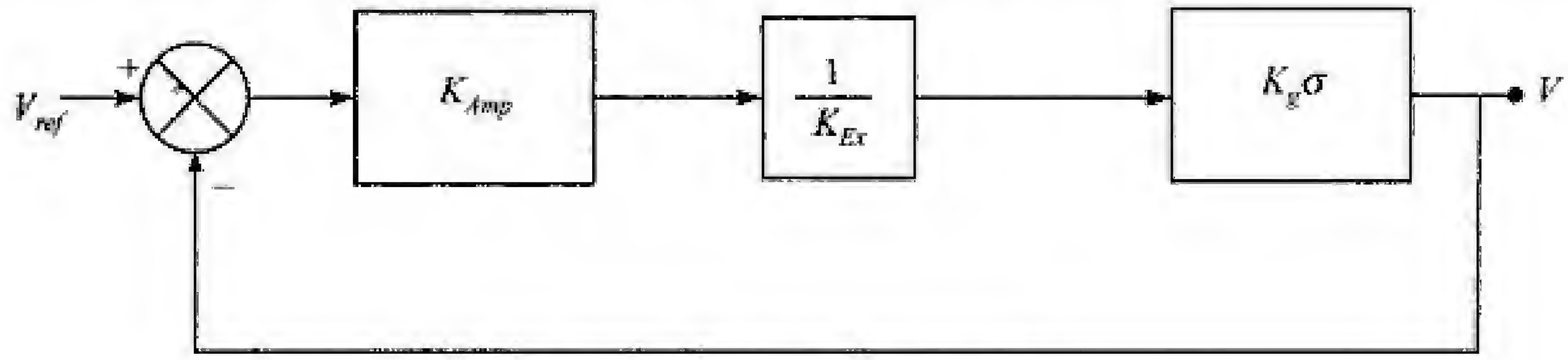


Fig. 2.14 Simplified block diagram.

2.5 MODELLING OF REGULATING TRANSFORMERS (RT)

Let the *regulating transformer* (Fig. 2.15) be placed in a two-bus system with a *complex transformation* ratio

$$n = |n| \angle \theta \tag{2.23}$$

The primary voltage and current will then be (nV_2) and (I_2/n^*), respectively.

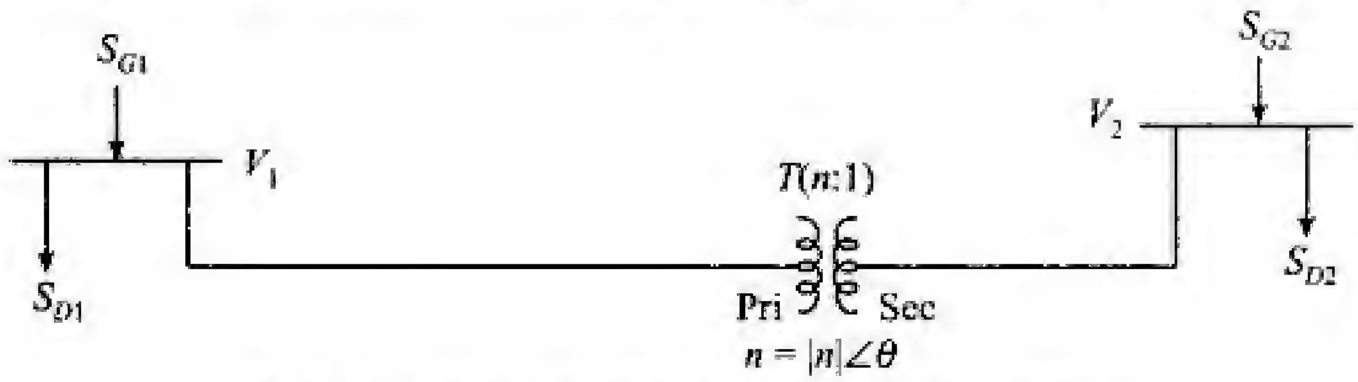


Fig. 2.15 Regulating transformer in a two-bus network.

The current balance equations can be written as

$$I_1 = V_1 Y_{sh} + (V_1 - nV_2)\, Y_{se} \tag{2.24}$$

$$I_2/n^* = nV_2 Y_{sh} + (nV_2 - V_1)\, Y_{se} \tag{2.25}$$

[The equivalent circuit has been shown in Fig. 2.16.]

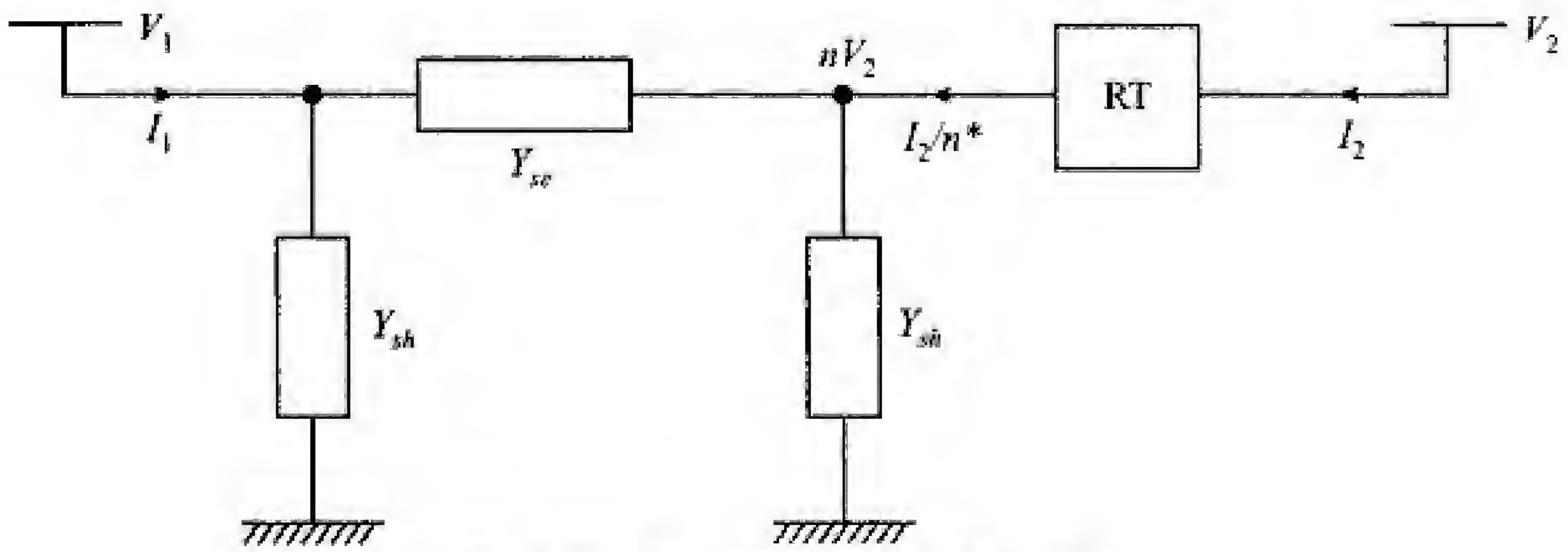


Fig. 2.16 Equivalent circuit of Fig. 2.15.

Equation (2.25) can be rewritten as

$$I_2 = -n^*V_1Y_{se} + nn^*(Y_{sh} + Y_{se})V_2$$

or,

$$I_2 = (-n^*Y_{se})V_1 + nn^*(Y_{sh} + Y_{se})V_2 \tag{2.26}$$

Also from the equation (2.24),

$$I_1 = (Y_{sh} + Y_{se})V_1 + (-nY_{se})V_2 \tag{2.27}$$

Hence, from equations (2.27) and (2.26),

$$Y_{Bus} = \begin{bmatrix} (Y_{sh} + Y_{se}) & -nY_{se} \\ -n^*Y_{se} & nn^*(Y_{sh} + Y_{se}) \end{bmatrix} \tag{2.28}$$

In practice, RT is either a *voltage magnitude control transformer* or a *phase angle control transformer*. In the former case, $\angle\theta = 0°$ and in the latter case, $|n|$ is a constant.

2.6 THREE-PHASE MODELLING

In a three-phase network, the three nodes are mostly associated together in their interconnections. This network is then termed as a *compound network* and the admittances are represented by *compound admittances*. Laws and equations that are valid for ordinary networks are also valid for compound network by simply replacing single quantities by appropriate matrices.

Figure 2.17 represents six *mutually* coupled single admittances. The node currents can be linked by admittance matrix to the branch voltages as follows:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} & y_{15} & y_{16} \\ y_{21} & y_{22} & y_{23} & y_{24} & y_{25} & y_{26} \\ y_{31} & y_{32} & y_{33} & y_{34} & y_{35} & y_{36} \\ y_{41} & y_{42} & y_{43} & y_{44} & y_{45} & y_{46} \\ y_{51} & y_{52} & y_{53} & y_{54} & y_{55} & y_{56} \\ y_{61} & y_{62} & y_{63} & y_{64} & y_{65} & y_{66} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} \tag{2.29}$$

Partitioning the above matrix,

$$\begin{bmatrix} [I_X] \\ [I_Y] \end{bmatrix} = \begin{bmatrix} [Y_{XX}] & [Y_{XY}] \\ [Y_{YX}] & [Y_{YY}] \end{bmatrix} \begin{bmatrix} [V_X] \\ [V_Y] \end{bmatrix} \tag{2.30}$$

where,

$$[I_X] = [I_1 I_2 I_3]^T$$
$$[I_Y] = [I_4 I_5 I_6]^T \tag{2.31}$$

$$[Y_{XX}] = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}, \; [Y_{XY}] = \begin{bmatrix} y_{14} & y_{15} & y_{16} \\ y_{24} & y_{25} & y_{26} \\ y_{34} & y_{35} & y_{36} \end{bmatrix}$$

$$[Y_{YX}] = \begin{bmatrix} y_{41} & y_{42} & y_{43} \\ y_{51} & y_{52} & y_{53} \\ y_{61} & y_{62} & y_{63} \end{bmatrix}, \; [Y_{YY}] = \begin{bmatrix} y_{44} & y_{45} & y_{46} \\ y_{54} & y_{55} & y_{56} \\ y_{64} & y_{65} & y_{66} \end{bmatrix} \tag{2.32}$$

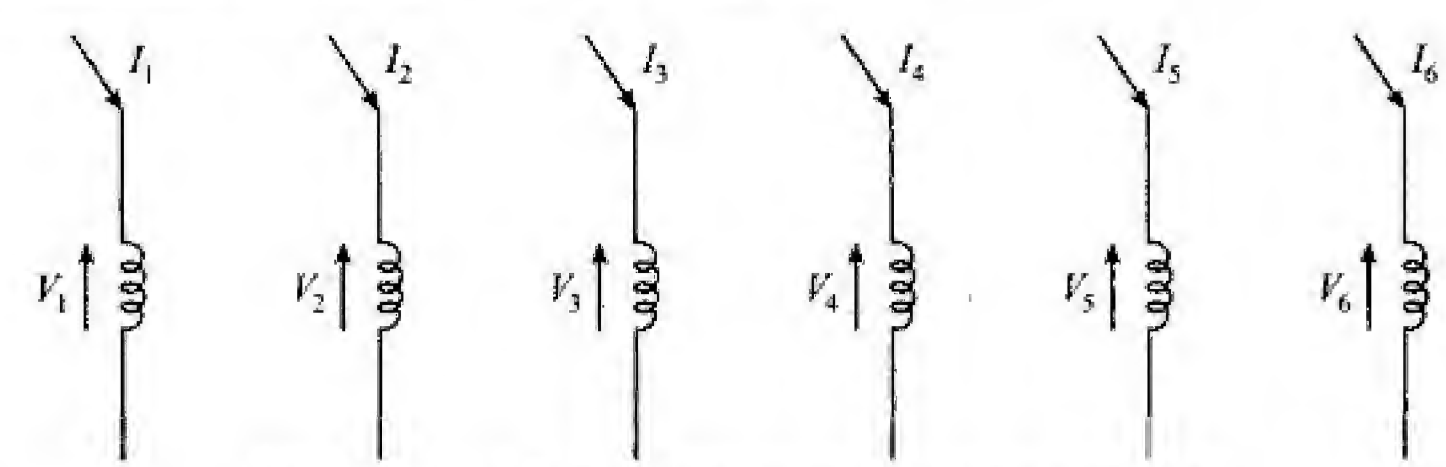


Fig. 2.17 Six coils with nodal currents and branch voltages.

Thus, the six coils can be represented by two *compound coils* (X and Y) consisting of three individual admittances. This has been shown in Fig. 2.18.

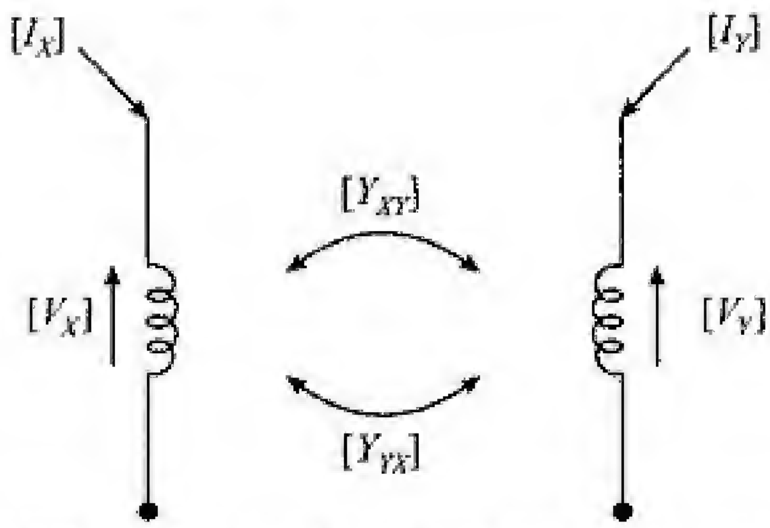


Fig. 2.18 Equivalent admittance coils and equivalent node currents and voltages.

Hence, equation (2.29) is finally represented in completed compound form as

$$\begin{bmatrix} [I_X] \\ [I_Y] \end{bmatrix} = \begin{bmatrix} [Y_{XX}] & [Y_{XY}] \\ [Y_{YX}] & [Y_{YY}] \end{bmatrix} \begin{bmatrix} [V_X] \\ [V_Y] \end{bmatrix}$$

or,

$$\begin{bmatrix} [I_X] \\ [I_Y] \end{bmatrix} = \begin{bmatrix} [Y_{XX}] & [Y_{XY}] \\ [Y_{XY}]^T & [Y_{YY}] \end{bmatrix} \begin{bmatrix} [V_X] \\ [V_Y] \end{bmatrix} \tag{2.33}$$

since

$$[Y_{XY}] = [Y_{XY}]^T$$

For a three-phase transformer, assuming y_p and y_s as the self-admittances of primary and secondary coils (being equivalent to $y_{11}, y_{22}, y_{33}, \ldots$) y'_m the mutual admittance between primary coils, y'''_m the mutual admittance between the secondary coils and y''_m the mutual admittance between primary and secondary coils on different cores, the nodal currents in the coils may be linked with the branch voltages as

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} y_p & y'_m & y'_m & -y_m & y''_m & y''_m \\ y'_m & y_p & y'_m & y''_m & -y_m & y''_m \\ y'_m & y'_m & y_p & y''_m & y''_m & -y_m \\ -y_m & y''_m & y''_m & y_s & y'''_m & y'''_m \\ y''_m & -y_m & y''_m & y'''_m & y_s & y'''_m \\ y''_m & y''_m & -y_m & y'''_m & y'''_m & y_s \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} \tag{2.34}$$

The primed values are effectively zero for three single phase units.

If transformer connections are to be incorporated, the Y_{BUS} is formed utilising the relation

$$[Y_{BUS}] = [c]^T[Y_{PRM}]\,[c] \tag{2.35}$$

where $[c]$ is the *connection matrix*, and $[Y_{PRM}]$ is the *primitive matrix*.

Table 2.1 represents the $[Y_{BUS}]$ matrix for common transformer connections assuming three individual units so that primed values vanish.

TABLE 2.1: Elements of transformer admittance matrices

Type of connection	Y_{XX} *(self, pri.)*	Y_{YY} *(self, sec.)*	Y_{XY} *or* Y_{YX} *(mutual)*
Y / Y (neutral solidly grounded)	Y_A	Y_A	$-Y_A$
Y Δ (Y-side neutral solidly grounded)	Y_A	Y_B	Y_C
Y Y	$Y_B/3$	$Y_B/3$	$-Y_C/3$
Δ Δ	Y_B	Y_B	$-Y_B$

where,

$$\left.\begin{aligned} Y_A &= \begin{bmatrix} y & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & y \end{bmatrix} \\ Y_A,\ Y_B &= \begin{bmatrix} 2y & -y & -y \\ -y & 2y & -y \\ -y & -y & 2y \end{bmatrix} \\ Y_C &= \begin{bmatrix} -y & y & 0 \\ 0 & -y & y \\ y & 0 & -y \end{bmatrix} \end{aligned}\right\} \tag{2.36}$$

It may be noted that any two winding transformers may be represented by two compound-linked admittances. The current voltage relationship is given by

$$\begin{bmatrix} I_{pri} \\ I_{sec} \end{bmatrix} = \begin{bmatrix} Y_{PP} & Y_{PS} \\ Y_{SP} & Y_{SS} \end{bmatrix} \begin{bmatrix} V_{pri} \\ V_{sec} \end{bmatrix} \tag{2.37}$$

where $[Y_{PS}] = [Y_{SP}]^T$ and is the same as equation (2.33). Y_{PP}, Y_{SS} indicate self admittances at primary and secondary buses while Y_{PS} and Y_{SP} represent the mutual admittance. In case the off-nominal tap ratio is to be included, Section 2.5 is to be followed in conjunction with the above modelling.

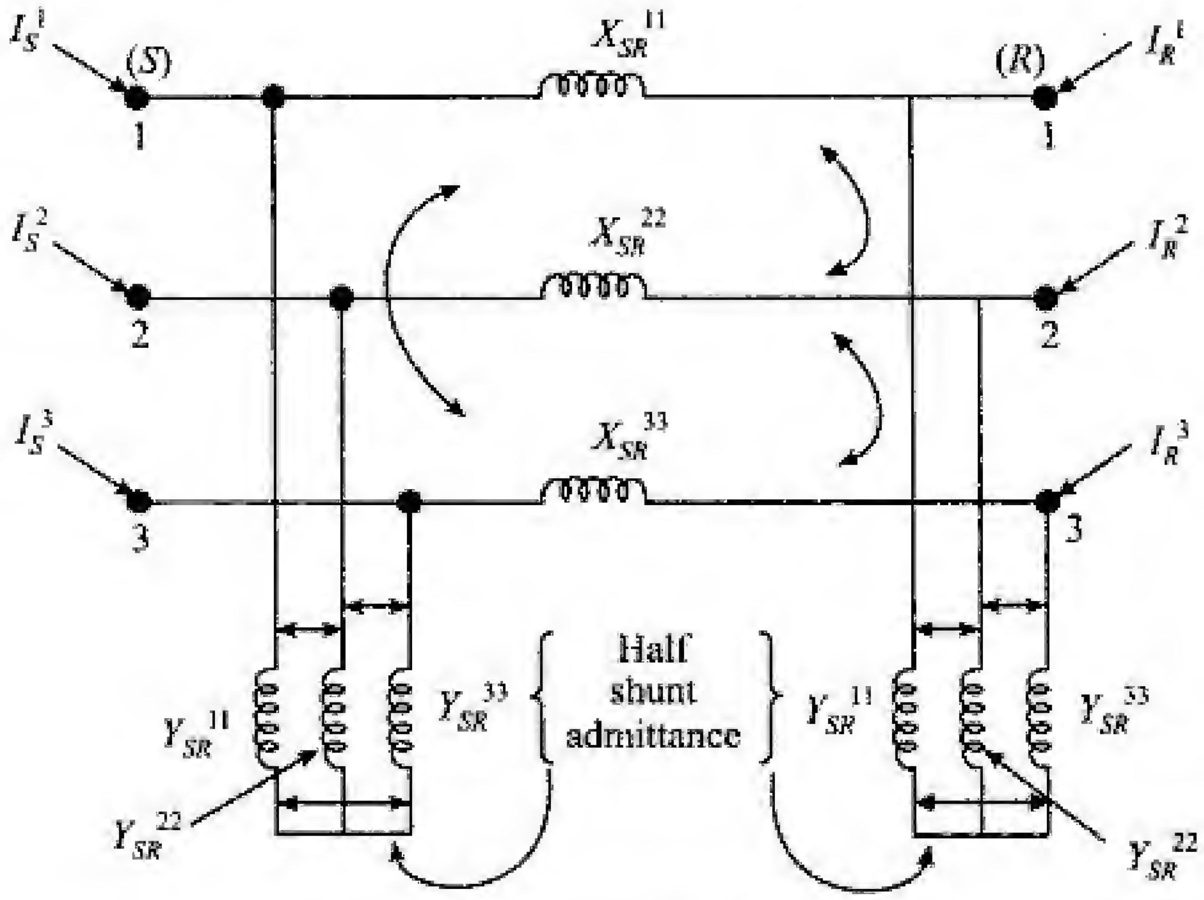
2.7 MODELLING OF THREE-PHASE SINGLE CIRCUIT TRANSMISSION LINE

Figure 2.19 represents the lumped parameter representation of a single circuit three-phase transmission line with suffix (S) as sending end and (R) as receiving end and superscripts 1, 2, 3 the phases. X is the series reactance while Y is the shunt admittance.

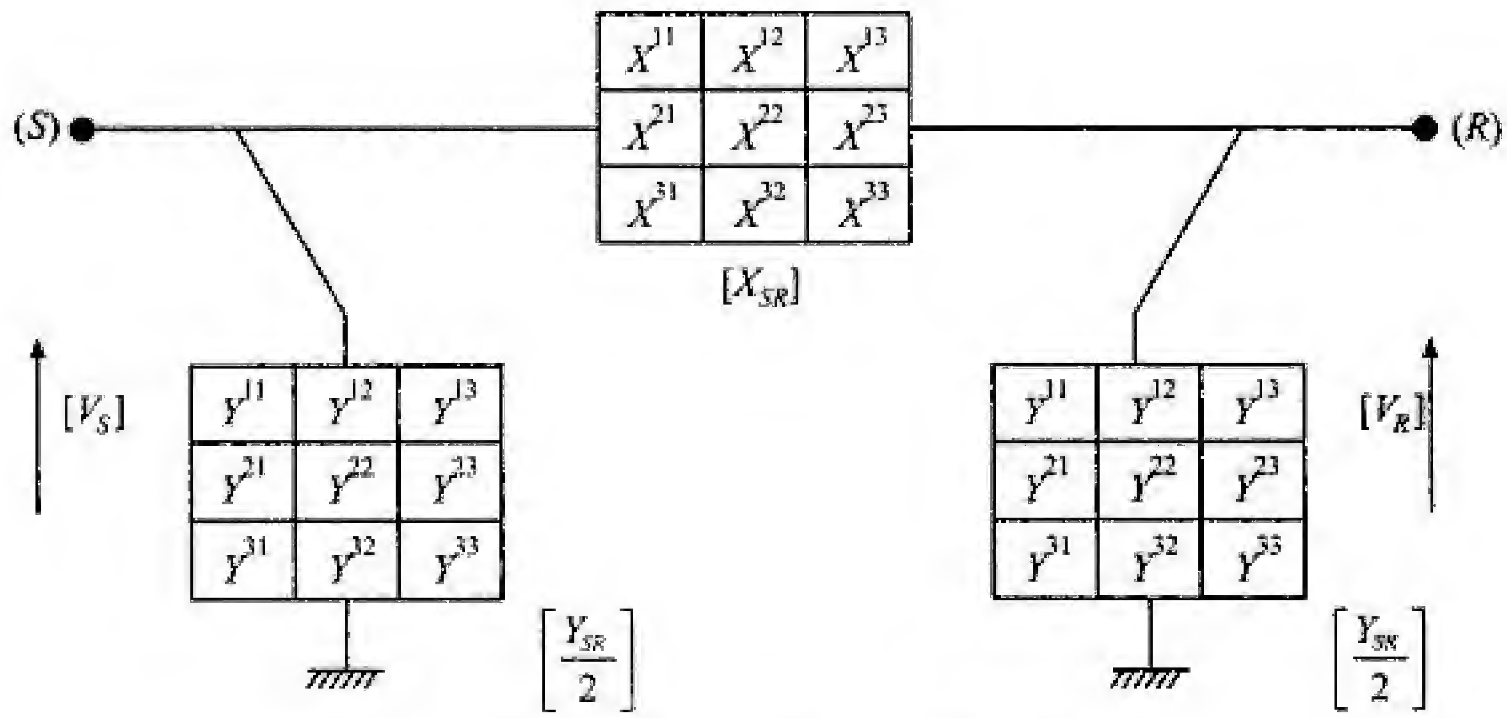
$$\begin{bmatrix}[I_S]\\ [I_R]\end{bmatrix}=\begin{bmatrix}[X_{SR}]^{-1}+{[Y_{SR}]}/{2} & -[X_{SR}]^{-1}\\ -[X_{SR}]^{-1} & [X_{SR}]^{-1}+{[Y_{SR}]}/{2}\end{bmatrix}\begin{bmatrix}[V_S]\\ [V_R]\end{bmatrix} \tag{2.38}$$

(6×1) matrix $\qquad$ (6×6) matrix $\qquad$ (6×1) matrix

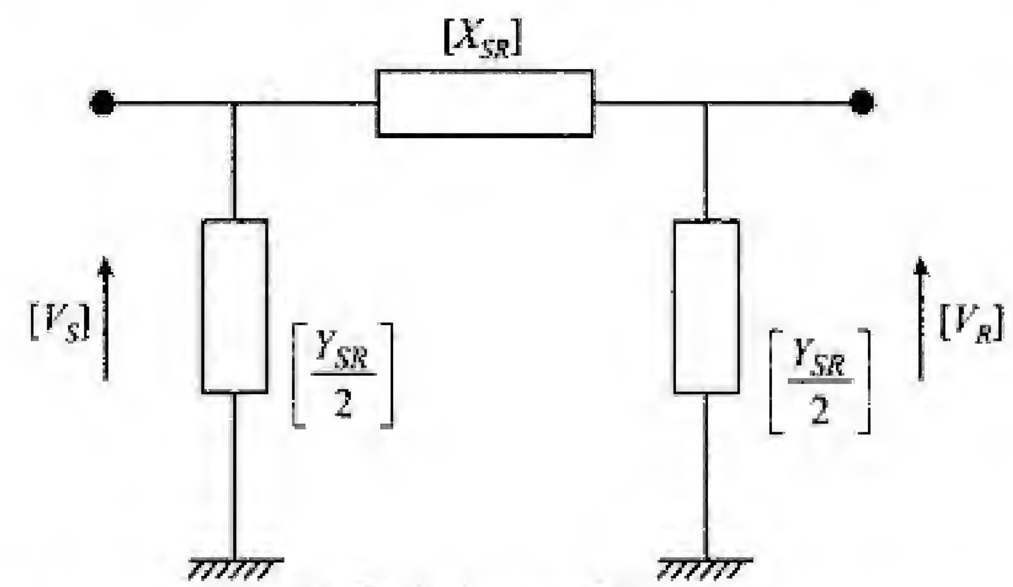
These primitive matrices in Fig. 2.19(a) can be represented by equivalent matrices in Figs. 2.19(b) and 2.19(c) utilising the techniques as described earlier. The current voltage relations are described in a generalised manner as under.



(a) Line model



(b) Equivalent modelling of components.



(c) Equivalent model

Fig. 2.19 Modelling of three-phase transmission line using π model.

2.8 MODELLING OF PAIR OF THREE-PHASE MUTUALLY COUPLED TRANSMISSION LINES

Figure 2.20 represents the equivalent of each of the two mutually coupled lines utilising π model. Here each admittance matrix element is a [3 × 3] matrix; the currents and voltages are related by the following relation:

$$\underset{\substack{[12\times 1]\\ \text{matrix}}}{\begin{bmatrix} I_{S_1} \\ I_{R_1} \\ I_{S_2} \\ I_{R_2} \end{bmatrix}} = \underset{\substack{[12\times 12]\\ \text{matrix}}}{\begin{bmatrix} [Y_{11}+Y_{33}] & [Y_{12}+Y_{34}] & [-Y_{11}] & [-Y_{12}] \\ [Y_{12}^T+Y_{34}^T] & [Y_{22}+Y_{44}] & [-Y_{12}] & [-Y_{22}] \\ [-Y_{11}] & [-Y_{12}] & [Y_{11}+Y_{55}] & [Y_{12}+Y_{56}] \\ [-Y_{12}^T] & [-Y_{22}] & [Y_{12}^T+Y_{56}^T] & [Y_{22}+Y_{66}] \end{bmatrix}} \underset{\substack{[12\times 1]\\ \text{matrix}}}{\begin{bmatrix} V_{S_1} \\ V_{R_1} \\ V_{S_2} \\ V_{R_2} \end{bmatrix}} \qquad (2.39)$$

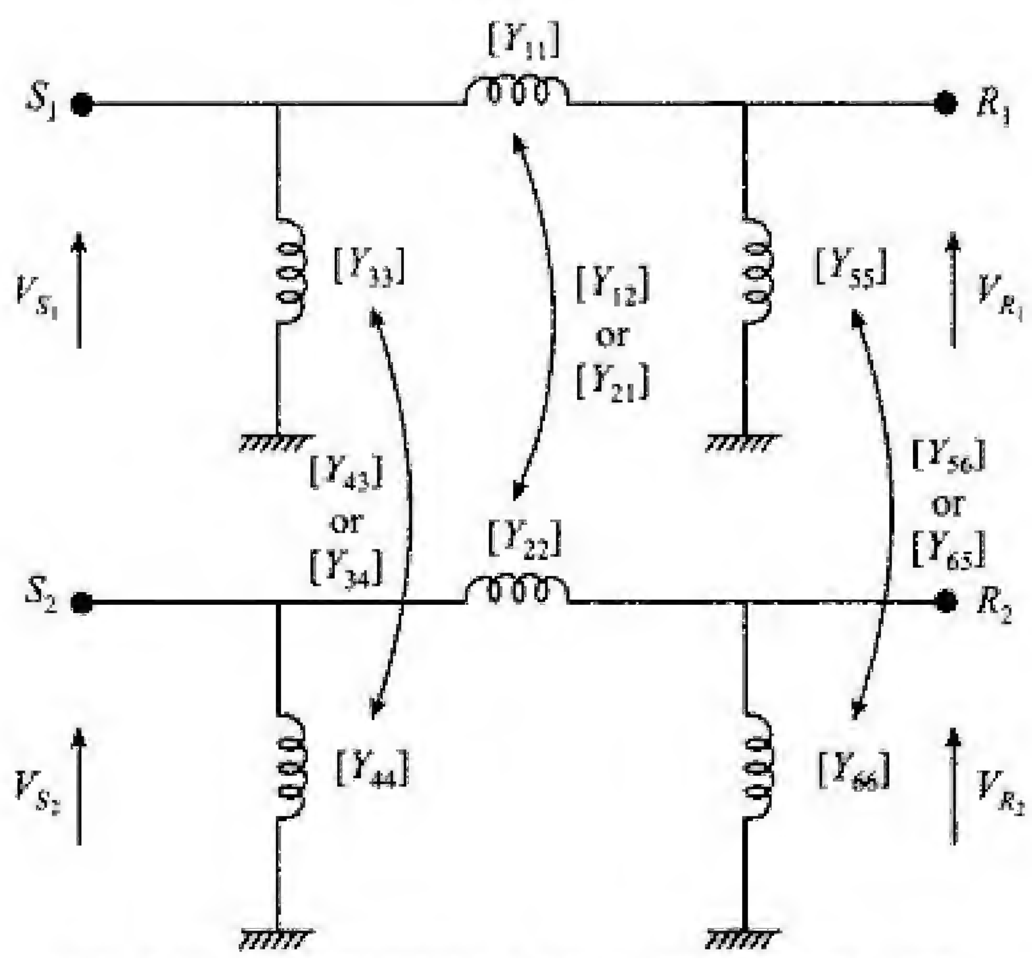


Fig. 2.20 Representation of two mutually coupled lines.

Figures 2.21(a) and 2.21(b) represent the compound admittance form of the matrix representation shown in equation (2.39) corresponding to Fig. 2.20.

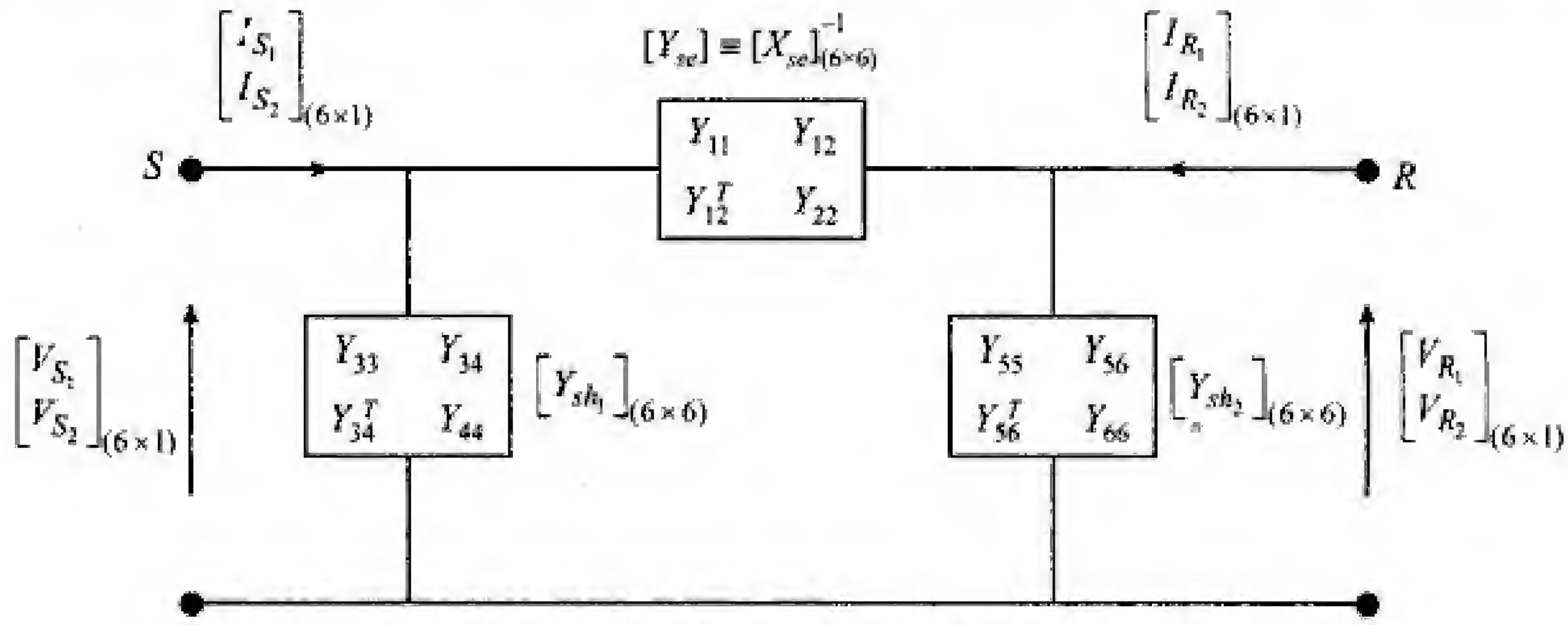


Fig. 2.21(a) (6 × 6) Matrix representation of Fig. 2.20.

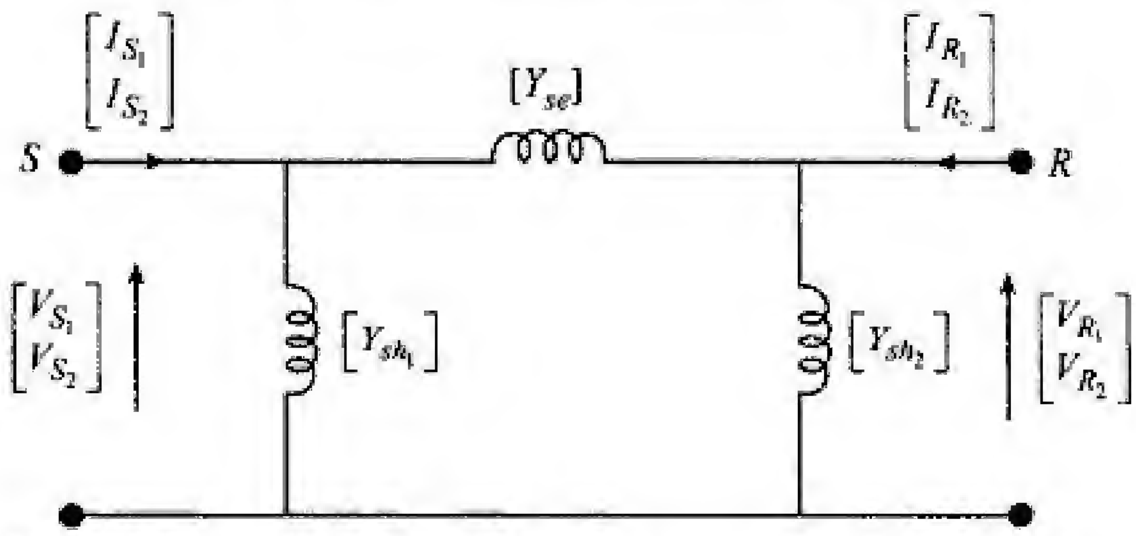


Fig. 2.21(b) (6 × 6) Compound matrix representation of Fig. 2.21(a).

Thus, the mutually coupled lines are finally represented as

$$\begin{bmatrix} \begin{bmatrix} I_{S_1} \\ I_{S_2} \end{bmatrix} \\ \begin{bmatrix} I_{R_1} \\ I_{R_2} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \left[\left[X_{se}\right]^{-1} + \left[Y_{sh_1}\right]\right] & -\left[X_{se}\right]^{-1} \\ -\left[X_{se_1}\right]^{-1} & \left[\left[X_{se}\right]^{-1} + \left[Y_{sh_2}\right]\right] \end{bmatrix} \begin{bmatrix} \begin{bmatrix} V_{S_1} \\ V_{S_2} \end{bmatrix} \\ \begin{bmatrix} V_{R_1} \\ V_{R_2} \end{bmatrix} \end{bmatrix} \tag{2.40}$$

[12 × 1] matrix   [12 × 12] matrix   [12 × 1] matrix

2.9 MODELLING OF A SHUNT CAPACITOR/INDUCTOR

For effective reactive power and bus voltage control, shunt capacitors and/or reactors are frequently used. Figure 2.22 represents a static shunt capacitor bank with its compound admittance representation.

As there is no coupling between the components of each phase, the Y matrix only contains the *diagonal* elements. In a similar way, the modelling of a shunt reactor can be done.

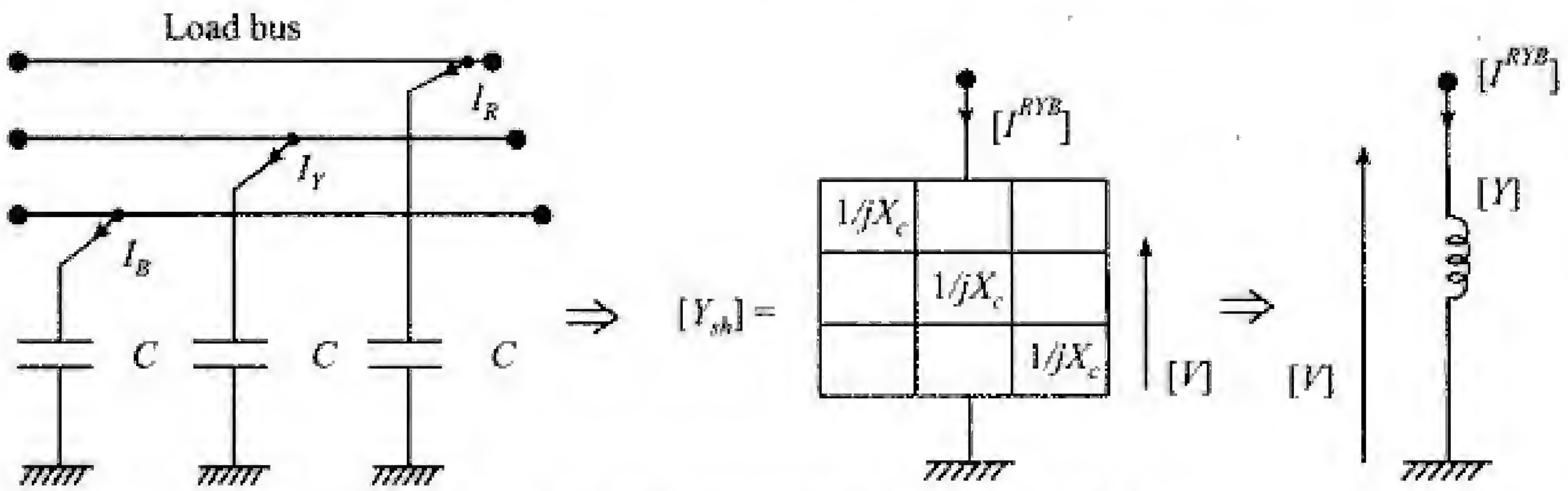


Fig. 2.22 Model representation of shunt capacitor.

2.10 MODELLING OF A SERIES CAPACITOR

The capacitive element is connected in series with the line and between two buses. The admittance matrix for this system has been written as

$$[Y] = \begin{bmatrix} [Y_{SE}] & -[Y_{SE}] \\ -[Y_{SE}] & [Y_{SE}] \end{bmatrix} \tag{2.41}$$

The shunt element does not exist.

Here,
$$[Y_{SE}] = \begin{bmatrix} 1/jX_{se} & & \\ & 1/jX_{se} & \\ & & 1/jX_{se} \end{bmatrix}$$

Figure 2.23 represents the modelling.

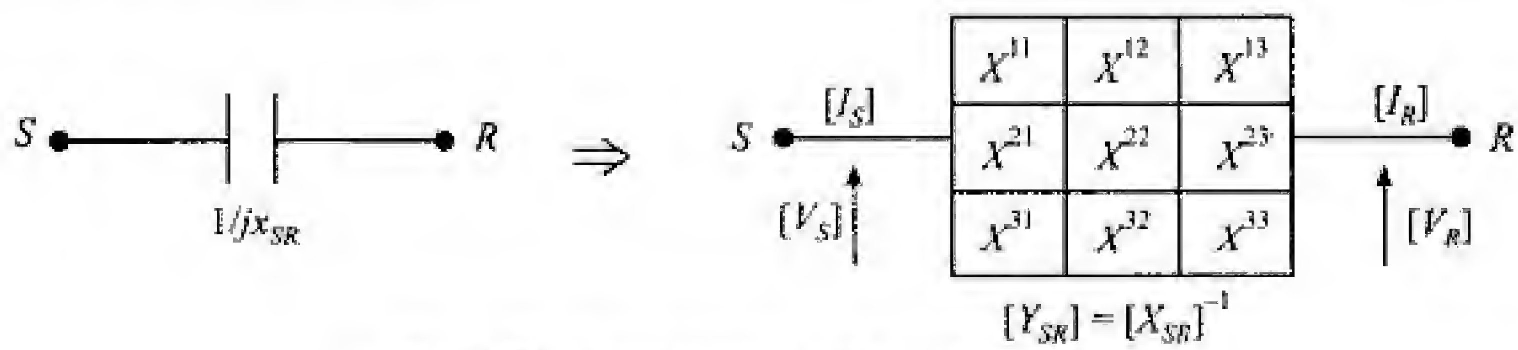


Fig. 2.23 Representation of series capacitor.

2.11 MODELLING OF STATIC VAR COMPENSATOR (SVC)

Let B_{SVC} be the shunt susceptance of the SVC corresponding to the MVAR loading of it. It is then added to the susceptance at the busbar. The total susceptance is given by B. A reduction in the controlling voltage V will cause the desired susceptance B to increase, causing MVAR output of SVC enhanced.

The SVC injected current into the bus is then given by

$$I = -VY \tag{2.42}$$

Here,
$$Y = G + jB \tag{2.43}$$

[G may be assumed to be zero here.]

The MVA output of the SVC is given by

$$S = VI^*_{SVC}$$

and
$$Q = |V|^2 (B_{SVC} + B_{Bus}) \tag{2.44}$$

2.12 MODELLING OF AN INDUCTION MOTOR

The slip of an induction motor is expressed in terms of its rotor speed and synchronous speed as

$$s = \frac{n_0 - n_r}{n_0} \times 100 \text{ (in percentage)} \tag{2.45}$$

The equation of motion for the shaft power is given by

$$\frac{d\delta}{dt} = (T_m - T_e)/2H \tag{2.46}$$

where H is the inertia constant, T_m the mechanical torque and T_e the electrical torque. However, the mechanical torque is equivalent to load torque and is commonly expressed as

$$T_m \equiv T_L \propto (\text{speed})^k \tag{2.47}$$

k is an exponent and is 1 for *fan* type of loads and 2 for *pump* type of loads. The electrical torque is given by

$$T_e = \text{Real}\,[EI^*]/2\pi f_0 \tag{2.48}$$

where E is the air gap voltage, I the stator current input and f_0 the base frequency.

The transient reactance X' has been defined as the *apparent reactance* seen through the equivalent circuit when the rotor is held locked and the slip is unity.

Thus, from Fig. 2.24, the equivalent circuit during transient operation, we obtain

$$X' = X_s + \frac{X_r X_m}{(X_r + X_m)} \tag{2.49}$$

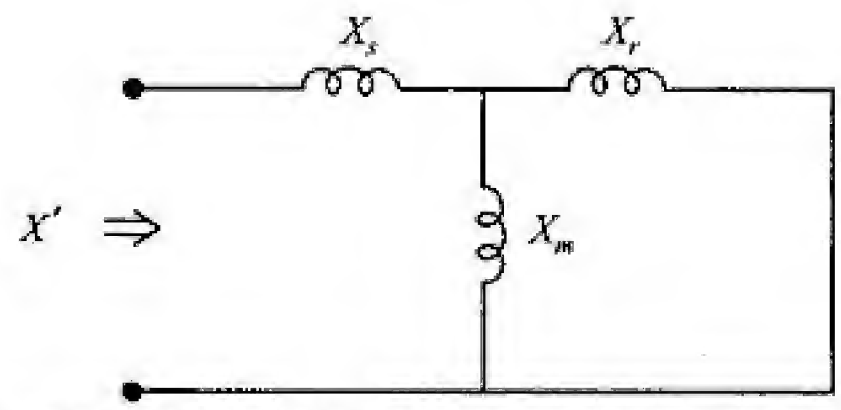


Fig. 2.24 Equivalent circuit of induction motor during transient state of operation.

The transient model of the induction motor has been assumed by a Thevein equivalent circuit of a voltage E' behind the transient reactance X' while the transient time constant T_0' is given by

$$T_0' = \frac{X_r + X_m}{2\pi \;\; f_0 R_r} \tag{2.50}$$

and the open circuit reactance X_0 is given by

$$X_0 = X_s + X_m \tag{2.51}$$

Assuming the stator resistance to be R_s, the governing equations of the model are given by

$$V_{re} - E_{re}' = I_{re} R_s - I_{im} X' \tag{2.52}$$

$$V_{im} - E_{im}' = I_{im} R_s + I_r X' \tag{2.53}$$

Here, the reactances are assumed to be unaffected by the rotor position and the model is analysed in the real (*re*) and imaginary (*im*) axes for the network.

The system model is described as

$$\begin{bmatrix} I_{re} \\ I_{im} \end{bmatrix} = \frac{1}{R_s^2 + X_s'^2} \begin{bmatrix} R_s & X' \\ -X' & X' \end{bmatrix} \begin{bmatrix} V_{re} - E'_{re} \\ V_{im} - E'_{im} \end{bmatrix} \tag{2.54}$$

The rotor reactance does not vary much with the variation of rotor resistance with slip, provided the saturation effect is neglected. Transient reactance X' varies with rotor reactance only and hence is almost constant at any slip.

The induction machine can also be modelled in terms of d–q axis as follows: the (p.u) voltage equations for a single rotor winding induction motor in d–q coordinate are given by

$$V_{ds} = R_s \ i_{ds} - \omega\psi_{qs} + \dot{\psi}_{ds} \tag{2.55}$$

$$V_{qs} = R_s \ i_{qs} + \omega\psi_{qs} + \dot{\psi}_{ds} \tag{2.56}$$

$$V_{dr} = R_r i_{dr} - \omega s\psi_{dr} + \dot{\psi}_{qr} \tag{2.57}$$

$$V_{qr} = R_r i_{qr} + \omega s\psi_{dr} + \dot{\psi}_{qr} \tag{2.58}$$

The corresponding flux linkages are

$$\psi_{ds} = L_s i_{ds} + L_m i_{dr} \tag{2.59}$$

$$\psi_{qs} = L_s i_{qs} + L_m i_{qr} \tag{2.60}$$

$$\psi_{dr} = L_m i_{ds} + L_r i_{dr} \tag{2.61}$$

$$\psi_{qr} = L_s i_{qs} + L_r i_{qr} \tag{2.62}$$

Neglecting stator transients and assuming the rotor short-circuited

$$\dot{\psi}_{ds} = 0 \text{ and } \dot{\psi}_{qs} = 0 \tag{2.63}$$

$$V_{dr} = 0 \text{ and } V_{qr} = 0 \tag{2.64}$$

$$V'_d = \left(\frac{-X_m}{L_r}\right)\psi_{qr}, \quad V'_q = \left(\frac{-X_m}{L_r}\right)\psi_{dr} \tag{2.64a}$$

$$X'_s = X_s - X_m^2 / X_r \tag{2.65}$$

Substituting equation (2.63) in (2.55), $\dot{\psi}_{ds}$ and $\dot{\psi}_{qs}$ are eliminated. ψ_{ds} and ψ_{qs} are also eliminated by substitution of equations (2.59) and (2.60) in (2.55) and (2.56); i_{dr} and i_{qr} are then eliminated using rearranged equations (2.61) and (2.62). ψ_{dr} and ψ_{qr} are eliminated using equation (2.64a). Using equation (2.65) in the final form, the resulting equation is

$$\begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix} = \begin{bmatrix} R_s & -X'_s \\ X'_s & R_s \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \begin{bmatrix} V'_{ds} \\ V'_{qs} \end{bmatrix} \tag{2.66}$$

or,

$$[V_s] = \left[R_s + jX'_s \right] [I_s] + [V'_m] \tag{2.67}$$

The state equations can be developed by substituting the value of V_{dr} and V_{qr} from equations (2.64) in (2.57) and (2.58). Substitution for i_{dr} and i_{qr} is done from equations (2.61) and (2.62). In its new form, ψ_{dr} and ψ_{qr} are replaced by V_q' and V_d' using equation (2.64a). In its final form, the derivative of V_d' and V_q' are taken to give:

$$\dot{V}_d' = \left(\frac{R_r}{L_r}\right)V_d' + s\omega V_q' - (L_m X_m R_r / L_r^2) i_{qs} \tag{2.68}$$

$$\dot{V}_q' = -s\omega V_d' - (R_r/L_r)V_q' + (L_m X_m R_r / L_r^2) i_{ds} \tag{2.69}$$

Expressing equations (2.68) and (2.69) in phasor form,

$$\dot{V}_m' = \left[-\frac{R_r}{X_r} - js\right]\omega V_m' + j(R_r/X_r)(X_s - X_s')\omega I_s \tag{2.70}$$

At steady state $\dot{V}_m' = 0$. Assuming $|I_s| = 1.0, X_r = X_s$,

$$I_s = \cos\theta - j\sin\theta \tag{2.71}$$

And from equation (2.70),

$$V_m' = jR_r(X_s - X_s')(\cos\theta - j\sin\theta)/(R_r + jsX_s) \tag{2.72}$$

Rationalising and taking the ratio of imaginary to real parts,

$$\frac{V_q'}{V_d'} = \frac{R_r\cos\theta - sX_s\sin\theta}{R_r\sin\theta - sX_s\cos\theta} \tag{2.73}$$

Similarly, substitution of equation (2.71) in (2.67) with $V_s = 1$, yields

$$\frac{V_q'}{V_d'} = \frac{R_s'\sin\theta - X_s'\cos\theta}{\left(1 - R_s'\cos\theta - X_s'\sin\theta\right)} \tag{2.74}$$

[θ is the motor p.f. angle]

2.13 POWER NETWORK MODELLING

Let $\quad I_i$ = injected current at node i ($i = 1, 2, \ldots, n$)

$V_i = |V_i| e^{j\delta_i}$ (voltage at node i)

The current I_i can be expressed as a function of the voltages. Thus,

$$I_i = y_{ii}\ V_i + \sum_{j\in\alpha(i)} |y_{ij}|(V_i - V_j),\ i = 1, 2, \ldots, n$$

$\alpha(i)$ designates the subset of the nodes connected to node i and

$$y_{ii} = \sum_{j\in\alpha(i)} y_{ij}';\ y_{ij} = \frac{1}{z_{ij}} = |y_{ij}|\ e^{(-j\theta_{ij})\dagger}$$

† A line or cable connecting two buses i and j can be modelled by a "pi" equivalent circuit having series impedance z_{ij} and shunt admittance y_{ij}, where $z_{ij} = r_{ij} + ix_{ij}$ and $y_{ij} = g_{ij} + jh_{ij}$. Since the "$pi$" circuit of the line is symmetrical, we assume $g_{ij} = g_{ji} = 0$; $h_{ij} = h_{ji} = \dfrac{\omega C_{ij}}{2}$.

In general form, the preceding equations can be written as

$$[I]=[Y][V], \tag{2.75}$$

where $Y_{ij}=G_{ij}+jH_{ij};\ Y_{ii}=G_{ii}+jH_{ii}$ and $G_{ij}=-\frac{r_{ij}}{z_{ij}^2};\ H_{ij}=\frac{x_{ij}}{z_{ij}^2}$

$$G_{ii}=\sum_{j\in\alpha(i)}(g_{ij}-G_{ij}),\ H_{ii}=\sum_{j\in\alpha(i)}\left(h_{ij}-H_{ij}\right)$$

Also, at node i, $P_i-jQ_i=V_i^*I_i$

However, V_i^* is complex conjugate of V_i and hence

$$P_i=\text{Real}\left\{V_i^*\left[\ y_{ii}V_i+\sum_{k\in\alpha(i)}y_{ij}(V_i-V_j)\ \right]\right\} \tag{2.76}$$

and

$$Q_i=\text{Im}\left\{V_i^*\left[\ y_{ii}V_i+\sum_{k\in\alpha(i)}y_{ij}(V_i-V_j)\ \right]\right\} \tag{2.77}$$

Simplification yields

$$P_i=V_i^2\sum_{j\in\alpha(i)}(y_{ij}\cos\ \theta_{ij}+g_{ij})-V_i\sum_{j\in\alpha(i)}V_jy_{ij}\cos\ (\theta_{ij}+\delta_i-\delta_j)=\sum_{j\in\alpha(i)}P_{ij} \tag{2.78}$$

and

$$Q_i=V_i^2\sum_{j\in\alpha(i)}(y_{ij}\sin\theta_{ij}-h_{ij})-V_i\sum_{j\in\alpha(i)}V_jy_{ij}\sin\ (\theta_{ij}+\delta_i-\delta_j)=\sum_{j\in\alpha(i)}Q_{ij} \tag{2.79}$$

where P_{ij} and Q_{ij} denote the active and reactive powers through the line connecting the ith and jth nodes.

Obviously,

$$P_{ij}=V_i^2\left(y_{ij}\cos\theta_{ij}+g_{ij}\right)-V_iV_jy_{ij}\cos\left(\theta_{ij}+\delta_i-\delta_j\right) \tag{2.80}$$

and

$$P_{ji}=V_j^2\left(y_{ji}\cos\theta_{ij}+g_{ij}\right)-V_iV_jy_{ij}\cos\left(\theta_{ij}+\delta_j-\delta_i\right) \tag{2.81}$$

$$Q_{ij}=V_i^2\left(y_{ij}\sin\theta_{ij}-h_{ij}\right)-V_iV_jy_{ij}\sin\left(\theta_{ij}+\delta_i-\delta_j\right) \tag{2.82}$$

$$Q_{ji}=V_j^2\left(y_{ij}\sin\theta_{ij}-h_{ij}\right)-V_iV_jy_{ij}\sin\left(\theta_{ij}+\delta_j-\delta_i\right) \tag{2.83}$$

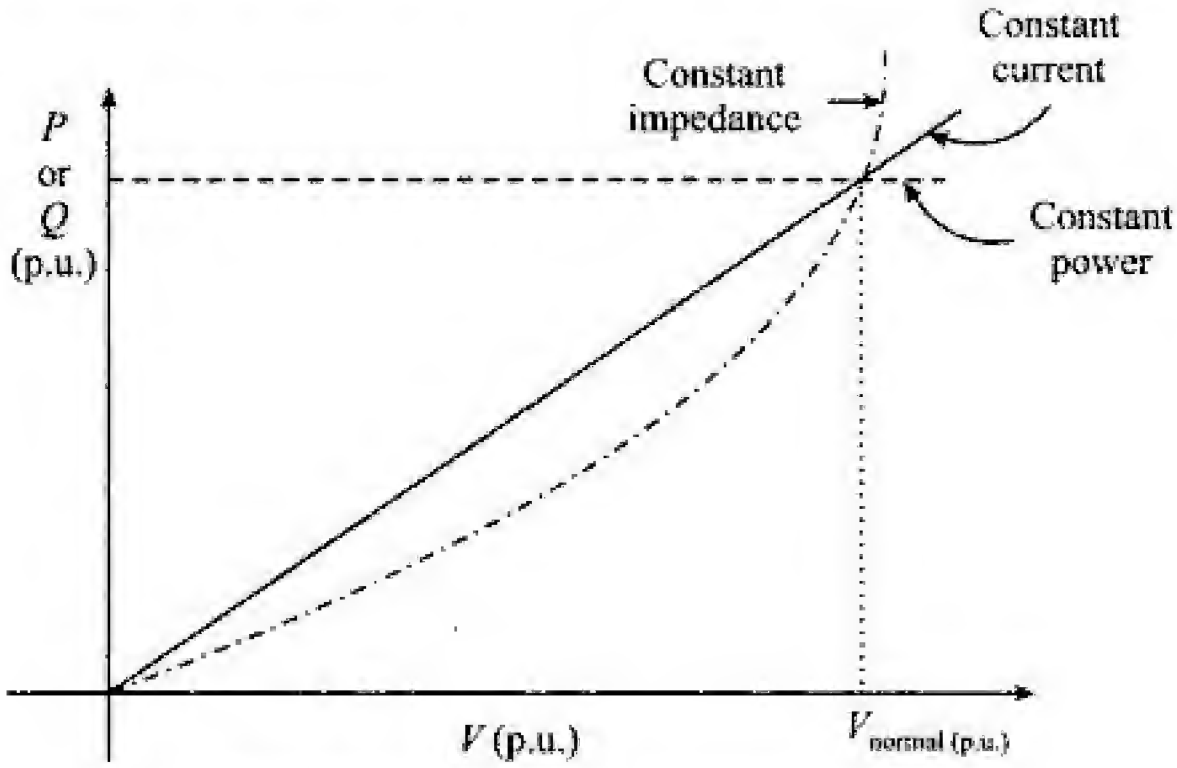
(*Note*: Conventionally $g_{ij}=0$ and $h_{ij}=\frac{\omega C_{ij}}{2}$. It may be noted that power flow equation have been dealt in detail in Chapter 4 where we replaced the notation of susceptance h by b.)

2.14 MODELLING OF LOAD

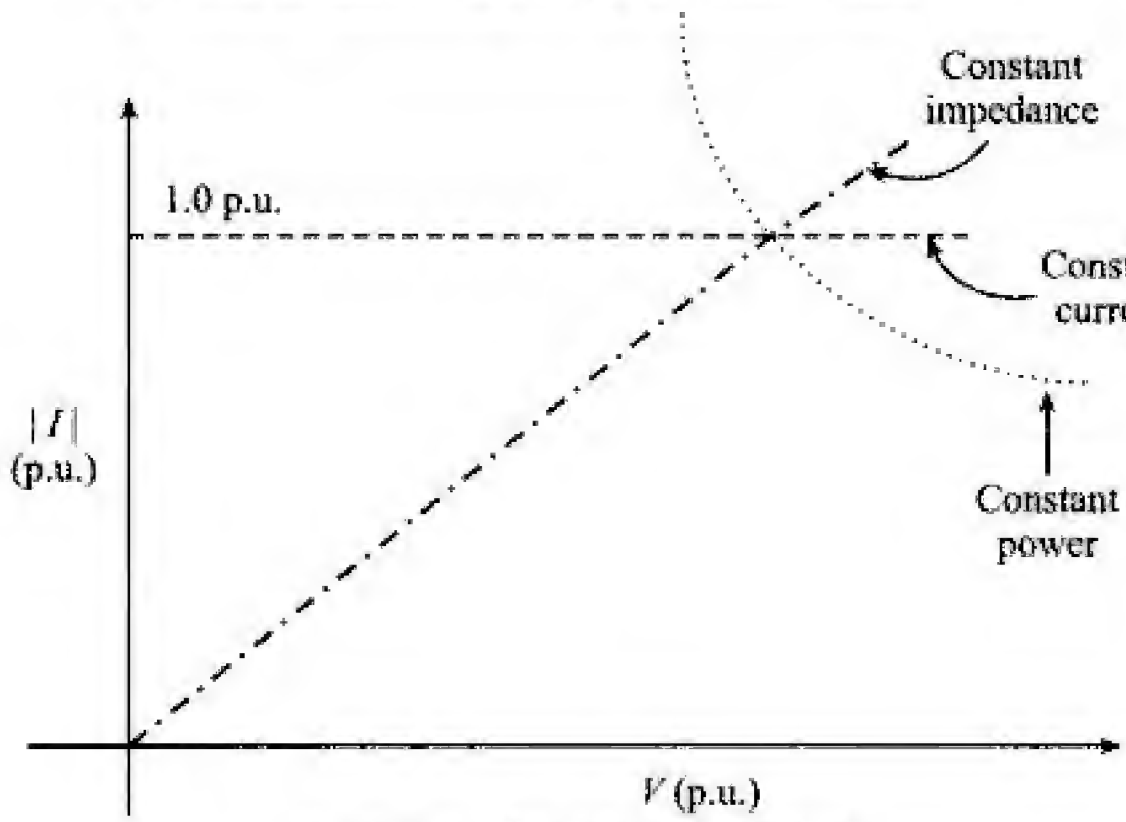
Load drawn by the consumers is the toughest parameter to be assumed scientifically. The magnitude of load, in fact, changes continually so that the load forecasting problem is truly a statistical one. The loads are generally composed of

Lighting and Heating	20–30%
Induction Motors	55–75%
Synchronous Motors	05–15%

The loads are mostly of composite character and it is prudent to represent them by P–V or Q–V characteristics (Figs. 2.25(a) and (b)).



(a) General characteristics of power system loads.



(b) Current characteristics of loads.

Fig. 2.25 Characteristics of loads.

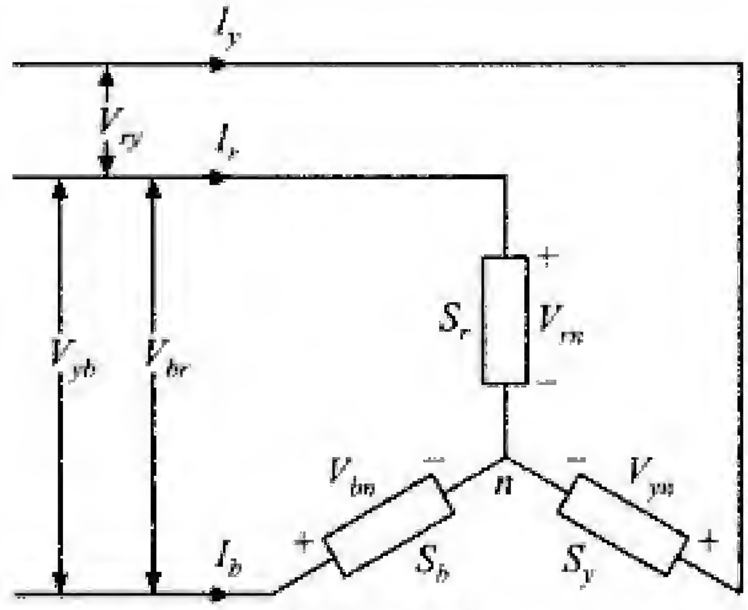
Loads in power system can be represented in the following categories:

(i) Constant real and reactive power (PQ type)
(ii) Constant current type
(iii) Constant impedance type, and
(iv) Mixed type.

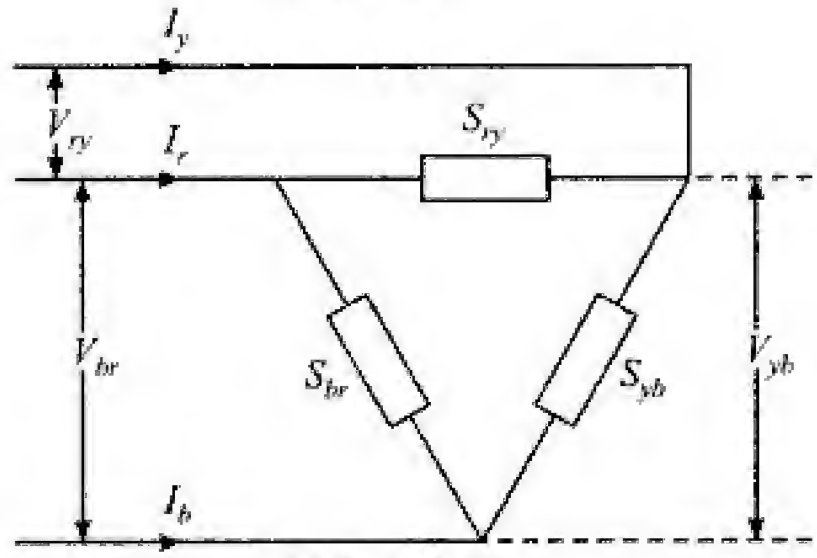
The actual loads may be connected to single phase or three phase power supply (balanced or unbalanced). If the load configuration is star, loads are connected between each phase and neutral; for delta configuration, the loads in each phase are connected across phases.

The load modelling is required primarily for load flow studies. The models are developed as per defined categories as mentioned above and are defined by a complex power per phase and either line to neutral (for star load) or a line to line voltage (for delta load). The units for complex power can be VA or p.u. VA while that of voltage can be volts or p.u. volts.

For both the star and delta connected loads, the basic requirement is to find the load component of the line currents entering the loads. Let us now describe the load models first for star load (Fig. 2.26(a)) and then for delta load (Fig. 2.26(b)).



(a) Star load



(b) Delta load

Fig. 2.26 Three phase load modelling.

For star load

Let us note down first the specified complex powers and voltages as follows:

Phase "r": $|S_r|\angle\phi_r = P_r + jQ_r$

and $|V_{rn}|\angle\delta_r$

Phase "y": $|S_y|\angle\phi_y = P_y + jQ_y$

and $|V_{yn}|\angle\delta_y$

Phase "b": $|S_b|\angle\phi_b = P_b + jQ_b$

and $|V_{bn}|\angle\delta_b$

(i) *Constant real and reactive power loads:*

$$I_r = \left[\frac{S_r}{V_{rn}}\right]^* = \frac{|S_r|}{|V_{rn}|}\angle\delta_r - \phi_r = \frac{P_r - jQ_r}{V_{rn}^*} = |I_r|\angle\alpha_r \tag{2.84a}$$

$$I_y = \left[\frac{S_y}{V_{yn}}\right]^* = \frac{|S_y|}{|V_{yn}|}\angle\delta_y - \phi_y = \frac{P_y - jQ_y}{V_{yn}^*} = |I_y|\angle\alpha_y \tag{2.84b}$$

$$I_b = \left[\frac{S_b}{V_{bn}}\right]^* = \frac{|S_b|}{|V_{bn}|}\angle\delta_b - \phi_b = \frac{P_b - jQ_b}{V_{bn}^*} = |I_b|\angle\alpha_b \tag{2.84c}$$

In the iterative process, the complex power(s) remains constant while the voltages will change during each iteration until convergence is achieved. In practical life, induction motor is a constant power load.

(ii) *Constant impedance loads:* In this model, the load impedance is first obtained from the specified complex power and line to neutral voltage.

$$Z_r = \frac{|V_{rn}|^2}{S_r^*} = \frac{|V_{rn}|^2}{|S_r|}\angle\phi_r = \frac{|V_{rn}|^2}{P_r - jQ_r} = |Z_r|\angle\phi_r \tag{2.85a}$$

$$Z_y = \frac{|V_{yn}|^2}{S_y^*} = \frac{|V_{yn}|^2}{|S_y|}\angle\phi_y = \frac{|V_{yn}|^2}{P_y - jQ_y} = |Z_y|\angle\phi_y \tag{2.85b}$$

$$Z_b = \frac{|V_{bn}|^2}{S_b^*} = \frac{|V_{bn}|^2}{|S_b|}\angle\phi_b = \frac{|V_{bn}|^2}{P_b - jQ_b} = |Z_b|\angle\phi_b \tag{2.85c}$$

$$\therefore \quad I_r = \frac{V_{rn}}{Z_r} = \frac{|V_{rn}|}{|Z_r|}\angle\delta_r - \phi_r = |I_r|\angle\alpha_r \tag{2.86a}$$

$$I_y = \frac{V_{yn}}{Z_y} = \frac{|V_{yn}|}{|Z_y|}\angle\delta_y - \phi_y = |I_y|\angle\alpha_y \tag{2.86b}$$

$$I_b = \frac{V_{bn}}{Z_b} = \frac{|V_{bn}|}{|Z_b|}\angle\delta_b - \phi_b = |I_b|\angle\alpha_b \tag{2.86c}$$

In this model, impedance Z_r, Z_y and Z_b will remain constant and the line to neutral voltage will change during each iteration until convergence is achieved. Static impedances such as resistors, inductors and capacitors are constant impedance loads. Domestic loads and incandescent loads are constant impedance loads.

(iii) *Constant current loads:* In this model, the magnitudes of the currents are computed first and they are held constant while the angle of the voltage (δ) changes during each iteration. This makes the load power factor constant $\left[\cos\phi = \cos\left(\tan^{-1}\frac{Q}{P}\right)\right]$

$$I_r = |I_r|\angle\delta_r - \phi_r \tag{2.87a}$$

$$I_y = |I_y|\angle\delta_y - \phi_y \tag{2.87b}$$

$$I_b = |I_b|\angle\delta_b - \phi_b \tag{2.87c}$$

Rectifier loads and fluorescent lamps are constant current loads.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

(iii) *Constant current loads:* In this modelling concept, the magnitudes of the currents are computed first and then held constant while the angle (δ) changes during each iteration. The power factor of load remains constant.

$$I_{ry} = |I_{ry}| \angle \delta_{ry} - \phi_{ry} \tag{2.91a}$$

$$I_{yb} = |I_{yb}| \angle \delta_{yb} - \phi_{yb} \tag{2.91b}$$

$$I_{br} = |I_{br}| \angle \delta_{br} - \phi_{br} \tag{2.91c}$$

(iv) *Mixed loads:* Mixed loads can be modelled by assigning a percentage of the total load to each of the above three load models. The delta arm currents are the summation of these three components in each arm.

The line currents of the delta load are expressed by the following relationship:

$$\begin{bmatrix} I_r \\ I_y \\ I_b \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_{ry} \\ I_{yb} \\ I_{br} \end{bmatrix} \tag{2.92}$$

EXERCISES

1. What is 'modelling of electrical components' and why it is required?
2. Explain the analytical concept behind different models concepts of an isolated synchronous generator.
3. How would you analytically model a regulating transformer in power network?
4. Explain the concept of 'three-phase modelling'.
5. Analytically model the following:
   (i) a three-phase single circuit transmission line,
   (ii) a pair of three-phase mutually coupled transmission lines,
   (iii) a shunt capacitor,
   (iv) a series capacitor.
6. What is SVC? How would you model it?
7. How would you model an induction motor in d–q reference frame?
8. Write a short note on "modelling of electrical loads".

Chapter 3

Power Network Matrix Operations

3.1 INTRODUCTION TO $[Y_{BUS}]$ FORMULATION

The large, interconnected AC power system (network) consists of numerous power stations, transmission lines, transformers, shunt reactors and/or capacitors and distribution networks through which loads are supplied. All this leads to a high voltage, largely interconnected AC power transmission system and the assessment of the steady state behaviour of all the components of the network acting together as a system requires computer-based large-scale system analysis of the *network model*. In computer-based power system analysis, the network model takes on the form of *Bus Admittance Matrix* $[Y_{Bus}]$. $[Y_{Bus}]$ is often used in solving *load flow* (or *complex power flow*) problems. Its widespread application in power system computations is due to its simplicity in data preparation and the ease with which it can be formed and modified for any network change (e.g. addition or tripping of line etc.). $[Y_{Bus}]$ matrix is highly *sparse* and facilitates minimum computer storage as well as reduces *computer operation time*. There are different methods of formulation of $[Y_{Bus}]$ matrix and a few of them are reviewed here which are easily amenable to computer programming and easy to grasp.

3.2 NODAL METHOD FOR DEVELOPMENT OF $[Y_{BUS}]$

In this method of $[Y_{Bus}]$ formation, the variables include the *complex load voltages* being treated as *node voltages* (the reference is the 'ground' for designating the magnitudes of bus voltages and for *voltage angles*, the reference is one of the bus (or node) voltages which are usually fixed at a datum value (say, zero)). The *node current* being the other variable, it is the net current injected into the network at a given node (from a source and/or load external to the network). When the current enters the network from a node, the sign of the current is assumed to be positive, while for the current leaving the network, the sign is negative; the net nodal current being the algebraic sum of these node currents.

In the *nodal method* it is usual to use branch admittances rather than branch impedances. For an isolated branch y_{ij} (Fig. 3.1), the node voltage being V_i and V_j at the buses i and j, respectively, current flowing from node i to node j is given by

$$I_{ij} = y_{ij}\left(V_i - V_j\right) \tag{3.1}$$

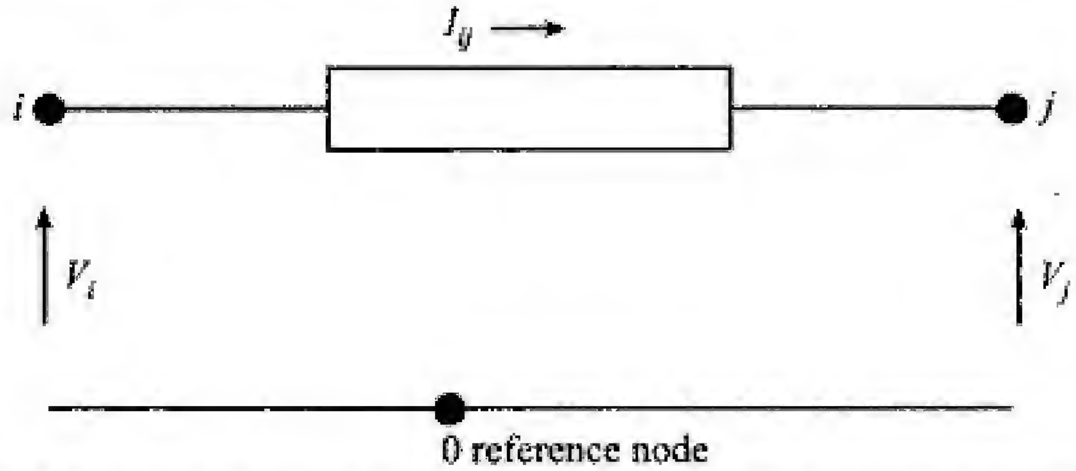


Fig. 3.1 Nodal relationship between node voltages and branch currents.

In a complex network the nodes being numbered 0, 1, 2, ..., n, where node 0 indicates the reference node, by Kirchoff's current law, the injected current I_i being equal to the sum of all currents leaving node i; thus, we can write

$$I_i = \sum_{j=0}^{n} I_{ij} = \sum_{j=0}^{n} y_{ij}\left(V_i - V_j\right) \tag{3.2}$$

With no ground potential (i.e. with zero reference voltage), for a linear system,

$$I_i = \sum_{\substack{j=0 \\ j\neq i}}^{n} y_{ij}V_i - \sum_{\substack{j=1 \\ j\neq i}}^{n} y_{ij}V_j \tag{3.3}$$

This equation, for a n-bus network, in matrix form can be represented as:

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \tag{3.4}$$

or,
$$[I] = [Y_{Bus}]\ [V] \tag{3.5}$$

$[Y_{Bus}]$ is called *Bus Admittance matrix* and it has a well-defined structure. The elements of $[Y_{Bus}]$ are important and hence defined below:

Y_{ii}, the *diagonal element*, is called *self admittance* of node i, while Y_{ij}, the *off-diagonal element*, is called *mutual admittance* (or *transfer admittance*) between nodes i and j.

Obviously, and

$$\left.\begin{aligned} Y_{ii} &= \sum_{\substack{j=0 \\ j\neq i}}^{n} y_{ij} \\ Y_{ij} &= -y_{ij} \end{aligned}\right\} \tag{3.6}$$

The properties of the $[Y_{Bus}]$ matrix are as follows:

(i) $[Y_{Bus}]$ is a *square matrix* of order $n \times n$.

(ii) $[Y_{Bus}]$ is symmetrical, since $y_{ij} = y_{ji}$.

(iii) Only $\dfrac{(n\times n)-n}{2} + n$, i.e. $\dfrac{n(n+1)}{2}$ terms are to be *stored* for a n-bus power system.

(iv) The elements of $[Y_{Bus}]$ matrix are *complex* numbers; $[Y_{Bus}]$ matrix itself is thus complex.

(v) Each diagonal element $[Y_{ii}, Y_{jj}, \ldots]$ is the sum of the admittance of the branches which are linked with corresponding i-th, j-th nodes including branches to ground, while each off-diagonal element Y_{ij} is negative of the branch admittance between nodes i and j. In order to illustrate this property, let us assume a two-bus system (Fig. 3.2) where a transmission line is represented by series admittance y_{se} and shunt admittance y_{sh}.

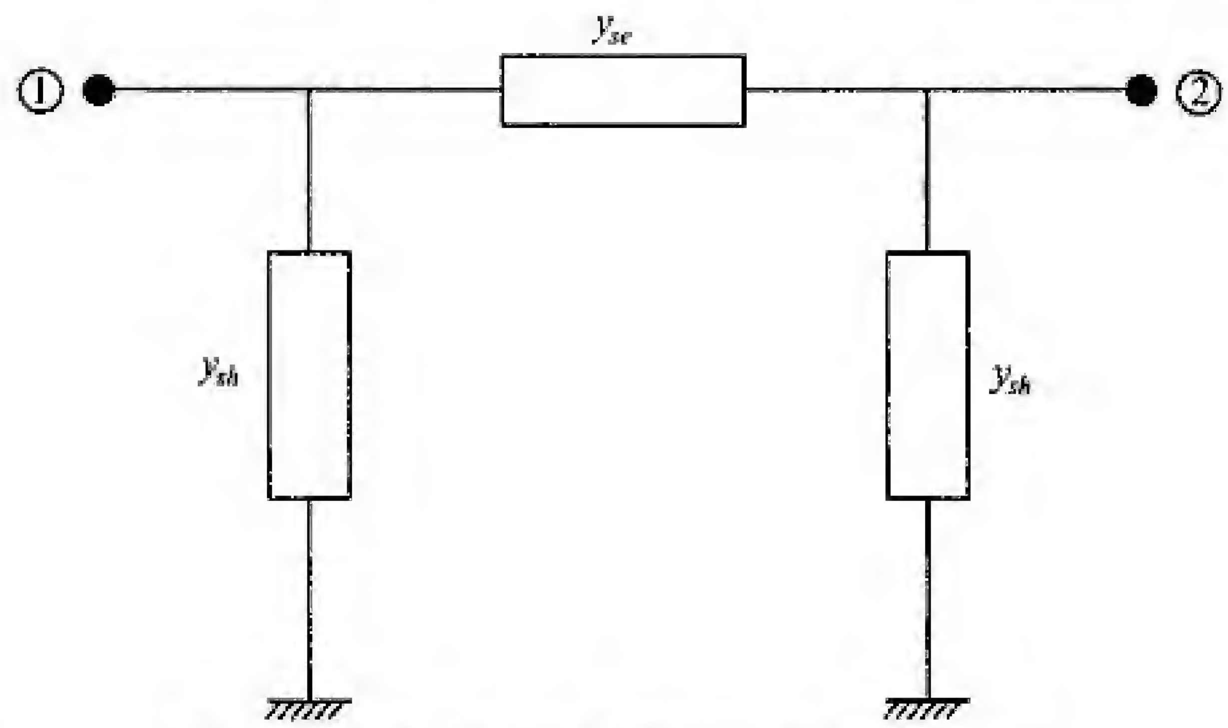


Fig. 3.2 Schematic representation of a two-bus system.

In this case the diagonal elements of $[Y_{Bus}]$ are given as

$$Y_{12} = Y_{21} = -y_{se}$$

$$\therefore \quad [Y_{Bus}] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} y_{se} + y_{sh} & -y_{se} \\ -y_{se} & y_{se} + y_{sh} \end{bmatrix} \tag{3.7}$$

(*vi*) $Y_{ij}\ (i \neq j) = 0$ if i-th bus and j-th buses are *not connected*.

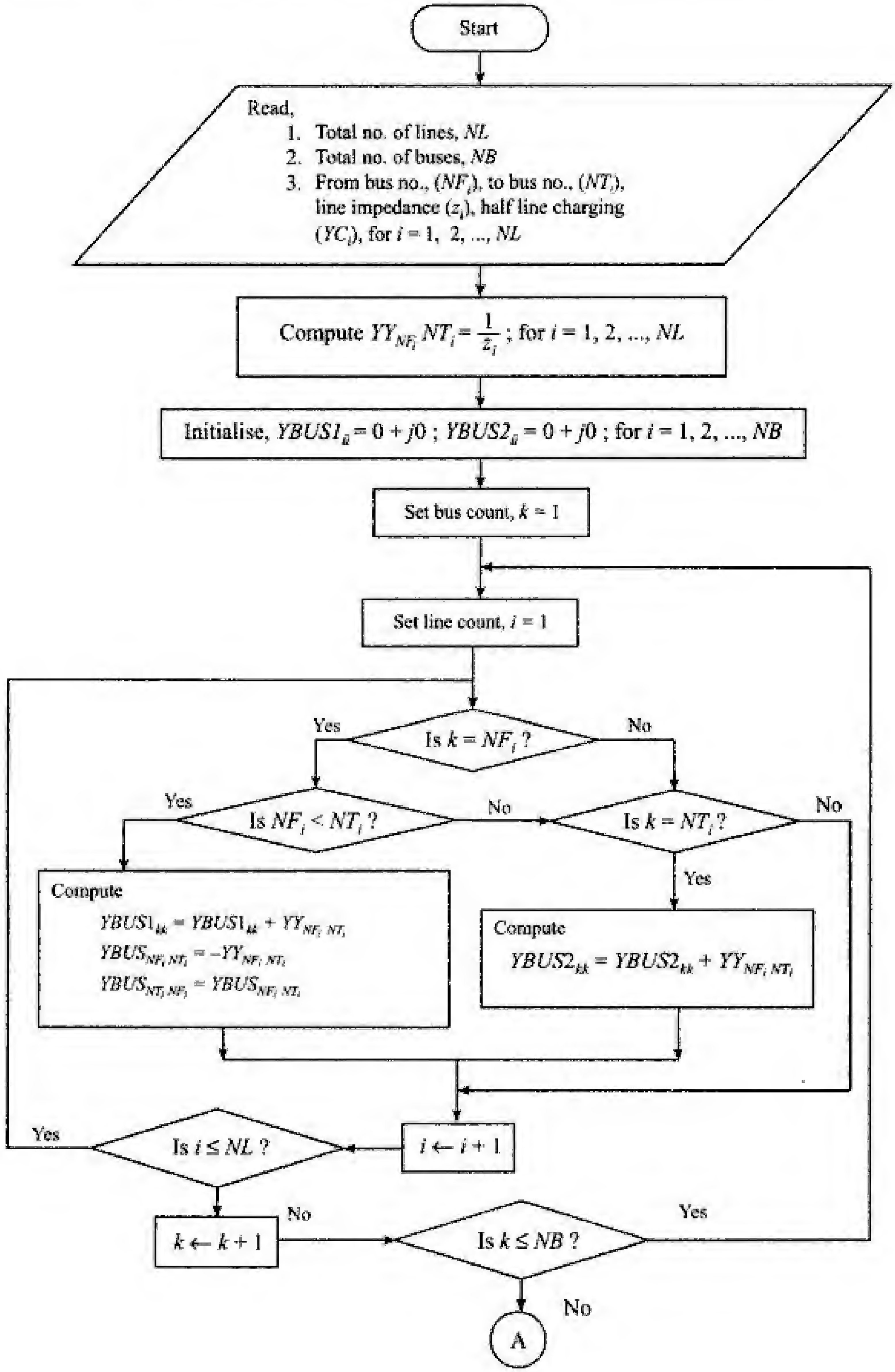
In actual systems lots of interconnections do not exist between a number of buses and hence the $[Y_{Bus}]$ matrix becomes highly *sparse* (containing number of zero elements in the matrix). This saves tremendous computer storage and memory requirements. The flowchart for obtaining $[Y_{Bus}]$ by nodal method is shown in Fig. 3.3.

Formation of $[Z_{Bus}]$ from $[Y_{Bus}]$

In this context, it may be noted here that formation of *bus impedance matrix* $[Z_{Bus}]$ is possible by inversion of $[Y_{Bus}]$ by using special algorithms.

$$[Z_{Bus}] = [Y_{Bus}]^{-1} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & & & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix} \tag{3.8}$$

In the $[Z_{Bus}]$ matrix the diagonal elements are *short circuit driving point impedances* while the off-diagonal elements are *short circuit transfer impedances*. $[Z_{Bus}]$ is symmetric provided $[Y_{Bus}]$ is symmetric, which is very much usual in power network structure. However, $[Z_{Bus}]$ is not *sparse* like $[Y_{Bus}]$ and is a full matrix containing *non-zero* elements (zero elements in $[Y_{Bus}]$ become non-zero elements in the corresponding $[Z_{Bus}]$).


Start
Read,
1. Total no. of lines, NL
2. Total no. of buses, NB
3. From bus no., (NF_i), to bus no., (NT_i), line impedance (z_i), half line charging (YC_i), for $i = 1, 2, ..., NL$
Compute $YY_{NF_i}NT_i = \frac{1}{z_i}$; for $i = 1, 2, ..., NL$
Initialise, $YBUS1_{ii} = 0 + j0$; $YBUS2_{ii} = 0 + j0$; for $i = 1, 2, ..., NB$
Set bus count, $k = 1$
Set line count, $i = 1$
Is $k = NF_i$?
Yes
No
Is $NF_i < NT_i$?
Yes
No
Is $k = NT_i$?
Yes
No
Compute
$YBUS1_{kk} = YBUS1_{kk} + YY_{NF_i NT_i}$
$YBUS_{NF_i NT_i} = -YY_{NF_i NT_i}$
$YBUS_{NT_i NF_i} = YBUS_{NF_i NT_i}$
Compute
$YBUS2_{kk} = YBUS2_{kk} + YY_{NF_i NT_i}$
Is $i \le NL$?
Yes
No
$i \leftarrow i + 1$
$k \leftarrow k + 1$
Is $k \le NB$?
Yes
No
A

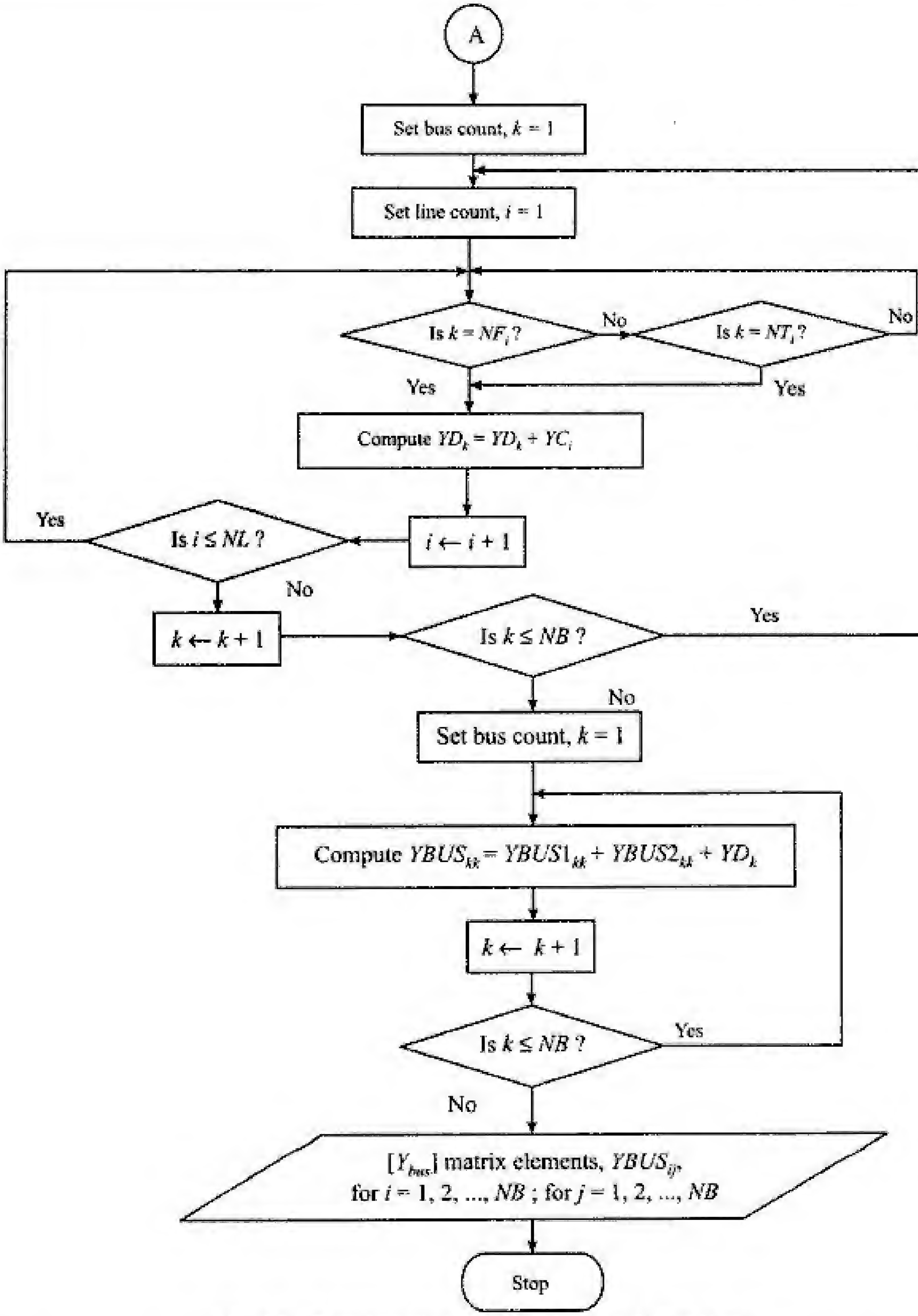


Fig. 3.3 Flowchart of $[Y_{Bus}]$ formation by nodal method.

Example 3.1: A three-bus system is shown in Fig. E3.1. Each line has a series impedance of $(0.05 + j0.15)$ p.u. while the shunt admittance is neglected. Find $[Y_{Bus}]$.

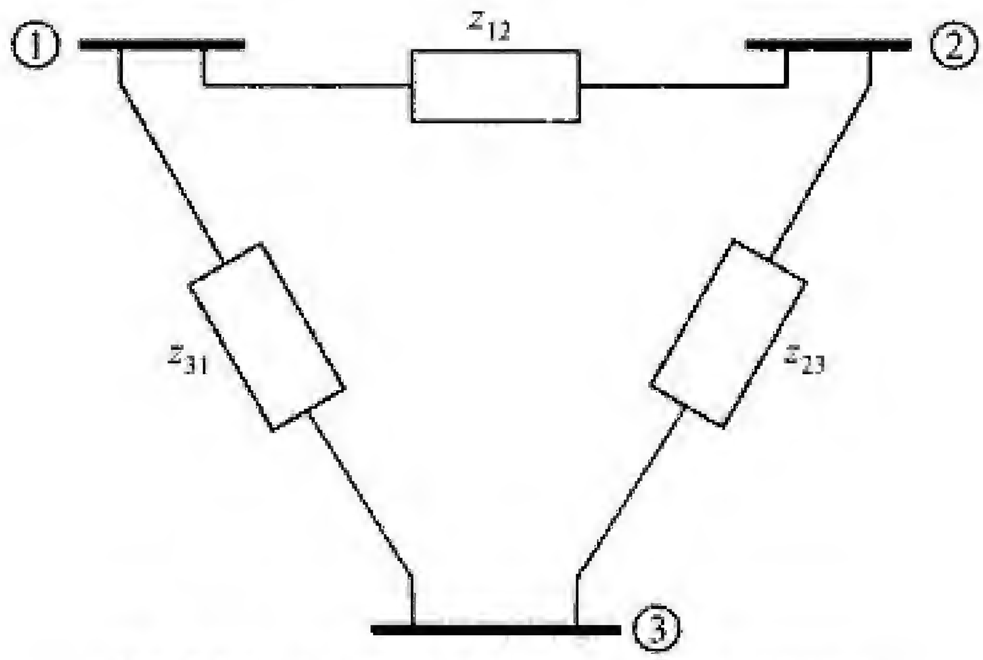


Fig. E3.1 A three-bus three-line power system.

Solution: Given: $z_{12} = z_{23} = z_{31} = (0.05 + j0.15)$ p.u.

$\therefore$ $y_{12} = y_{23} = y_{31}$ (series admittance of each line)

$$= \frac{1}{(0.05 + j0.15)} = (2 - j6) \text{ p.u.}$$

Since the given problem is a three-bus system hence $[Y_{Bus}]$ matrix would be a 3 × 3 matrix.

$$[Y_{Bus}] = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

where,
$$Y_{11} = y_{12} + y_{31};\ Y_{12} = Y_{21} = -y_{12}$$
$$Y_{22} = y_{12} + y_{23};\ Y_{13} = Y_{31} = -y_{31}$$
$$Y_{33} = y_{23} + y_{31};\ Y_{23} = Y_{32} = -y_{23}$$

Since,
$$y_{12} = y_{23} = y_{31} = (2 - j6) \text{ p.u.}$$

$\therefore$
$$Y_{11} = 2 - j6 + 2 - j6 = (4 - j12) \text{ p.u.}$$
$$Y_{12} = Y_{21} = (-2 + j6) \text{ p.u.}$$
$$Y_{22} = 2 - j6 + 2 - j6 = (4 - j12) \text{ p.u.}$$
$$Y_{23} = Y_{32} = (-2 + j6) \text{ p.u.}$$
$$Y_{33} = (4 - j12) \text{ p.u.}$$
$$Y_{13} = Y_{31} = (-2 + j6) \text{ p.u.}$$

$\therefore$
$$[Y_{Bus}] = \begin{bmatrix} (4 - j12) & (-2 + j6) & (-2 + j6) \\ (-2 + j6) & (4 - j12) & (-2 + j6) \\ (-2 + j6) & (-2 + j6) & (4 - j12) \end{bmatrix} \text{ p.u.}$$

Identical result is obtained by executing the $[Y_{Bus}]$ software following the flowchart presented in the text. The input and output of the result are shown below.

Execution of the computer program YBUS.FOR for Example 3.1

Line data: ZBUS0.DAT

```
3,3 [No. of lines, No. of buses]
1,2,(0.05,0.15),(0,0) [From bus, To bus, (R, X), (G/2, B/2)]
1,3,(0.05,0.15),(0,0)
2,3,(0.05,0.15),(0,0)
```

Output of YBUS.FOR: YBUS0.DAT

```
No. of buses = 3
Ybus matrix
Ybus(  1,   1 ) = (   4.000000, -12.000000 )  Y11
Ybus(  1,   2 ) = (  -2.000000,   6.000000 )  Y12
Ybus(  1,   3 ) = (  -2.000000,   6.000000 )
Ybus(  2,   1 ) = (  -2.000000,   6.000000 )
Ybus(  2,   2 ) = (   4.000000, -12.000000 )
Ybus(  2,   3 ) = (  -2.000000,   6.000000 )
Ybus(  3,   1 ) = (  -2.000000,   6.000000 )
Ybus(  3,   2 ) = (  -2.000000,   6.000000 )
Ybus(  3,   3 ) = (   4.000000, -12.000000 )  Y33
```

Example 3.2: In Example 3.1, for the same three-bus system (Fig. E3.1) let a new bus (bus no. 4) be added with bus no. 3 through a transmission line of p.u. z (= $0.1 + j0.3$). Obtain $[Y_{Bus}]$.

Solution: Let the bus no. 4 be added to bus no. 3 through a transmission line of $z = (0.1 + j0.3)$ p.u., i.e. $y_{32} = 1/(0.1 + j0.3) = (1 - j3)$ p.u. [Fig. E3.2]. Since the new element y_{34} is added with bus 3, entries of Y_{33} will change and new entries of Y_{34} and Y_{44} will appear in the new bus admittance matrix.

Obviously, due to presence of 4-bus system, this bus admittance matrix will be a 4×4 matrix.

$$Y_{33} = Y_{33\,(\text{old})} + (1 - j3) = (5 - j15) \text{ p.u.}$$

$$Y_{34} = Y_{43} = -\,y_{34} = (-1 + j3) \text{ p.u.}$$

$$Y_{44} = (1 - j3) \text{ p.u.}$$

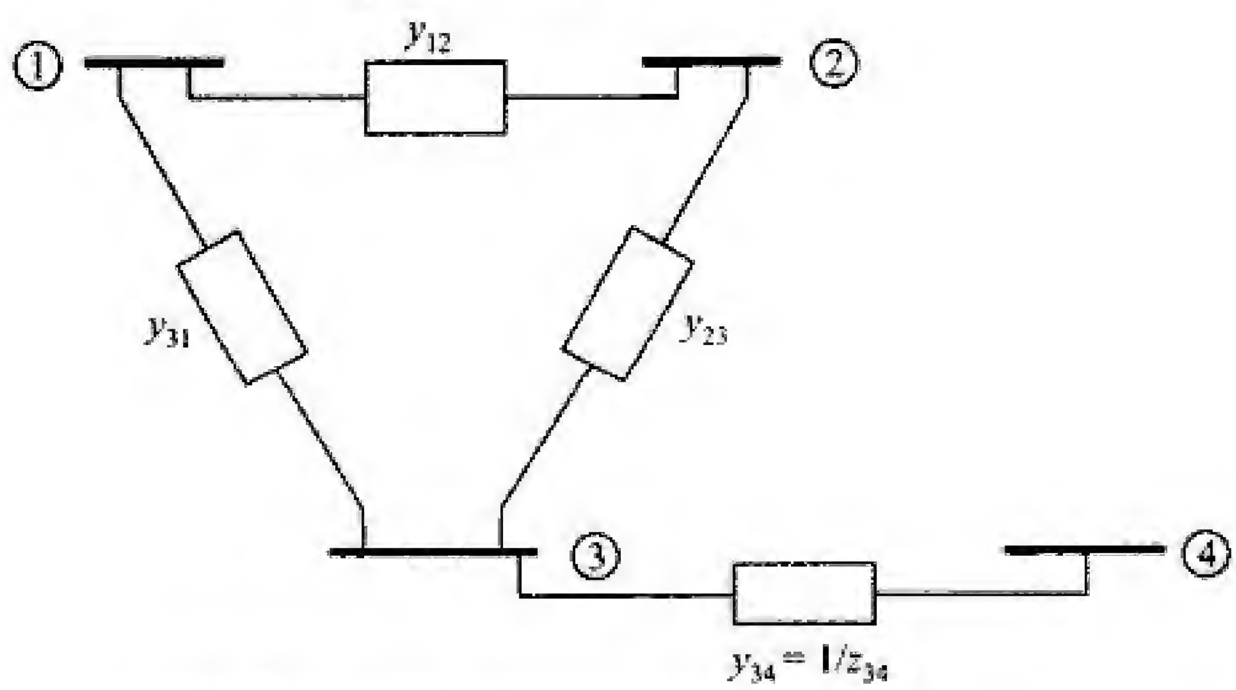


Fig. E3.2 A new bus added to three-bus system.

Since there is no connection of bus 4 with any other bus, except bus no. 3, hence,

$$Y_{14} = Y_{41} = 0; \quad Y_{24} = Y_{42} = 0.$$

Final $[Y_{Bus}]$ matrix thus becomes

$$[Y_{Bus}] = \begin{bmatrix} 4-j12 & -2+j6 & -2+j6 & 0 \\ -2+j6 & 4-j12 & -2+j6 & 0 \\ -2+j6 & -2+j6 & 5-j15 & -1+j3 \\ 0 & 0 & -1+j3 & 1-j3 \end{bmatrix} \text{ p.u.}$$

Execution of $[Y_{Bus}]$ software also yields the same result. It is left for the reader as an exercise.

Example 3.3: The following data refers to a six-bus ten-line power network

Line no.	*From bus*	*To bus*	*R (p.u.)*	*X (p.u.)*	*B/2 (p.u.)*
1	1	2	0.08	0.2	0.018
2	1	4	0.05	0.25	0.03
3	1	5	0.1	0.25	0.03
4	2	3	0.05	0.2	0.025
5	2	4	0.05	0.15	0.015
6	2	5	0.15	0.2	0.02
7	2	6	0.09	0.25	0.025
8	3	5	0.15	0.3	0.03
9	4	5	0.25	0.4	0.035
10	5	6	0.15	0.28	0.025

(a) Find $[Y_{Bus}]$

(b) Also find $[Y_{Bus}]$ when line 4-5 is tripped.

Solution:

(*a*) Since this problem involves six buses, $[Y_{Bus}]$ matrix will be a 6×6 matrix. Result of calculation of $[Y_{Bus}]$ using the developed software is shown below:
Here, the line/branch admittance being calculated first, diagonal elements (Y_{ii}) of bus no. 1 are first obtained followed by the calculation of off-diagonal elements of the same bus (Y_{ij}). The same scheme being executed for each of the buses, the final $[Y_{Bus}]$ array is obtained.

(*b*) For the second case, when line 4-5 is tripped, the system reduces to a 9-line system with 6 buses. With this input, new $[Y_{Bus}]$ is obtained.

Execution of the computer program YBUS.FOR for Example 3.3

Line data for Example 3.3a: ZBUS1A.DAT

```
10,6 [No. of lines, No. of buses]
1,2,(0.08,0.20),(0.0,0.018)     [From bus,  To bus,  (R, X),  (G/2, B/2)]
1,4,(0.05,0.25),(0.0,0.030)        |          |        |         |
1,5,(0.10,0.25),(0.0,0.030)        |          |        |         |
2,3,(0.05,0.20),(0.0,0.025)        |          |        |         |
2,4,(0.05,0.15),(0.0,0.015)        |          |        |         |
2,5,(0.15,0.20),(0.0,0.020)        |          |        |         |
2,6,(0.09,0.25),(0.0,0.025)        |          |        |         |
3,5,(0.15,0.30),(0.0,0.030)        |          |        |         |
4,5,(0.25,0.40),(0.0,0.035)        |          |        |         |
5,6,(0.15,0.28),(0.0,0.025)
```

Output of YBUS.FOR for Example 3.3a: YBUS1A.DAT

```
No.of buses =   6

Ybus matrix

Ybus(  1,   1 ) = (     3.872679,-11.526770 )  Y11
Ybus(  1,   2 ) = (    -1.724138,  4.310345 )  Y12
Ybus(  1,   3 ) = (      .000000,   .000000 )
Ybus(  1,   4 ) = (     -.769231,  3.846154 )
Ybus(  1,   5 ) = (    -1.379310,  3.448276 )
Ybus(  1,   6 ) = (      .000000,   .000000 )
Ybus(  2,   1 ) = (    -1.724138,  4.310345 )
Ybus(  2,   2 ) = (     8.575396,-21.654300 )
Ybus(  2,   3 ) = (    -1.176471,  4.705882 )
Ybus(  2,   4 ) = (    -2.000000,  6.000000 )
Ybus(  2,   5 ) = (    -2.400000,  3.200000 )
Ybus(  2,   6 ) = (    -1.274788,  3.541076 )
Ybus(  3,   1 ) = (      .000000,   .000000 )
Ybus(  3,   2 ) = (    -1.176471,  4.705882 )
Ybus(  3,   3 ) = (     2.509804, -7.317549 )
Ybus(  3,   4 ) = (      .000000,   .000000 )
Ybus(  3,   5 ) = (    -1.333333,  2.666667 )
Ybus(  3,   6 ) = (      .000000,   .000000 )
Ybus(  4,   1 ) = (     -.769231,  3.846154 )
Ybus(  4,   2 ) = (    -2.000000,  6.000000 )
Ybus(  4,   3 ) = (      .000000,   .000000 )
Ybus(  4,   4 ) = (     3.892826,-11.563910 )
Ybus(  4,   5 ) = (    -1.123595,  1.797753 )
Ybus(  4,   6 ) = (      .000000,   .000000 )
Ybus(  5,   1 ) = (    -1.379310,  3.448276 )
Ybus(  5,   2 ) = (    -2.400000,  3.200000 )
Ybus(  5,   3 ) = (    -1.333333,  2.666667 )
Ybus(  5,   4 ) = (    -1.123595,  1.797753 )
Ybus(  5,   5 ) = (     7.722860,-13.747720 )
Ybus(  5,   6 ) = (    -1.486620,  2.775025 )
Ybus(  6,   1 ) = (      .000000,   .000000 )
Ybus(  6,   2 ) = (    -1.274788,  3.541076 )
Ybus(  6,   3 ) = (      .000000,   .000000 )
Ybus(  6,   4 ) = (      .000000,   .000000 )
Ybus(  6,   5 ) = (    -1.486620,  2.775025 )
Ybus(  6,   6 ) = (     2.761408, -6.266101 )  Y66
```

Line data for Example 3.3b: ZBUS1B.DAT

```
9,6
1,2,(0.08,0.20),(0.0,0.018)
1,4,(0.05,0.25),(0.0,0.030)
1,5,(0.10,0.25),(0.0,0.030)
```

```
2,3,(0.05,0.20),(0.0,0.025)
2,4,(0.05,0.15),(0.0,0.015)
2,5,(0.15,0.20),(0.0,0.020)
2,6,(0.09,0.25),(0.0,0.025)
3,5,(0.15,0.30),(0.0,0.030)
5,6,(0.15,0.28),(0.0,0.025)
```

Output of YBUS.FOR for Example 3.3b: YBUS1B.DAT

```
No.of buses = 6

Ybus matrix

Ybus(  1,  1 ) = (   3.872679,-11.526770 )   Y11
Ybus(  1,  2 ) = (  -1.724138,  4.310345 )   Y12
Ybus(  1,  3 ) = (    .000000,   .000000 )
Ybus(  1,  4 ) = (   -.769231,  3.846154 )
Ybus(  1,  5 ) = (  -1.379310,  3.448276 )
Ybus(  1,  6 ) = (    .000000,   .000000 )
Ybus(  2,  1 ) = (  -1.724138,  4.310345 )
Ybus(  2,  2 ) = (   8.575396,-21.654300 )
Ybus(  2,  3 ) = (  -1.176471,  4.705882 )
Ybus(  2,  4 ) = (  -2.000000,  6.000000 )
Ybus(  2,  5 ) = (  -2.400000,  3.200000 )
Ybus(  2,  6 ) = (  -1.274788,  3.541076 )
Ybus(  3,  1 ) = (    .000000,   .000000 )
Ybus(  3,  2 ) = (  -1.176471,  4.705882 )
Ybus(  3,  3 ) = (   2.509804, -7.317549 )
Ybus(  3,  4 ) = (    .000000,   .000000 )
Ybus(  3,  5 ) = (  -1.333333,  2.666667 )
Ybus(  3,  6 ) = (    .000000,   .000000 )
Ybus(  4,  1 ) = (   -.769231,  3.846154 )
Ybus(  4,  2 ) = (  -2.000000,  6.000000 )
Ybus(  4,  3 ) = (    .000000,   .000000 )
Ybus(  4,  4 ) = (   2.769231, -9.801153 )
Ybus(  4,  5 ) = (    .000000,   .000000 )
Ybus(  4,  6 ) = (    .000000,   .000000 )
Ybus(  5,  1 ) = (  -1.379310,  3.448276 )
Ybus(  5,  2 ) = (  -2.400000,  3.200000 )
Ybus(  5,  3 ) = (  -1.333333,  2.666667 )
Ybus(  5,  4 ) = (    .000000,   .000000 )
Ybus(  5,  5 ) = (   6.599264,-11.984970 )
Ybus(  5,  6 ) = (  -1.486620,  2.775025 )
Ybus(  6,  1 ) = (    .000000,   .000000 )
Ybus(  6,  2 ) = (  -1.274788,  3.541076 )
Ybus(  6,  3 ) = (    .000000,   .000000 )
Ybus(  6,  4 ) = (    .000000,   .000000 )
Ybus(  6,  5 ) = (  -1.486620,  2.775025 )
Ybus(  6,  6 ) = (   2.761408, -6.266101 )   Y66
```

3.3 MODIFICATION OF [Y_{BUS}] DUE TO INCLUSION OF REGULATING TRANSFORMER BETWEEN TWO BUSES

In this section we will discuss about the formation of *admittance matrix* between two buses with inclusion of a two-winding transformer between them.

We take into account the p.u. transformer admittances (the series and shunt using π model) which are actually reciprocal of the p.u. impedance of the transformer that has the complex transformation ratio. The π equivalent line may be shown to the left as well as to the right of the transformer. We present here both of these representations for better understanding. We assume the regulating transformer to be situated at the j-th bus of the line when the π model is included at the left side of the transformer (Fig. 3.4), while i-th bus we assume the regulating transformer to be situated at the further of the line when the π model is included at the right side of the transformer (Fig. 3.5). Since the regulating transformers are usually provided at the end(s) of the line, we approach a realistic situation by assuming the transformer to be placed at the ends of the line and nearest to the buses.

Case A: When the regulating transformer is present between two buses and the line model is placed at the left side of the transformer (a:1).

Let us assume the regulating transformer is having a *complex transformation ratio* of a (= $|a| \angle\alpha$). Figure 3.4 represents the voltages and currents at the line and bus side for the transformer being included in the line.

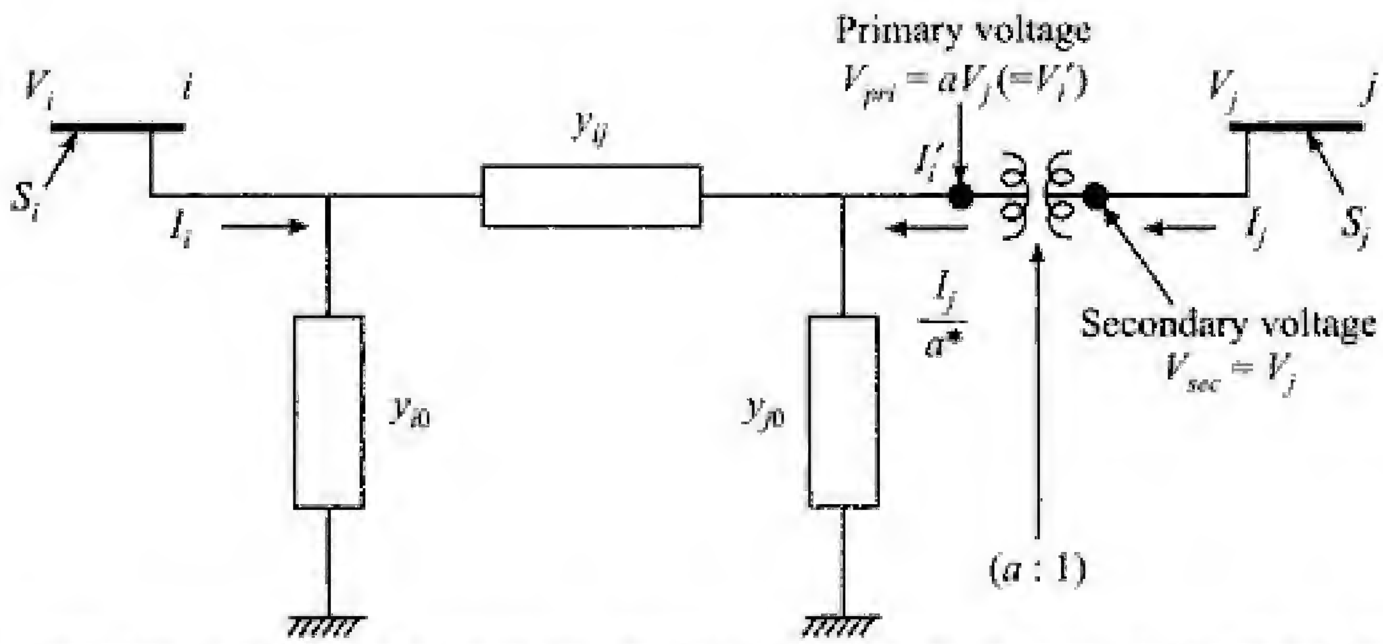


Fig. 3.4 Equivalent circuit for a line containing regulating transformer between two buses and placed at the j-th bus. [S_i and S_j are the injected complex powers at the i-th and j-th buses respectively, while V_i and V_j are respective bus voltages.

The transformer is assumed to be nearer to the j-th bus and has complex *off-nominal* tap ratio a : 1, which corresponds to $V_{pri} : V_{sec}$. Assuming the transformer to be loss-less,

$$\frac{V_i'}{V_j} = a = |a| \angle\alpha$$

or,
$$V_i' \left(\text{i.e., } V_{pri}\right) = aV_j \tag{3.9}$$

Also, input power being equal to output power,

$$V_i' I_i'^* = V_j I_j^* \text{ or, } \frac{V_i'}{V_j} = a = \frac{I_j^*}{I_i'^*} \quad [\, I_i' \text{ is the secondary current of transformer}]$$

$$\therefore \qquad I_i'^* = \frac{I_j^*}{a} \text{ or } I_i' = \frac{I_j}{a^*} \tag{3.10}$$

Next we consider the current balance at two buses by the following two equations:

$$I_i = \frac{S_i^*}{V_i^*} = V_i y_{i0} + (V_i - aV_j) y_{ij} = V_i y_0 + (V_i - aV_j) y_{ij} \tag{3.11}$$

and
$$\frac{I_j}{a^*} = \frac{S_j^*}{(aV_j)^*} = aV_j y_{j0} + (aV_j - V_i) y_{ij} = aV_j y_0 + (aV_j - V_i) y_{ij}$$

[assuming $y_{i0} = y_{j0} = y_0$]

$$\therefore \qquad I_j = (-a^* y_{ij}) \; V_i + aa^*(y_0 + y_{ij}) V_j \tag{3.12}$$

Let us now rewrite equations (3.11) and (3.12) in pair form as follows:

$$I_i = (y_0 + y_{ij}) \; V_i + (-ay_{ij}) V_j$$

and
$$I_j = (-a^* y_{ij}) \; V_i + aa^*(y_0 + y_{ij}) V_j$$

In matrix form these two equations can be represented as

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} y_0 + y_{ij} & -ay_{ij} \\ -a^* y_{ij} & aa^*(y_0 + y_{ij}) \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix}$$

or
$$[I] = [Y][V]$$

where
$$[Y] = \begin{bmatrix} y_0 + y_{ij} & -ay_{ij} \\ -a^* y_{ij} & aa^*(y_0 + y_{ij}) \end{bmatrix} \tag{3.13}$$

It may be noted that a is *complex* and $[Y]$ is *not symmetric*.

If a is a *real quantity*, i.e. $a = (\text{KV})_{\text{base}}/(\text{KV})_{\text{tap}}$, then

$$[Y] = \begin{bmatrix} y_0 + y_{ij} & -ay_{ij} \\ -ay_{ij} & a^2(y_0 + y_{ij}) \end{bmatrix} \tag{3.14}$$

The matrix $[Y]$ then becomes *symmetric*.

Case B: When the regulating transformer is present between two buses and the line model is placed at the right side of the transformer (1:a).

With reference to Fig. 3.5,
$$\frac{V_j'}{V_i} = a = |a| \angle\alpha, \text{ i.e. } V_j' = aV_i \tag{3.15}$$

Also
$$V_i I_i^* = (aV_i) I_i'^*$$

[Power being equal at the transformer input and output, while I_j' is the secondary current of transformer]

or,
$$\frac{I_j'^*}{I_i^*} = \frac{1}{a}$$

or,
$$I_i = a^* I_j' \tag{3.16}$$

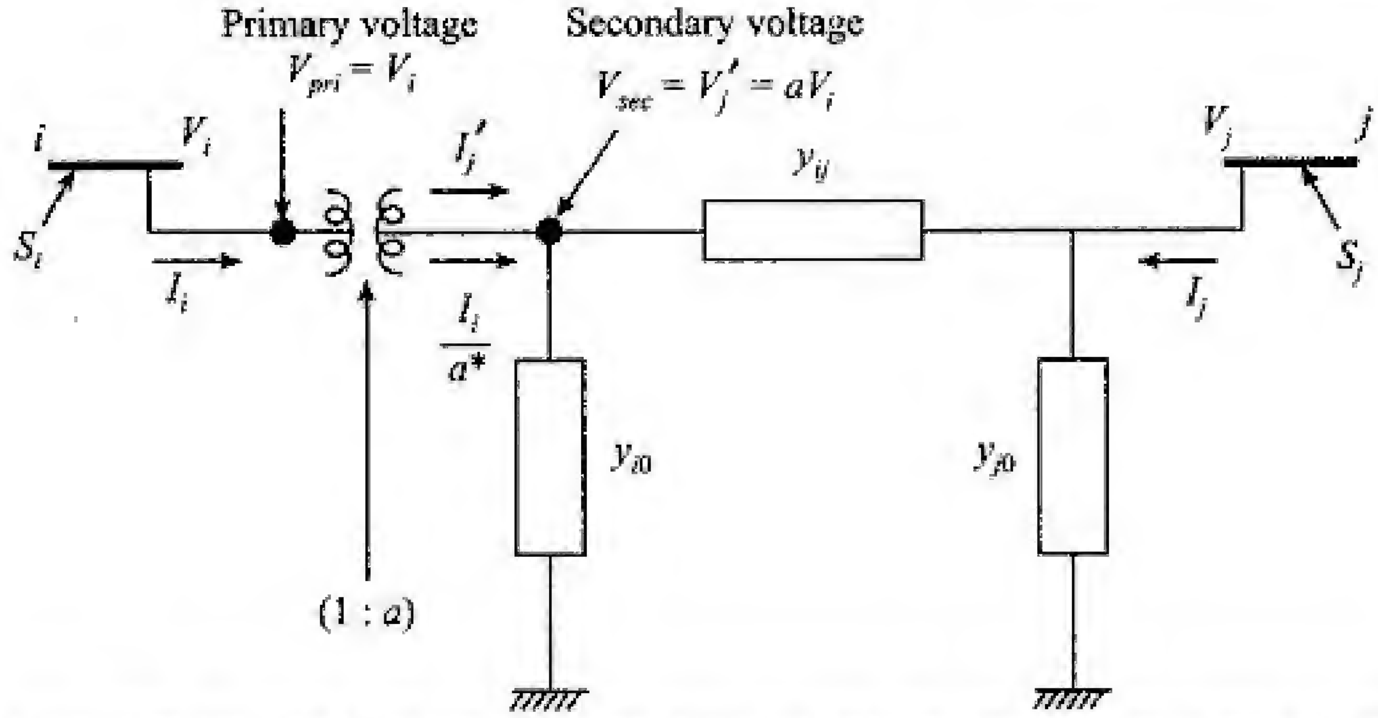


Fig. 3.5 Equivalent circuit for a line containing regulating transformer between two buses and placed at the i-th bus.

At bus i we can write,

$$I_i = a^* I_j' = a^* \left[aV_i y_0 + \left(aV_i - V_j \right) y_{ij} \right]$$

or

$$I_i = aa^* y_0 V_i + aa^* y_{ij} V_i - a^* y_{ij} V_j$$

$$\therefore \quad I_i = aa^* \left(y_0 + y_{ij} \right) \; V_i - a^* y_{ij} V_j \tag{3.17}$$

Also at bus j we can write,

$$I_j = y_0 V_j + \left(V_j - aV_i \right) y_{ij}$$

$$\therefore \quad I_j = \left(-ay_{ij} \right) \; V_i + \left(y_0 + y_{ij} \right) \; V_j \tag{3.18}$$

In matrix form equations (3.17) and (3.18) can be rearranged as

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} aa^* \left(y_0 + y_{ij} \right) & -a^* y_{ij} \\ -ay_{ij} & y_0 + y_{ij} \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} \tag{3.19}$$

Here

$$[Y] = \begin{bmatrix} aa^* \left(y_0 + y_{ij} \right) & -a^* y_{ij} \\ -ay_{ij} & y_0 + y_{ij} \end{bmatrix} \tag{3.20}$$

It may be noted that a being *real*,

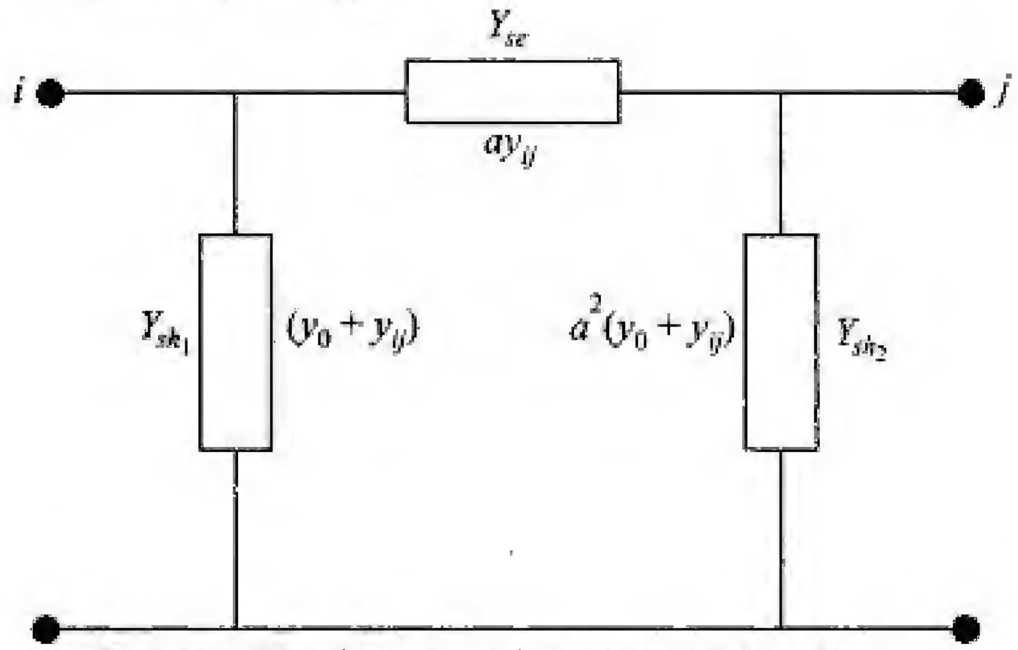
$$[Y] = \begin{bmatrix} a^2 \left(y_0 + y_{ij} \right) & -ay_{ij} \\ -ay_{ij} & y_0 + y_{ij} \end{bmatrix} \tag{3.21}$$

and the $[Y]$ matrix becomes *symmetrical*.

[In practical cases, the regulating transformer is designed for either *voltage magnitude* or *phase angle control*. In the former case $\alpha = 0$ and $|a|$ can be changed in discrete steps of $\Delta\,|a|$. In the latter case, $|a|$ is constant and α is changed in discrete steps of $\Delta\alpha$. In both the cases, we have also assumed the line model to be placed at the non-unity side of the transformer.]

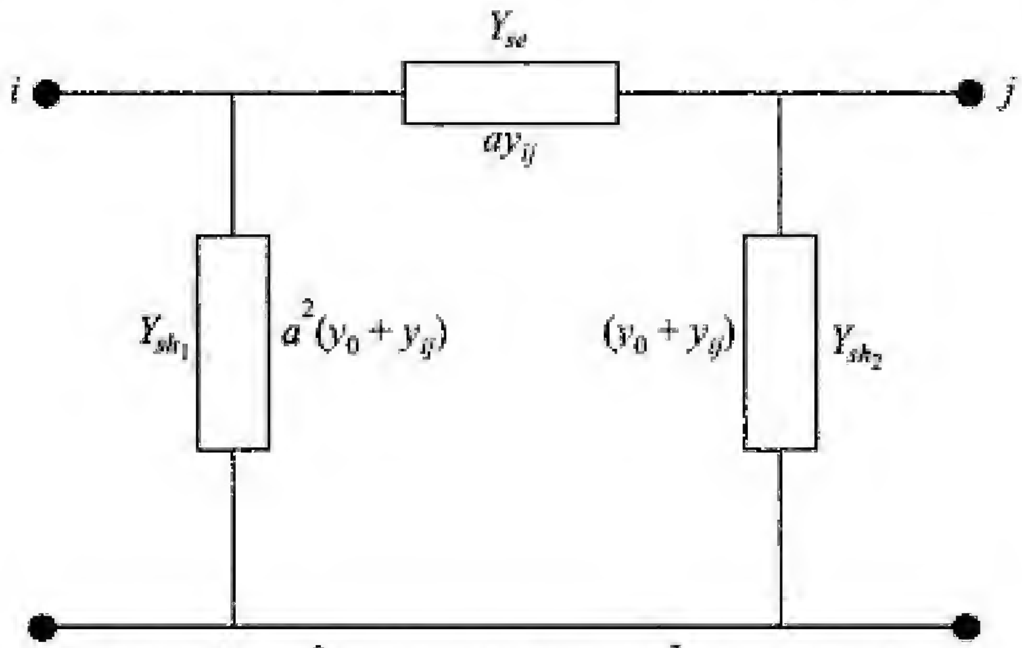
3.4 FORMATION OF $[Y_{BUS}]$ WITH TRANSFORMER PRESENT IN THE LINE

Once the modelling of the branch with the transformer installed between these two buses is done, we proceed to modify $[Y_{Bus}]$. We are now in a position to draw the equivalent π-circuit of the line transformer system connected between bus i and j. Figure 3.6(a) represents the π equivalent circuit when the transformer is placed at the j-th bus, while Fig. 3.6(b) represents the π equivalent circuit when the transformer is placed at the i-th bus. It may be noted here that Y_{se} and Y_{sh_1} or Y_{sh_2} in Figs. 3.6(a) and (b) are used to form Y_{12} (or Y_{21}) and Y_{11} or Y_{22} respectively.



$Y_{ij} = ay_{ij}$; $Y_{ii} = (y_0 + y_{ij}) + (-ay_{ij}) = y_0 + (1 - a)y_{ij}$;
$Y_{jj} = a^2(y_0 + y_{ij}) + (-ay_{ij}) = a^2y_0 + a(a - 1)y_{ij}$

Fig. 3.6(a) π Equivalent circuit for the transformer shown in Fig. 3.4.



$Y_{ij} = ay_{ij}$; $Y_{ii} = a^2(y_0 + y_{ij}) + (-ay_{ij}) = a^2 y_0 + a(a - 1)y_{ij}$;
$Y_{jj} = (y_0 + y_{ij}) + (- ay_{ij}) = y_0 + (1 - a)y_{ij}$

Fig. 3.6(b) π Equivalent circuit for the transformer shown in Fig. 3.5.

Then the $[Y_{Bus}]$ matrix can be modified with inclusion of the transformer at either end with revised form of its self (diagonal) and transfer (off-diagonal) elements as shown below:

(*a*) *For the case when the transformer is at the j-th bus and line model is at the left of it*

$$\begin{aligned} Y_{ii_{(new)}} &= y_{i0} + \cdots + y_0 + (1-a)\,y_{ij} + ay_{ij} + \cdots + y_{in} \\ &= y_{i0} + \cdots + y_0 + y_{ij} + \cdots + y_{in} \end{aligned} \tag{3.22}$$

Since the given problem is a three-bus system, hence $[Y_{Bus}]$ matrix will be a 3 × 3 matrix.

∴ $[Y_{Bus}]$ matrix before considering the line with transformer (as explained earlier) is given by [see Fig. E3.3(b)].

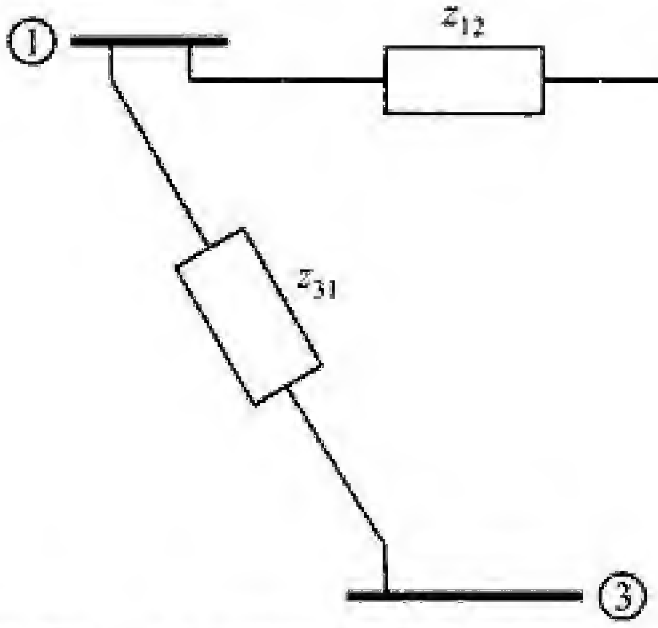


Fig. E3.3(b) The system before considering the line with transformer.

$$[Y_{Bus}] = \begin{bmatrix} (4-j12) & (-2+j6) & (-2+j6) \\ (-2+j6) & (2-j6) & (0+j0) \\ (-2+j6) & (0+j0) & (2-j6) \end{bmatrix} \text{ p.u.}$$

This is the case of regulating transformer placed at the i-th bus $(V_3'/V_2 = 1.02)$, i.e. $a = 1.02$ [see Fig. E3.3(c)].

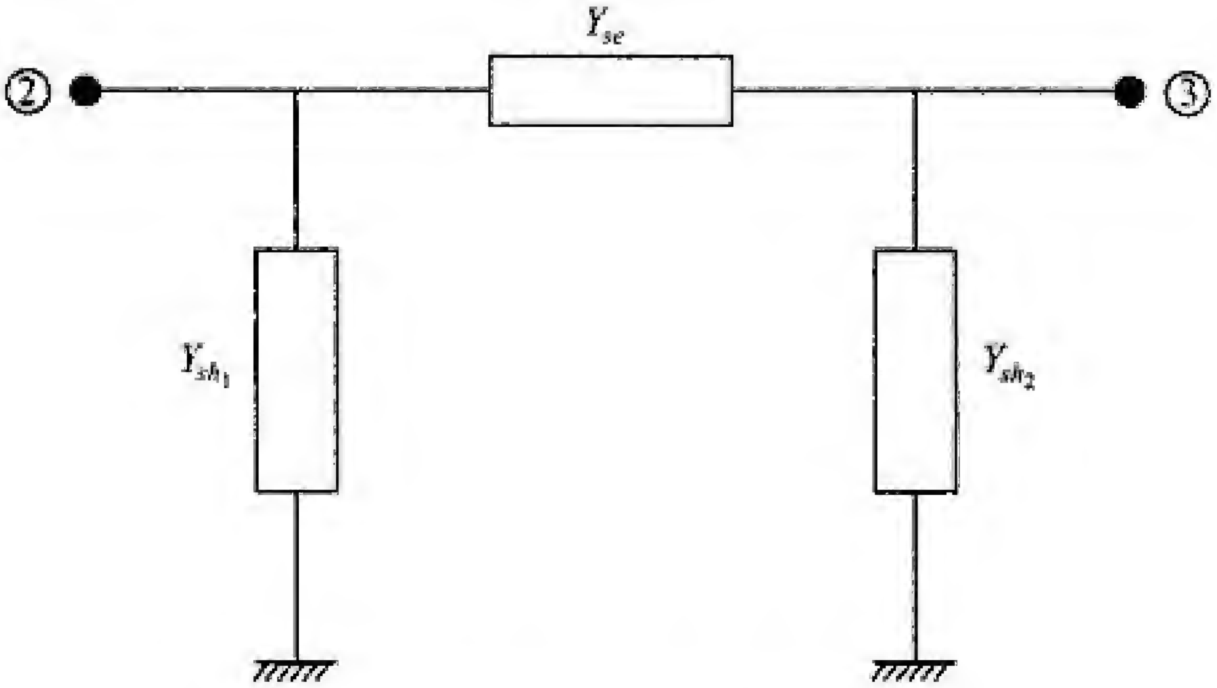


Fig. E3.3(c) π model of the line with transformer.

$$\because \qquad y_{23} = \frac{1}{j0.5} = -j2 \text{ p.u.}$$

$$\therefore \qquad Y_{23} = ay_{23} = 1.02\times(-j2) = -j2.04 \text{ p.u.}$$

$$Y_{22} = a(a-1)y_{23} \ [\because y_0 = 0 + j0]$$

$$= 1.02 \times (1.02-1) \times (-j2) = -j0.0408 \text{ p.u.}$$

$$Y_{33} = (1-a)y_{23} = (1-1.02) \times (-j2) = j0.04 \text{ p.u.}$$

$\therefore$ For the transformer in line 2–3,

$$Y_{Bus_{22}} = Y_{Bus_{22_{old}}} + (-j2.04) + (-j0.0408)$$

$$= (2-j6) - j2.0808 = 2 - j8.0808 \text{ p.u.}$$

$$Y_{Bus_{33}} = Y_{Bus_{33_{old}}} + (-j2.04) + (j0.04)$$

$$= (2-j6) - j2 = (2-j8) \text{ p.u.}$$

$$Y_{Bus_{23}} = Y_{Bus_{32}} = -(-j2.04) = j2.04 \text{ p.u.}$$

$$\therefore \qquad [Y_{Bus}] = \begin{bmatrix} (4-j12) & (-2+j6) & (-2+j6) \\ (-2+j6) & (2-j8.0808) & j2.04 \\ (-2+j6) & j2.04 & (2-j8) \end{bmatrix} \text{ p.u.}$$

Execution of the computer program TMYBUS.FOR for Example 3.4

Line data: ZBUS11.DAT

`3,3,1`	[No. of lines, No. of buses, No. of transformers]
`1,2,(0.05,0.15),(0,0)`	[From bus, To bus, (*R*, *X*), (*G*/2, *B*/2)]
`1,3,(0.05,0.15),(0,0)`	[From bus, To bus, (*R*, *X*), (*G*/2, *B*/2)]
`2,3,(0,0.5),(1.02,0)`	[From bus, To bus, (*R*, *X*), Rectangular form of off-nominal tap ratio of transformer]

Output of TMYBUS.FOR: YBUS11.DAT

The output furnished the $[Y_{Bus}]$ *matrix first considering no transformer and then considering the said transformer.*

```
No. of buses =  3
Before considering line transformer
Ybus matrix
Ybus (  1,   1 ) = (   4.00000,-12.00000 )  Y11
Ybus (  1,   2 ) = (  -2.00000,  6.00000 )  Y12
Ybus (  1,   3 ) = (  -2.00000,  6.00000 )
Ybus (  2,   1 ) = (  -2.00000,  6.00000 )
Ybus (  2,   2 ) = (   2.00000, -6.00000 )
Ybus (  2,   3 ) = (    .00000,   .00000 )
Ybus (  3,   1 ) = (  -2.00000,  6.00000 )
Ybus (  3,   2 ) = (    .00000,   .00000 )
Ybus (  3,   3 ) = (   2.00000, -6.00000 )  Y33
```

```
For Transformer in line - 3
series admittance of line       (0.000000E+00,-2.040000)
shunt admittance of from bus    (0.000000E+00,-4.079996E-02)
shunt admittance of to bus      (0.000000E+00,3.999996E-02)
Ybus matrix after considering line transformer

Ybus (  1,   1 ) = (    4.00000, -12.00000 )  Y11
Ybus (  1,   2 ) = (   -2.00000,   6.00000 )  Y12
Ybus (  1,   3 ) = (   -2.00000,   6.00000 )
Ybus (  2,   1 ) = (   -2.00000,   6.00000 )
Ybus (  2,   2 ) = (    2.00000,  -8.08080 )
Ybus (  2,   3 ) = (     .00000,   2.04000 )
Ybus (  3,   1 ) = (   -2.00000,   6.00000 )
Ybus (  3,   2 ) = (     .00000,   2.04000 )
Ybus (  3,   3 ) = (    2.00000,  -8.00000 )  Y33
```

Example 3.5: A five-bus system is shown in Fig. E3.4. Assume an ideal transformer to be connected between buses 1 and 2 in series with a line of reactance $j0.4$ p.u. If the off-nominal tap ratio be 1:1.05, find $[Y_{Bus}]$ using computer program. Assume line model on nonunity side.

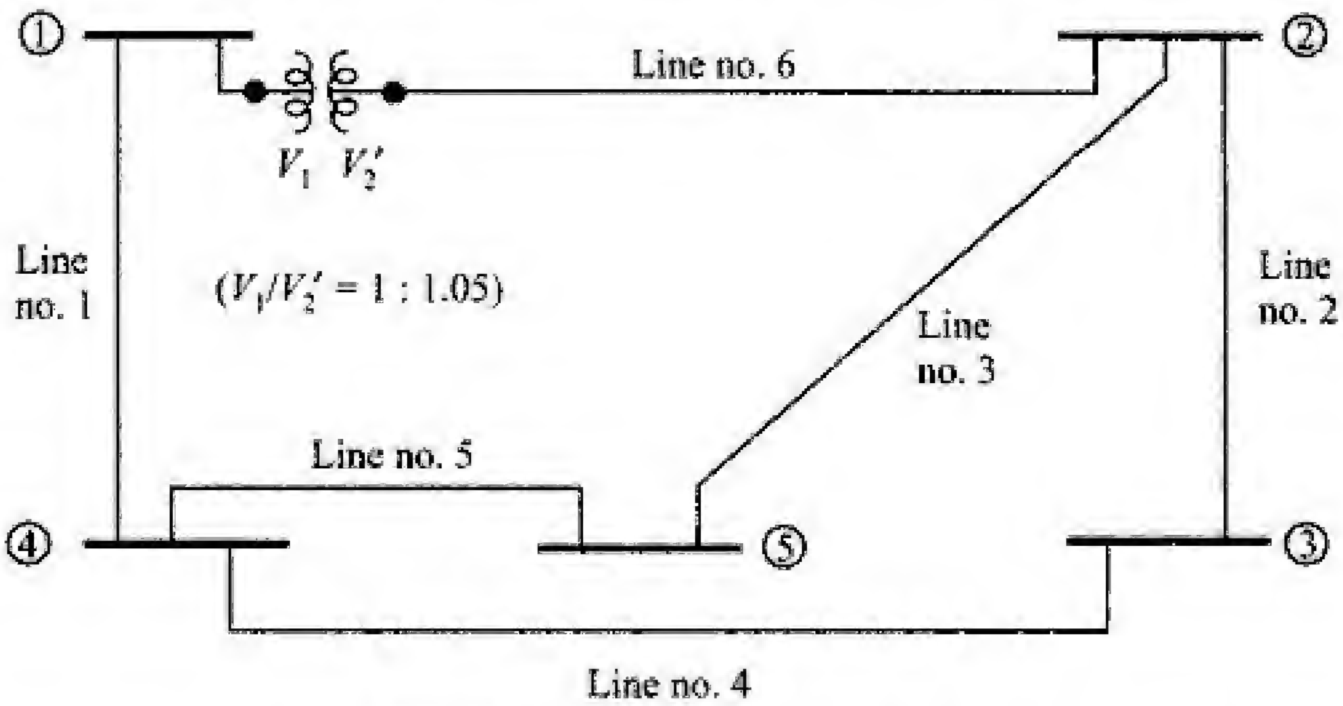


Fig. E3.4 A five-bus power system.

The line data for the given system is shown below (neglect shunt charging effect)

Line no.	*From bus*	*To bus*	*R (in p.u.)*	*X (in p.u.)*	*Off-nominal tap ratio of transformer*
1	1	4	0.10	0.30	
2	2	3	0.10	0.28	
3	2	5	0.075	0.18	
4	3	4	0.15	0.35	
5	4	5	0.15	0.40	
6	1	2	0	0.4	1:1.05

Solution:

Execution of the computer program YBUS.FOR for Example 3.5

The computer program following the flowchart furnished in the text for determining $[Y_{Bus}]$ *for the power network having transformer is executed with the following line data (input).*

Line data: ZBUS2A.DAT

```
6,5,1 [No. of lines, No. of buses, No. of transformers]
1,4,(0.10,0.30),(0.0,0.0)   [From bus, To bus, (R, X), (G/2, B/2)]
2,3,(0.10,0.28),(0.0,0.0)
2,5,(0.075,0.18),(0.0,0.0)
3,4,(0.15,0.35),(0.0,0.0)
4,5,(0.15,0.40),(0.0,0.0)
1,2,(0.0,0.4),(1.05,0.0)    [From bus, To bus, Rectangular form of off-nominal tap
                             ratio of transformer]
```

The output furnished the $[Y_{Bus}]$ *matrix first considering no transformer and then considering the said transformer connected near to i-th bus and having ratio 1:a, the line model is at nonunity side.* (Model of Fig. 3.6(b))

Output of YBUS.FOR: YBUS2A.DAT

```
No. of buses = 5
Before considering line transformer
Ybus matrix
Ybus ( 1, 1 ) = (   1.00000,  -3.00000 )   Y11
Ybus ( 1, 2 ) = (    .00000,    .00000 )   Y12
Ybus ( 1, 3 ) = (    .00000,    .00000 )
Ybus ( 1, 4 ) = (  -1.00000,   3.00000 )
Ybus ( 1, 5 ) = (    .00000,    .00000 )
Ybus ( 2, 1 ) = (    .00000,    .00000 )
Ybus ( 2, 2 ) = (   3.10361,  -7.90115 )
Ybus ( 2, 3 ) = (  -1.13122,   3.16742 )
Ybus ( 2, 4 ) = (    .00000,    .00000 )
Ybus ( 2, 5 ) = (  -1.97239,   4.73373 )
Ybus ( 3, 1 ) = (    .00000,    .00000 )
Ybus ( 3, 2 ) = (  -1.13122,   3.16742 )
Ybus ( 3, 3 ) = (   2.16570,  -5.58121 )
Ybus ( 3, 4 ) = (  -1.03448,   2.41379 )
Ybus ( 3, 5 ) = (    .00000,    .00000 )
Ybus ( 4, 1 ) = (  -1.00000,   3.00000 )
Ybus ( 4, 2 ) = (    .00000,    .00000 )
Ybus ( 4, 3 ) = (  -1.03448,   2.41379 )
Ybus ( 4, 4 ) = (   2.85640,  -7.60557 )
Ybus ( 4, 5 ) = (   -.82192,   2.19178 )
Ybus ( 5, 1 ) = (    .00000,    .00000 )
```

Solution:

Execution of the computer program YBUS.FOR for Example 3.6

Line data: ZBUS2B.DAT

```
6,5,1 [No. of lines, No. of buses, No. of transformers]
1,4,(0.10,0.30),(0.0,0.030) [From bus, To bus, (R, X), (G/2, B/2)]
2,3,(0.10,0.28),(0.0,0.030)
2,5,(0.075,0.18),(0.0,0.03)
3,4,(0.15,0.35),(0.0,0.030) [From bus, To bus, (R, X), (G/2, B/2)]
4,5,(0.15,0.40),(0.0,0.030) [From bus, To bus, (R, X), (G/2, B/2)]
1,2,(0.0,0.4),(1.05,0.0)    [From bus, To bus, Transf. imp., Rectangular form of
                             off-nominal tap ratio of transformer]
```

Output of YBUS.FOR: YBUS2B.DAT

```
No. of buses =  5
Before considering line transformer
Ybus matrix
Ybus (  1,   1 ) = (    1.00000,  -2.97000 )   Y11
Ybus (  1,   2 ) = (     .00000,    .00000 )   Y12
Ybus (  1,   3 ) = (     .00000,    .00000 )
Ybus (  1,   4 ) = (   -1.00000,   3.00000 )
Ybus (  1,   5 ) = (     .00000,    .00000 )
Ybus (  2,   1 ) = (     .00000,    .00000 )
Ybus (  2,   2 ) = (    3.10361,  -7.84115 )
Ybus (  2,   3 ) = (   -1.13122,   3.16742 )
Ybus (  2,   4 ) = (     .00000,    .00000 )
Ybus (  2,   5 ) = (   -1.97239,   4.73373 )
Ybus (  3,   1 ) = (     .00000,    .00000 )
Ybus (  3,   2 ) = (   -1.13122,   3.16742 )
Ybus (  3,   3 ) = (    2.16570,  -5.52121 )
Ybus (  3,   4 ) = (   -1.03448,   2.41379 )
Ybus (  3,   5 ) = (     .00000,    .00000 )
Ybus (  4,   1 ) = (   -1.00000,   3.00000 )
Ybus (  4,   2 ) = (     .00000,    .00000 )
Ybus (  4,   3 ) = (   -1.03448,   2.41379 )
Ybus (  4,   4 ) = (    2.85640,  -7.51557 )
Ybus (  4,   5 ) = (    -.82192,   2.19178 )
Ybus (  5,   1 ) = (     .00000,    .00000 )
Ybus (  5,   2 ) = (   -1.97239,   4.73373 )
Ybus (  5,   3 ) = (     .00000,    .00000 )
Ybus (  5,   4 ) = (    -.82192,   2.19178 )
Ybus (  5,   5 ) = (    2.79430,  -6.86551 )   Y55
```

```
For Transformer in line no. - 6
series admittance of line      (0.000000E+00,-2.625003)
shunt admittance of from bus   (0.000000E+00,-1.312528E-01)
shunt admittance of to bus     (0.000000E+00,1.250026E-01)
Ybus matrix after considering line transformer
Ybus (  1,   1 ) = (    1.00000,  -5.72626 )   Y11
Ybus (  1,   2 ) = (     .00000,   2.62500 )   Y12
Ybus (  1,   3 ) = (     .00000,    .00000 )   Y13
Ybus (  1,   4 ) = (   -1.00000,   3.00000 )
Ybus (  1,   5 ) = (     .00000,    .00000 )
Ybus (  2,   1 ) = (     .00000,   2.62500 )
Ybus (  2,   2 ) = (    3.10361, -10.34115 )
Ybus (  2,   3 ) = (   -1.13122,   3.16742 )
Ybus (  2,   4 ) = (     .00000,    .00000 )
Ybus (  2,   5 ) = (   -1.97239,   4.73373 )
Ybus (  3,   1 ) = (     .00000,    .00000 )
Ybus (  3,   2 ) = (   -1.13122,   3.16742 )
Ybus (  3,   3 ) = (    2.16570,  -5.52121 )
Ybus (  3,   4 ) = (   -1.03448,   2.41379 )
Ybus (  3,   5 ) = (     .00000,    .00000 )
Ybus (  4,   1 ) = (   -1.00000,   3.00000 )
Ybus (  4,   2 ) = (     .00000,    .00000 )
Ybus (  4,   3 ) = (   -1.03448,   2.41379 )
Ybus (  4,   4 ) = (    2.85640,  -7.51557 )
Ybus (  4,   5 ) = (    -.82192,   2.19178 )
Ybus (  5,   1 ) = (     .00000,    .00000 )
Ybus (  5,   2 ) = (   -1.97239,   4.73373 )
Ybus (  5,   3 ) = (     .00000,    .00000 )
Ybus (  5,   4 ) = (    -.82192,   2.19178 )
Ybus (  5,   5 ) = (    2.79430,  -6.86551 )   Y55
```

Example 3.7: Obtain $[Z_{Bus}]$ matrix for the system described in Example 3.6.

Solution:

$[Z_{Bus}]$ matrix is obtained by inverting the $[Y_{Bus}]$ matrix obtained in Example 3.6.

Execution of the computer program for matrix inversion MINV.FOR for Example 3.7

Input data: YBUS2B.DAT ($[Y_{Bus}]$ matrix)

Output of MINV.FOR: ZBUS2C.DAT

```
No. of buses = 5
Zbus matrix is
```

```
Zbus ( 1, 1 ) = (  .04924, -3.07140 )   Z11
Zbus ( 1, 2 ) = (  .01753, -3.33256 )   Z12
Zbus ( 1, 3 ) = (  .00257, -3.33543 )
Zbus ( 1, 4 ) = (  .01317, -3.26784 )
Zbus ( 1, 5 ) = (  .00668, -3.33713 )
Zbus ( 2, 1 ) = (  .01753, -3.33256 )
Zbus ( 2, 2 ) = (  .01607, -3.35777 )
Zbus ( 2, 3 ) = ( -.00602, -3.41489 )
Zbus ( 2, 4 ) = ( -.00791, -3.41450 )
Zbus ( 2, 5 ) = ( -.00212, -3.40069 )
Zbus ( 3, 1 ) = (  .00256, -3.33543 )
Zbus ( 3, 2 ) = ( -.00602, -3.41489 )
Zbus ( 3, 3 ) = (  .04346, -3.27301 )
Zbus ( 3, 4 ) = ( -.00351, -3.37645 )
Zbus ( 3, 5 ) = ( -.01535, -3.42836 )
Zbus ( 4, 1 ) = (  .01317, -3.26784 )
Zbus ( 4, 2 ) = ( -.00791, -3.41450 )
Zbus ( 4, 3 ) = ( -.00351, -3.37645 )
Zbus ( 4, 4 ) = (  .03559, -3.25728 )
Zbus ( 4, 5 ) = ( -.00328, -3.39083 )
Zbus ( 5, 1 ) = (  .00668, -3.33713 )
Zbus ( 5, 2 ) = ( -.00212, -3.40069 )
Zbus ( 5, 3 ) = ( -.01535, -3.42836 )
Zbus ( 5, 4 ) = ( -.00328, -3.39083 )
Zbus ( 5, 5 ) = (  .03806, -3.29810 )   Z55
```

Example 3.8: For Example 3.5, assume the transformer to be a phase angle regulator where $V_2'/V_1 = \varepsilon^{-j5^\circ}$. Find $[Y_{Bus}]$ matrix using computer method.

Solution:
The developed algorithm is executed with the following input data.
Here a (off-nominal tap ratio of transformer) is a complex quantity.

$$a = (\cos 5^\circ - j\sin 5^\circ) = (0.996195 - j0.087156) \qquad \text{(approx.)}$$

Execution of the computer program TMYBUS.FOR for Example 3.8

Line data: ZBUS2D.DAT

6,5,1 [No. of lines, No. of buses, No. of transformers]

1,4,(0.10,0.30),(0.0,0.0) [From bus, To bus, (R, X), (G/2, B/2)]

2,3,(0.10,0.28),(0.0,0.0)

2,5,(0.075,0.18),(0.0,0.0)

```
3,4,(0.15,0.35),(0.0,0.0)
4,5,(0.15,0.40),(0.0,0.0)
1,2,(0.0,0.4),(0.996195,-0.087156)  [From bus, To bus, Transf. Imp., Rectangular
                                     form of off-nominal tap ratio of transformer]
```

Output of TMYBUS.FOR: YBUS2D.DAT

```
No. of buses =  5
Before considering line transformer
Ybus matrix
Ybus (  1,   1 ) = (    1.00000,  -3.00000 )   Y11
Ybus (  1,   2 ) = (     .00000,    .00000 )   Y12
Ybus (  1,   3 ) = (     .00000,    .00000 )
Ybus (  1,   4 ) = (   -1.00000,   3.00000 )
Ybus (  1,   5 ) = (     .00000,    .00000 )
Ybus (  2,   1 ) = (     .00000,    .00000 )
Ybus (  2,   2 ) = (    3.10361,  -7.90115 )
Ybus (  2,   3 ) = (   -1.13122,   3.16742 )
Ybus (  2,   4 ) = (     .00000,    .00000 )
Ybus (  2,   5 ) = (   -1.97239,   4.73373 )
Ybus (  3,   1 ) = (     .00000,    .00000 )
Ybus (  3,   2 ) = (   -1.13122,   3.16742 )
Ybus (  3,   3 ) = (    2.16570,  -5.58121 )
Ybus (  3,   4 ) = (   -1.03448,   2.41379 )
Ybus (  3,   5 ) = (     .00000,    .00000 )
Ybus (  4,   1 ) = (   -1.00000,   3.00000 )
Ybus (  4,   2 ) = (     .00000,    .00000 )
Ybus (  4,   3 ) = (   -1.03448,   2.41379 )
Ybus (  4,   4 ) = (    2.85640,  -7.60557 )
Ybus (  4,   5 ) = (    -.82192,   2.19178 )
Ybus (  5,   1 ) = (     .00000,    .00000 )
Ybus (  5,   2 ) = (   -1.97239,   4.73373 )
Ybus (  5,   3 ) = (     .00000,    .00000 )
Ybus (  5,   4 ) = (    -.82192,   2.19178 )
Ybus (  5,   5 ) = (    2.79430,  -6.92551 )   Y55

For Transformer in line no. -  6

series admittance of line       (-2.178898E-01,-2.490486)
shunt admittance of from bus    (-2.162314E-01,2.846828E-02)
shunt admittance of to bus      (2.178898E-01,-9.514092E-03)
```

```
Ybus matrix after considering line transformer
Ybus (  1,  1 ) = (    .56588,   -5.46202 )  Y11
Ybus (  1,  2 ) = (    .21789,    2.49049 )  Y12
Ybus (  1,  3 ) = (    .00000,     .00000 )
Ybus (  1,  4 ) = (  -1.00000,    3.00000 )
Ybus (  1,  5 ) = (    .00000,     .00000 )
Ybus (  2,  1 ) = (    .21789,    2.49049 )
Ybus (  2,  2 ) = (   3.10361,  -10.40115 )
Ybus (  2,  3 ) = (  -1.13122,    3.16742 )
Ybus (  2,  4 ) = (    .00000,     .00000 )
Ybus (  2,  5 ) = (  -1.97239,    4.73373 )
Ybus (  3,  1 ) = (    .00000,     .00000 )
Ybus (  3,  2 ) = (  -1.13122,    3.16742 )
Ybus (  3,  3 ) = (   2.16570,   -5.58121 )
Ybus (  3,  4 ) = (  -1.03448,    2.41379 )
Ybus (  3,  5 ) = (    .00000,     .00000 )
Ybus (  4,  1 ) = (  -1.00000,    3.00000 )
Ybus (  4,  2 ) = (    .00000,     .00000 )
Ybus (  4,  3 ) = (  -1.03448,    2.41379 )
Ybus (  4,  4 ) = (   2.85640,   -7.60557 )
Ybus (  4,  5 ) = (   -.82192,    2.19178 )
Ybus (  5,  1 ) = (    .00000,     .00000 )
Ybus (  5,  2 ) = (  -1.97239,    4.73373 )
Ybus (  5,  3 ) = (    .00000,     .00000 )
Ybus (  5,  4 ) = (   -.82192,    2.19178 )
Ybus (  5,  5 ) = (   2.79430,   -6.92551 )  Y55
```

Example 3.9: For Example 3.5, assume the transformer to be a magnitude regulator with ratio $V_1/V_2' = 1.02{:}1$. Find $[Y_{Bus}]$ matrix using computer method.

Solution:

Here $V_1/V_2' = 1.02$. The developed algorithm is executed with the input data as given below [transformer at j-th bus, turns ratio a:1]:

Execution of the computer program TMYBUS.FOR for example 3.9

Line data: ZBUS2E.DAT

6,5,1 [No. of lines, No. of buses, No. of transformers]

```
1,4,(0.10,0.30),(0.0,0.0)      [From bus, To bus, (R, X), (G/2, B/2)]
2,3,(0.10,0.28),(0.0,0.0)
2,5,(0.075,0.18),(0.0,0.0)
```

```
3,4,(0.15,0.35),(0.0,0.0)
4,5,(0.15,0.40),(0.0,0.0)
1,2,(0.0,0.4),(1.02,0.0)
```

[From bus, To bus, transformer impedance, a]

Output of TMYBUS.FOR: YBUS2E.DAT

```
No. of buses =  5
Before considering line transformer
Ybus matrix
Ybus (  1,   1 ) = (     1.00000,  -3.00000 )  Y11
Ybus (  1,   2 ) = (      .00000,    .00000 )  Y12
Ybus (  1,   3 ) = (      .00000,    .00000 )
Ybus (  1,   4 ) = (    -1.00000,   3.00000 )
Ybus (  1,   5 ) = (      .00000,    .00000 )
Ybus (  2,   1 ) = (      .00000,    .00000 )
Ybus (  2,   2 ) = (     3.10361,  -7.90115 )
Ybus (  2,   3 ) = (    -1.13122,   3.16742 )
Ybus (  2,   4 ) = (      .00000,    .00000 )
Ybus (  2,   5 ) = (    -1.97239,   4.73373 )
Ybus (  3,   1 ) = (      .00000,    .00000 )
Ybus (  3,   2 ) = (    -1.13122,   3.16742 )
Ybus (  3,   3 ) = (     2.16570,  -5.58121 )
Ybus (  3,   4 ) = (    -1.03448,   2.41379 )
Ybus (  3,   5 ) = (      .00000,    .00000 )
Ybus (  4,   1 ) = (    -1.00000,   3.00000 )
Ybus (  4,   2 ) = (      .00000,    .00000 )
Ybus (  4,   3 ) = (    -1.03448,   2.41379 )
Ybus (  4,   4 ) = (     2.85640,  -7.60557 )
Ybus (  4,   5 ) = (     -.82192,   2.19178 )
Ybus (  5,   1 ) = (      .00000,    .00000 )
Ybus (  5,   2 ) = (    -1.97239,   4.73373 )
Ybus (  5,   3 ) = (      .00000,    .00000 )
Ybus (  5,   4 ) = (     -.82192,   2.19178 )
Ybus (  5,   5 ) = (     2.79430,  -6.92551 )  Y55

For Transformer in line -   6

series admittance of line       (0.000000E+00,4.99995E-02)
shunt admittance of from bus    (0.000000E+00,-5.00000E-02)
shunt admittance of to bus      (0.000000E+00,-5.099995E-02)
```

```
Ybus matrix after considering line transformer
Ybus ( 1, 1 ) = (    1.00000,  -5.50000 ) Y11
Ybus ( 1, 2 ) = (     .00000,   2.55102 ) Y12
Ybus ( 1, 3 ) = (     .00000,    .00000 ) Y13
Ybus ( 1, 4 ) = (   -1.00000,   3.00000 ) Y14
Ybus ( 1, 5 ) = (     .00000,    .00000 )
Ybus ( 2, 1 ) = (     .00000,   2.55102 )
Ybus ( 2, 2 ) = (    3.10361, -10.50215 )
Ybus ( 2, 3 ) = (   -1.13122,   3.16742 )
Ybus ( 2, 4 ) = (     .00000,    .00000 )
Ybus ( 2, 5 ) = (   -1.97239,   4.73373 )
Ybus ( 3, 1 ) = (     .00000,    .00000 )
Ybus ( 3, 2 ) = (   -1.13122,   3.16742 )
Ybus ( 3, 3 ) = (    2.16570,  -5.58121 )
Ybus ( 3, 4 ) = (   -1.03448,   2.41379 )
Ybus ( 3, 5 ) = (     .00000,    .00000 )
Ybus ( 4, 1 ) = (   -1.00000,   3.00000 )
Ybus ( 4, 2 ) = (     .00000,    .00000 )
Ybus ( 4, 3 ) = (   -1.03448,   2.41379 )
Ybus ( 4, 4 ) = (    2.85640,  -7.60557 )
Ybus ( 4, 5 ) = (    -.82192,   2.19178 )
Ybus ( 5, 1 ) = (     .00000,    .00000 )
Ybus ( 5, 2 ) = (   -1.97239,   4.73373 )
Ybus ( 5, 3 ) = (     .00000,    .00000 )
Ybus ( 5, 4 ) = (    -.82192,   2.19178 )
Ybus ( 5, 5 ) = (    2.79430,  -6.92551 ) Y55
```

3.5 DEVELOPMENT OF [Y_{BUS}] USING SINGULAR TRANSFORMATION

In order to develop [Y_{Bus}] by the concept of *singular transformation*, the basic building block would be the *primitive network* and its corresponding matrix. *A network is said to be primitive when the network elements are not interconnected with other part of the whole network.* The performance equations of such a network are described below:

$V + E = zI$ (in impedance form referring Fig. 3.7(a))

$I + i = yV$ (in admittance form referring Fig. 3.7(b))

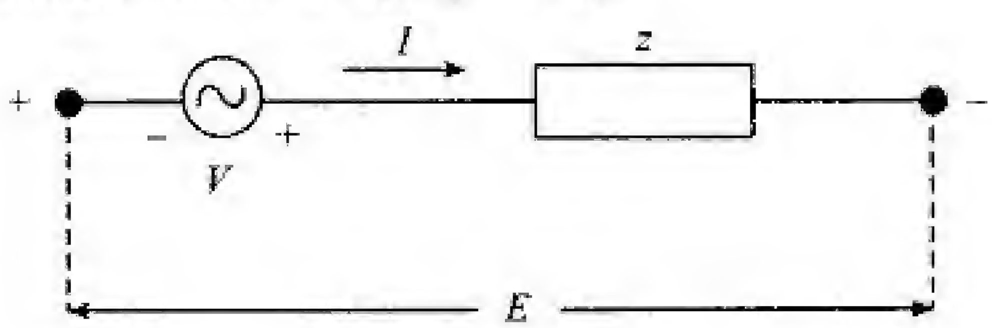


Fig. 3.7(a) Primitive network/branch in impedance form.

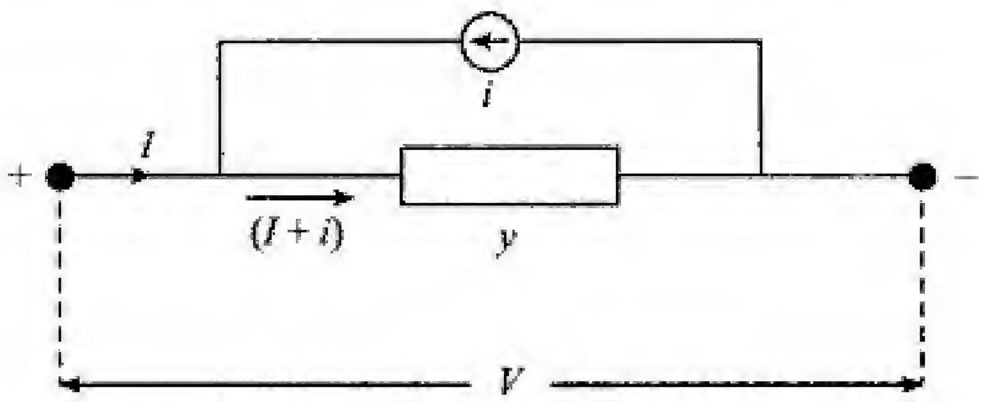


Fig. 3.7(b) Primitive network/branch in admittance form.

In both the networks V and I are the branch voltage and current vectors, while E and i are source vectors, respectively. z and y refer to primitive impedance and admittance matrices ($Z = Y^{-1}$). Mutual coupling between branches is neglected.

The drawback of the primitive matrices is that they themselves do not offer any information regarding the way in which various branches in a network are interconnected. For this reason the information regarding connection of the elements is arranged in a matrix form in correspondence with proper nodes (or buses) to which these branches are connected. The connection matrix is a $(b \times n)$ matrix, with b representing the number of branches and n the number of nodes or buses. This is termed as *node incidence matrix* (A') and is a rectangular type singular matrix. As per standard convention of circuit theory, the branch orientations are recorded in the matrix as under:

$A'_{ij} = +1$ when the current in the branch/element i is directed away from node j,

$A'_{ij} = -1$ if the current in the branch/element i enters towards node j,

$A'_{ij} = 0$ if branch/element i is not connected to node j.

Since $(I + i) = yV$, for branch b,

$$I_b = yV_b - i_b \qquad \text{[}y\text{ being the branch admittance]}$$

Pre-multiplying by A^T, transpose of the reduced incidence matrix*,

$$A^T I_b = A^T yV_b - A^T i_b \tag{3.28}$$

However, as the algebraic sum of the currents entering each bus is zero,

$$A^T I_b = 0$$

This gives
$$0 = A^T yV_b - A^T i_b \text{ or, } A^T i_b = A^T yV_b \tag{3.29}$$

Again $A^T i_b$ is a vector sum of all source currents entering each bus and can be designated as I_S and obviously is of order $(n \times 1)$.

$$\therefore \quad I_S = A^T yV_b = A^T yAV_S \quad [V_S = \text{source/bus voltage}]$$

i.e.
$$[I] = [A^T yA][V] \tag{3.30}$$

Thus we finally obtain the bus admittance matrix $[Y_{Bus}]$ as

$$[Y_{Bus}] = [A^T]\,[y]\,[A] \tag{3.31}$$

$[Y_{Bus}]$ is a $n \times n$ matrix.

*The reduced incidence matrix is the incidence matrix deprived of reference node.

An illustration

Consider the system shown in Fig. 3.8(a). we are to develop $[Y_{Bus}]$ using singular transformation.

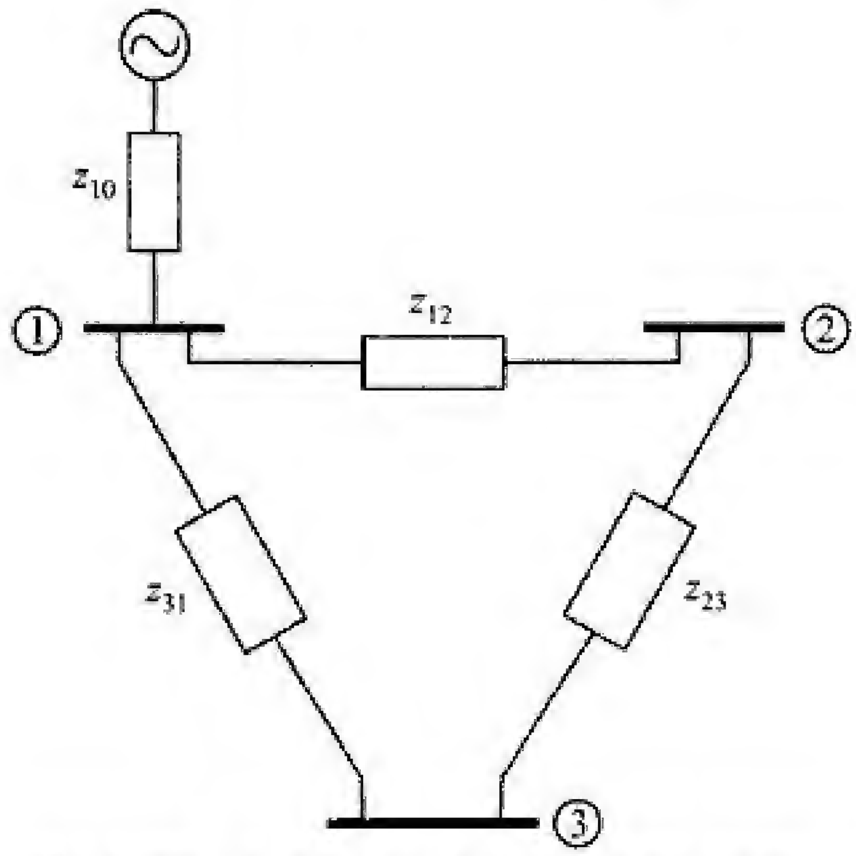


Fig. 3.8(a) A three-bus three-line power system.

The oriented/connected graph of the system is shown below (for mutually coupled branch, node near to *dot* is considered having outgoing current). See Fig. 3.8(b).

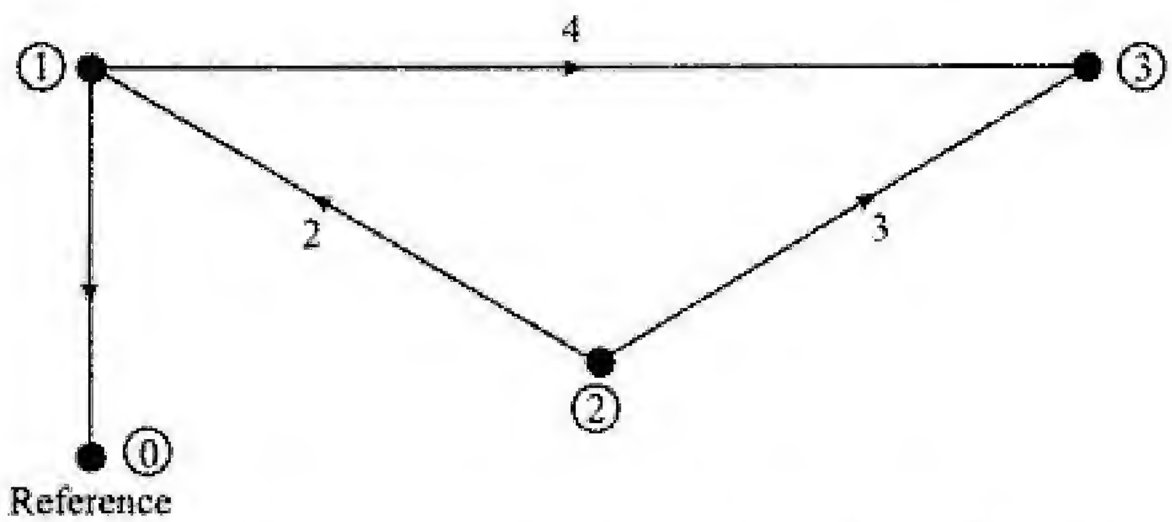


Fig. 3.8(b) Oriented/connected graph of the system shown in Fig. 3.8(a).

The node incidence matrix of the system is

$$
\begin{array}{ll}
 & \text{nodes} \rightarrow \\
 & \begin{array}{cccc} \;\;1 & \;\;2 & \;\;3 & \;\;0 \end{array} \\
\begin{array}{r} \text{branches} \;\; 1 \\ \downarrow \quad\;\; 2 \\ 3 \\ 4 \end{array} &
\begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \end{bmatrix}
\end{array}
$$

Deleting the column corresponding to reference node, the reduced incidence matrix is,

$$[A] = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

Primitive impedance matrix is

$$[z] = \begin{array}{c} \text{branches} \\ \downarrow \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \text{branches} \rightarrow \\ \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} z_{10} & 0 & 0 & 0 \\ 0 & z_{12} & 0 & 0 \\ 0 & 0 & z_{23} & 0 \\ 0 & 0 & 0 & z_{13} \end{bmatrix} \end{array}$$

$\therefore$ Primitive admittance matrix,

$$[y] = [z]^{-1} = \begin{bmatrix} y_{10} & 0 & 0 & 0 \\ 0 & y_{12} & 0 & 0 \\ 0 & 0 & y_{23} & 0 \\ 0 & 0 & 0 & y_{13} \end{bmatrix}$$

$\therefore$

$$[y][A] = \begin{bmatrix} y_{10} & 0 & 0 & 0 \\ 0 & y_{12} & 0 & 0 \\ 0 & 0 & y_{23} & 0 \\ 0 & 0 & 0 & y_{13} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} y_{10} & 0 & 0 \\ -y_{12} & y_{12} & 0 \\ 0 & y_{23} & -y_{23} \\ y_{13} & 0 & -y_{13} \end{bmatrix}$$

$\therefore$

$$[A^T][y][A] = \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} y_{10} & 0 & 0 \\ -y_{12} & y_{12} & 0 \\ 0 & y_{23} & -y_{23} \\ y_{13} & 0 & -y_{13} \end{bmatrix}$$

i.e.

$$[Y_{Bus}] = \begin{bmatrix} (y_{10} + y_{12} + y_{13}) & -y_{12} & -y_{13} \\ -y_{12} & (y_{12} + y_{23}) & -y_{23} \\ -y_{13} & -y_{23} & (y_{23} + y_{13}) \end{bmatrix}$$

Figure 3.9 represents the flowchart for the development of $[Y_{Bus}]$ matrix using singular transformation.

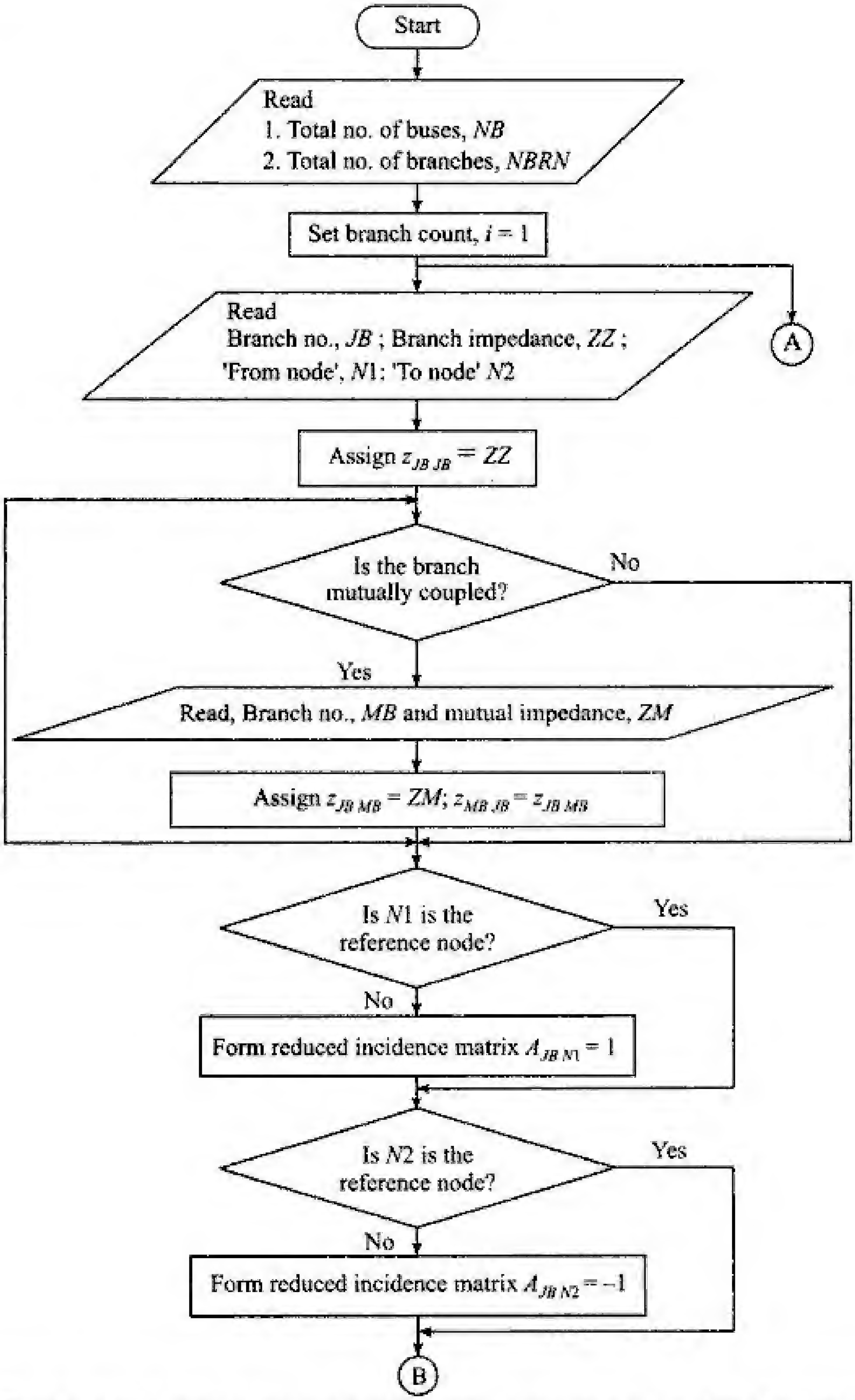


Fig. 3.9 Flowchart for development of $[Y_{Bus}]$ matrix using singular transformation *(Contd.)*...

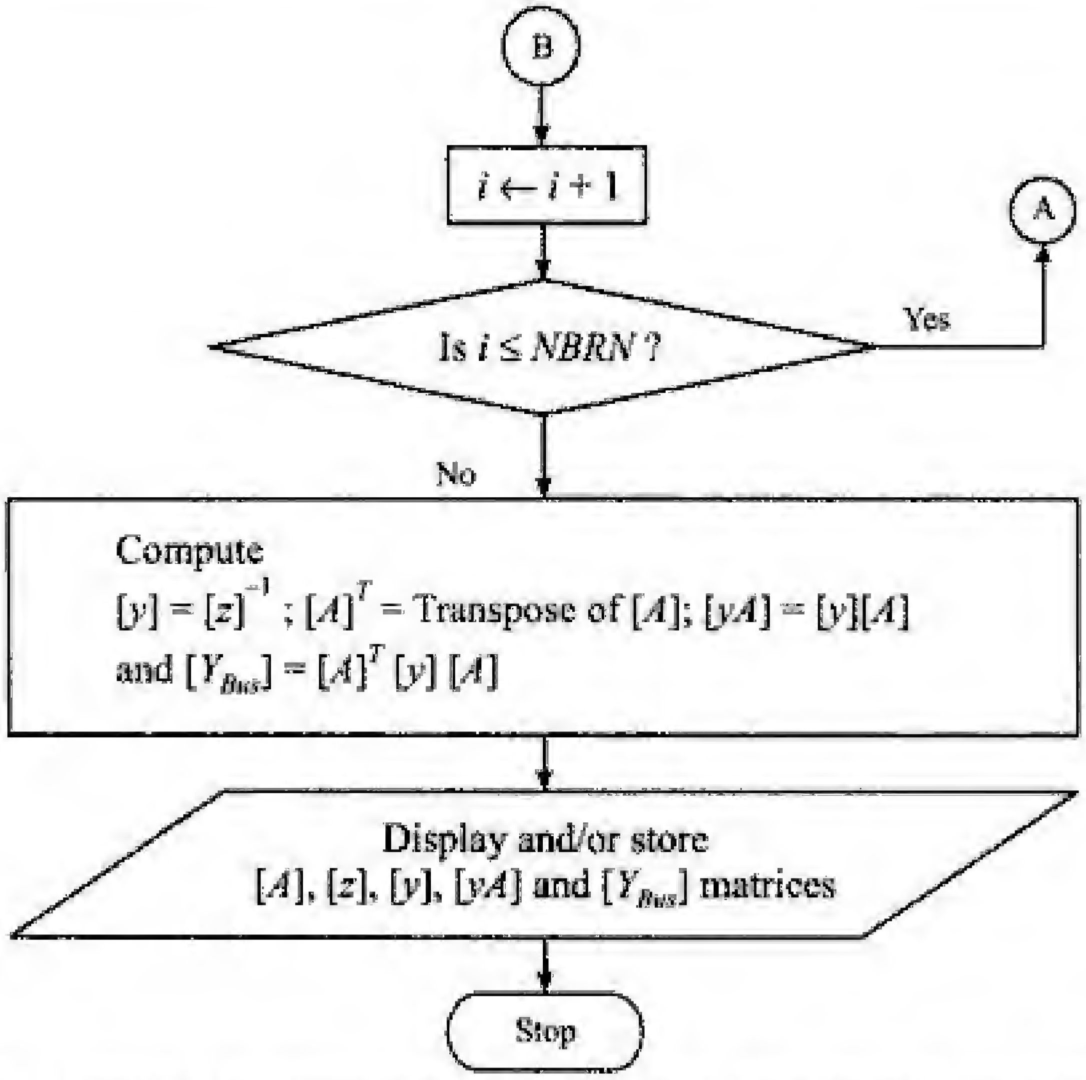


Fig. 3.9 Flowchart for development of $[Y_{Bus}]$ matrix using singular transformation.

Example 3.10: In a portion of a power system network (Fig. E3.5(a), two branches 1-2 and 2-3 are mutually coupled through z_m ($= j0.2$ p.u.). Find the bus admittance matrix using singular transformation.

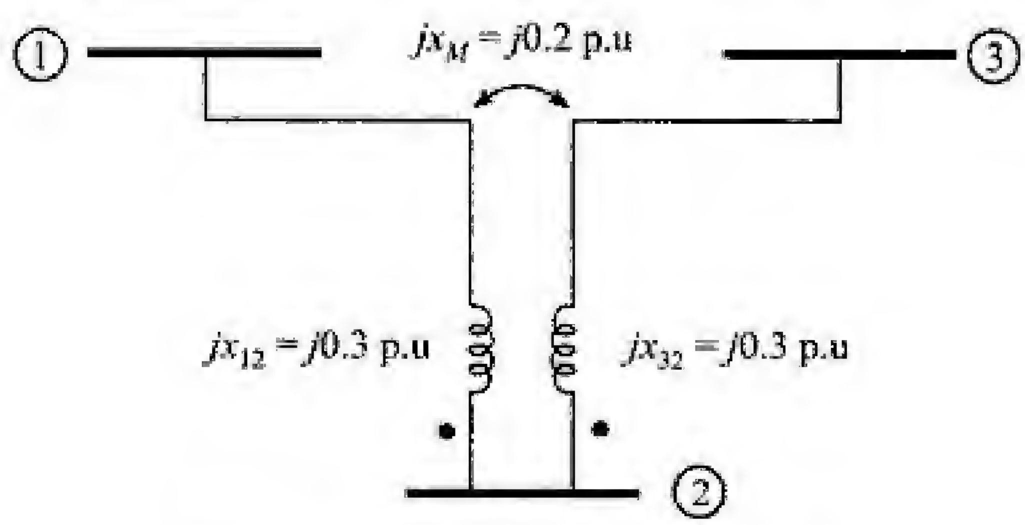


Fig. E3.5(a) Mutually coupled networks.

Solution: The oriented/connected graph of the system is shown in Fig. E3.5(b).

Fig. E3.5(b) Oriented/connected graph of the system shown in Fig. E3.5(a).

(Since there is a mutual coupling between branches 1 and 2, and for both branches the dots are towards bus 2, bus 2 is taken as from *node* for both of these two branches.)

Reduced incidence matrix is given by

$$[A] = \begin{array}{c} \text{branches} \\ \downarrow \end{array} \begin{array}{c} \\ 1 \\ 2 \end{array} \overset{\begin{array}{c} \text{nodes} \rightarrow \\ 1 \quad 2 \quad 3 \end{array}}{\begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}}$$

However, primitive impedance matrix is given by

$$[z] = \begin{array}{c} \\ 1 \\ 2 \end{array} \overset{\begin{array}{cc} 1 & 2 \end{array}}{\begin{bmatrix} j0.3 & j0.2 \\ j0.2 & j0.3 \end{bmatrix}}$$

i.e. the primitive admittance matrix becomes,

$$[y] = [z]^{-1} = \begin{array}{c} \\ 1 \\ 2 \end{array} \overset{\begin{array}{cc} 1 & 2 \end{array}}{\begin{bmatrix} -j6 & j4 \\ j4 & -j6 \end{bmatrix}}$$

$$\therefore \qquad [y][A] = \begin{bmatrix} -j6 & j4 \\ j4 & -j6 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} j6 & (-j6+j4) & -j4 \\ -j4 & (j4-j6) & j6 \end{bmatrix}$$

$$= \begin{bmatrix} j6 & -j2 & -j4 \\ -j4 & -j2 & j6 \end{bmatrix}$$

Now

$$[A^T] = \begin{bmatrix} -1 & 0 \\ 1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$\therefore \qquad [Y_{Bus}] = [A^T] \; [y][A] = \begin{bmatrix} -1 & 0 \\ 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} j6 & -j2 & -j4 \\ -j4 & -j2 & j6 \end{bmatrix}$$

$$= \begin{bmatrix} (-j6) & (j2) & (j4) \\ (j6-j4) & (-j2-j2) & (-j4+j6) \\ (j4) & (j2) & (-j6) \end{bmatrix} = \begin{bmatrix} -j6 & j2 & j4 \\ j2 & -j4 & j2 \\ j4 & j2 & -j6 \end{bmatrix}$$

Execution of the computer program ATYA.FOR for Example 3.10

Input required for computer simulation:

No. of buses = 3; No. of branches = 2

For branch-1 : Branch no. = 1; branch impedance = $j0.3$ p.u.;

From node = 2; to node = 1 [since the dot is towards node 2]

```
Ybus(  2,   1 ) = (   .000000,  2.000000 )
Ybus(  2,   2 ) = (   .000000, -4.000000 )
Ybus(  2,   3 ) = (   .000000,  2.000000 )
Ybus(  3,   1 ) = (   .000000,  4.000000 )
Ybus(  3,   2 ) = (   .000000,  2.000000 )
Ybus(  3,   3 ) = (   .000000, -6.000000 )   Y33
```

3.6 DEVELOPMENT OF $[Y_{BUS}]$ MATRIX USING COEFFICIENT MATRIX

Let us consider Fig. 3.10 using *current injection vectors*, I_i and I_j; following the sign convention outlined above, we can write

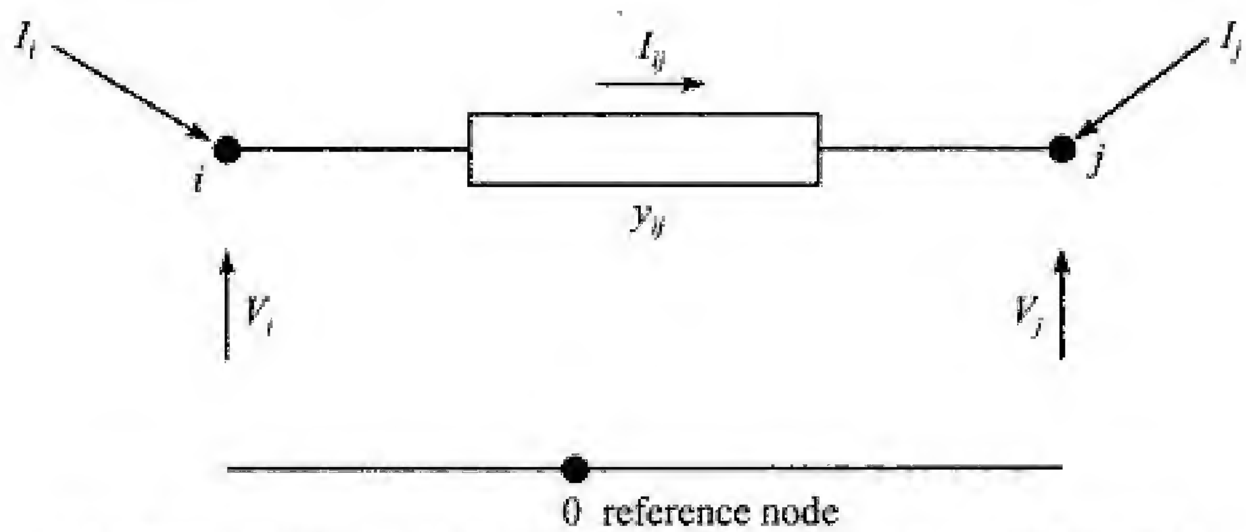


Fig. 3.10 Nodal currents injected into a single branch two-node circuit.

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{array}{c} i \\ j \end{array}\begin{bmatrix} +1 \\ -1 \end{bmatrix} I_{ij} \tag{3.32}$$

by arranging the respective nodal current equations in the vector form. Equation (3.32) indicates the *direction* of I_{ij} from i to j-th node with +1 and −1 entries being designated as rows i and j. Assuming voltage drop across y_{ij} to be V_{ij} and being *directed towards* I_{ij}, the node voltage governing equation $V_{ij} = V_i - V_j$ can be represented similarly in the vector form as

$$V_{ij} = \begin{bmatrix} +1 & -1 \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} \tag{3.33}$$

However, from the basic knowledge of circuit theory,

$$y_{ij} V_{ij} = I_{ij} \quad \text{or,} \quad y_{ij} \begin{bmatrix} +1 & -1 \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} = I_{ij} \tag{3.34}$$

Pre-multiplying both sides of equation (3.34) by the column of equation (3.32), we find

$$\begin{array}{c} i \\ j \end{array}\begin{bmatrix} +1 \\ -1 \end{bmatrix} y_{ij} \begin{bmatrix} +1 & -1 \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} = \begin{bmatrix} I_i \\ I_j \end{bmatrix} \tag{3.35}$$

i.e.

$$\begin{bmatrix} y_{ij} & -y_{ij} \\ -y_{ij} & y_{ij} \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} = \begin{bmatrix} I_i \\ I_j \end{bmatrix} \tag{3.36}$$

or,

$$[Y]\ [V] = [I] \tag{3.37}$$

It may be observed that the sum of the elements in each column of [Y] in equation (3.37) is a *singular matrix* and hence the rows of the matrix are linearly dependent. However, if one of the buses is selected as *reference* (or *slack* or *swing*), the matrix [Y] becomes *nonsingular* and it becomes easier to handle it mathematically. (A singular matrix has no inverse and with power system singular [Y] matrix, it is not possible to express bus voltages in the form $[V] = [Y]^{-1}\,[I]$, the set of equations given by $[I] = [Y][V]$ becomes unsolvable.) By taking one of the buses as reference, the corresponding row and column are deleted from [Y] matrix and hence the reduced matrix becomes nonsingular. Thus, provided one of the branches is connected to the reference (slack) node, the nonsingular form of [Y] matrix is obtained and is designated by $[Y_{Bus}]$.

[In this discussion let node j be selected as *slack* node; then equation (3.36) reduces to

$$[y_{ij}]V_i = I_i \tag{3.38)]}$$

Let us now concentrate on the building of $[Y_{Bus}]$ for a general system with multiple buses and interconnection by generalising the two node models under discussion.

The coefficient matrix relating the node voltages and currents in equation (3.36) can be written as

$\begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}$. Relating this coefficient matrix of equation (3.36) with those of equation (3.35), we obtain

$$\begin{array}{c} i \\ j \end{array}\begin{bmatrix} +1 \\ -1 \end{bmatrix} \begin{array}{c} \begin{array}{cc} i & j \end{array} \\ [+1 \;\; -1] \end{array} = \begin{array}{cc} & \begin{array}{cc} i & j \end{array} \\ \begin{array}{c} i \\ j \end{array} & \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} \end{array} \tag{3.39}$$

This concept is very important *building block* for representing general networks. The *row* and *column pointers i, j* identify each entry in the coefficient matrix by *nodal numbers*. The following example will further exemplify the concept:

Let us assume a three-bus equivalent circuit of a power network as shown in Fig. 3.11.

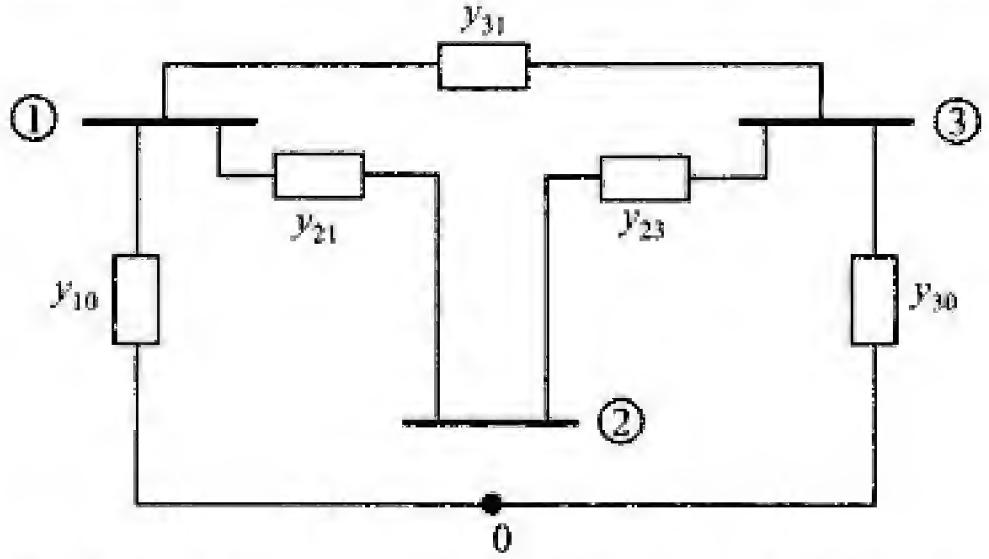


Fig. 3.11 A three-bus network equivalent in admittance form.

In Fig. 3.11, the admittance of each branch is equal to the reciprocal of the branch impedance. Two branches y_{10} and y_{30} are connected directly to the reference node 0 and are governed by equation (3.38) while the three other branches, y_{12}, y_{23}, y_{13}, are characterised by equation (3.36).

Let us develop the *individual building blocks* first for each branch following the *coefficient matrix technique* outlined above. Here the individual blocks (matrices) are:

$$\begin{array}{cc} & 1 \\ 1 & [+1] \end{array} y_{10}\,; \quad \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} \end{array} y_{21}\,; \quad \begin{array}{cc} & \begin{array}{cc} 1 & 3 \end{array} \\ \begin{array}{c} 1 \\ 3 \end{array} & \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} \end{array} y_{31}\,;$$

$$\begin{array}{c|cc} & 3 & 2 \\ \hline 3 & +1 & -1 \\ 2 & -1 & +1 \end{array}\; y_{23}\,; \qquad \begin{array}{c|c} & 3 \\ \hline 3 & +1 \end{array}\; y_{30} \qquad \begin{bmatrix} \text{Pointers are always placed} \\ \text{“from” “to” mode.} \end{bmatrix}$$

Combining these elements of the above matrices having *identical* row and column pointers/labels, we obtain $[Y_{Bus}]$ as

$$[Y_{Bus}] = \begin{bmatrix} y_{10} + y_{21} + y_{31} & -y_{21} & -y_{31} \\ -y_{21} & y_{21} + y_{23} & -y_{23} \\ -y_{31} & -y_{23} & y_{30} + y_{31} + y_{23} \end{bmatrix} \tag{3.40}$$

The bus admittance matrix obtained here follows the usual rules (or properties) detailed in the text of development of $[Y_{Bus}]$ by *Nodal Method. This method of developing* $[Y_{Bus}]$ *is advantageous when extended to the development of* $[Y_{Bus}]$ *for the networks having mutually coupled branches.* Before we proceed to that part, let us illustrate the procedure developed here for a typical system and formulate the steps of the algorithm.

3.6.1 Steps of Algorithm to Develop $[Y_{Bus}]$ When there is no Mutual Coupling between Branches (Using Coefficient Matrix)

Step 1: Obtain the admittance of each branch (primitive admittance) of the equivalent network which is the reciprocal of the corresponding branch impedances. In case any bus is having generator or motor and/or transformer, the respective positive sequence reactances of the generator or motor and/or transformers should be combined for each bus and the combined reactances and generated emfs are replaced by the equivalent current sources and shunt admittances as illustrated below (Fig. 3.12(a))

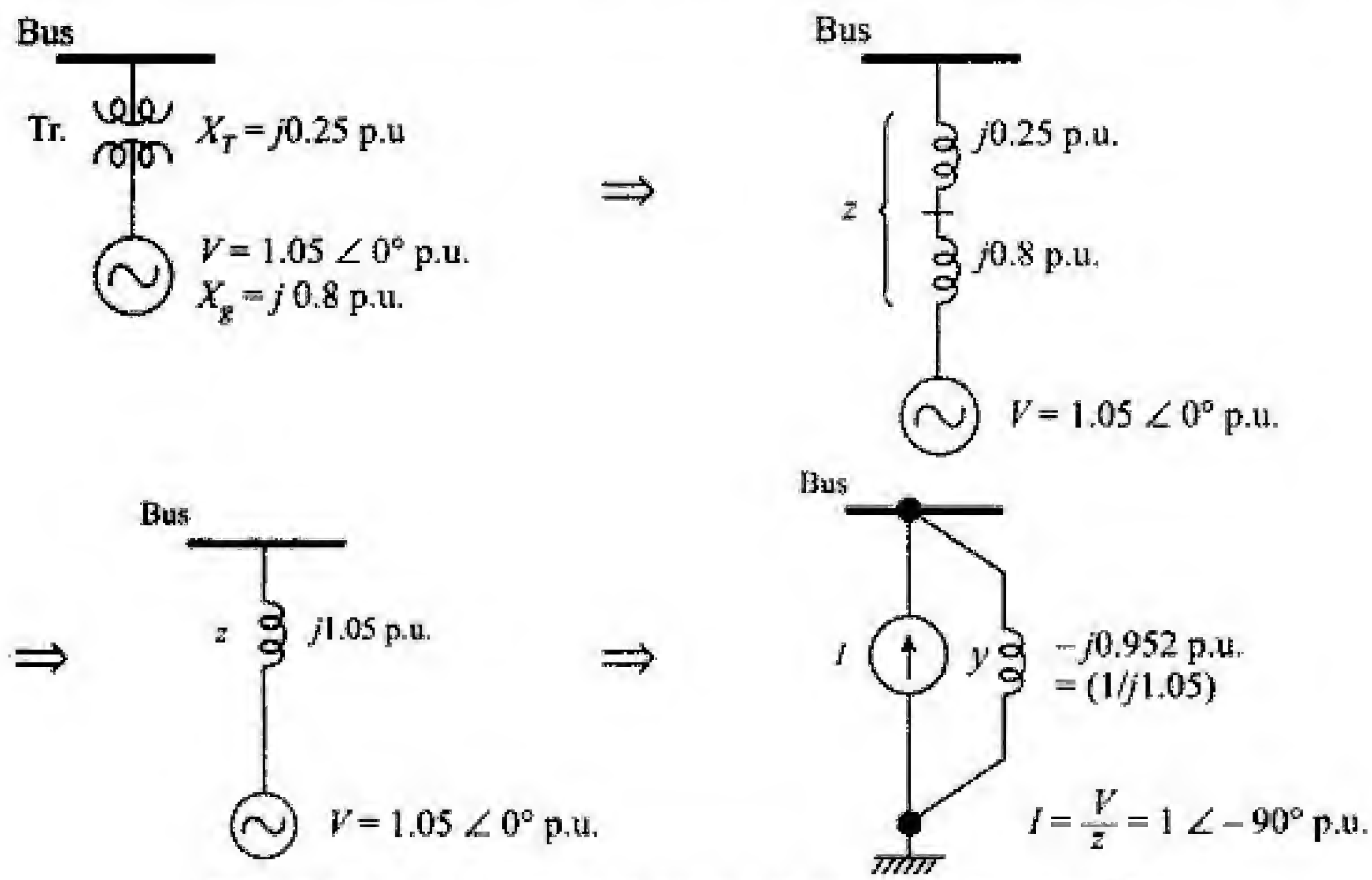


Fig. 3.12(a) Development of shunt admittance for any node/bus.

Step 2: Insert coefficient matrix $\begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}$ with each primitive branch admittance and designate the pointers as shown with respect to Fig. 3.12(b).

$$\begin{array}{c c} & \begin{array}{cc} i & j \end{array} \\ \begin{array}{c} i \\ j \end{array} & \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} y_{ij} \end{array}$$

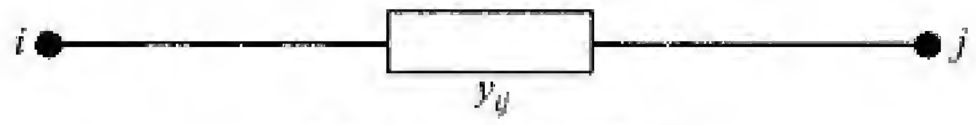
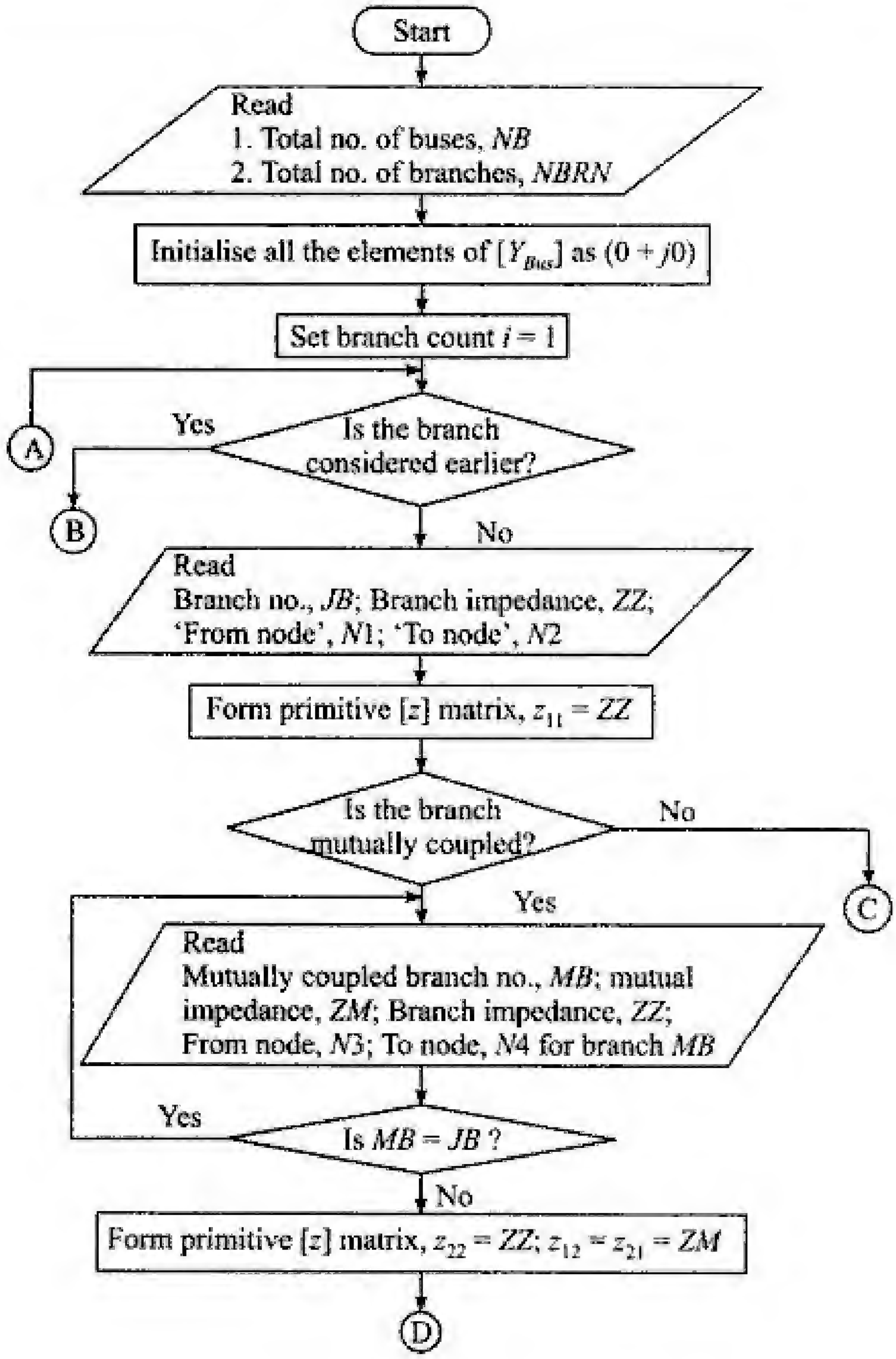


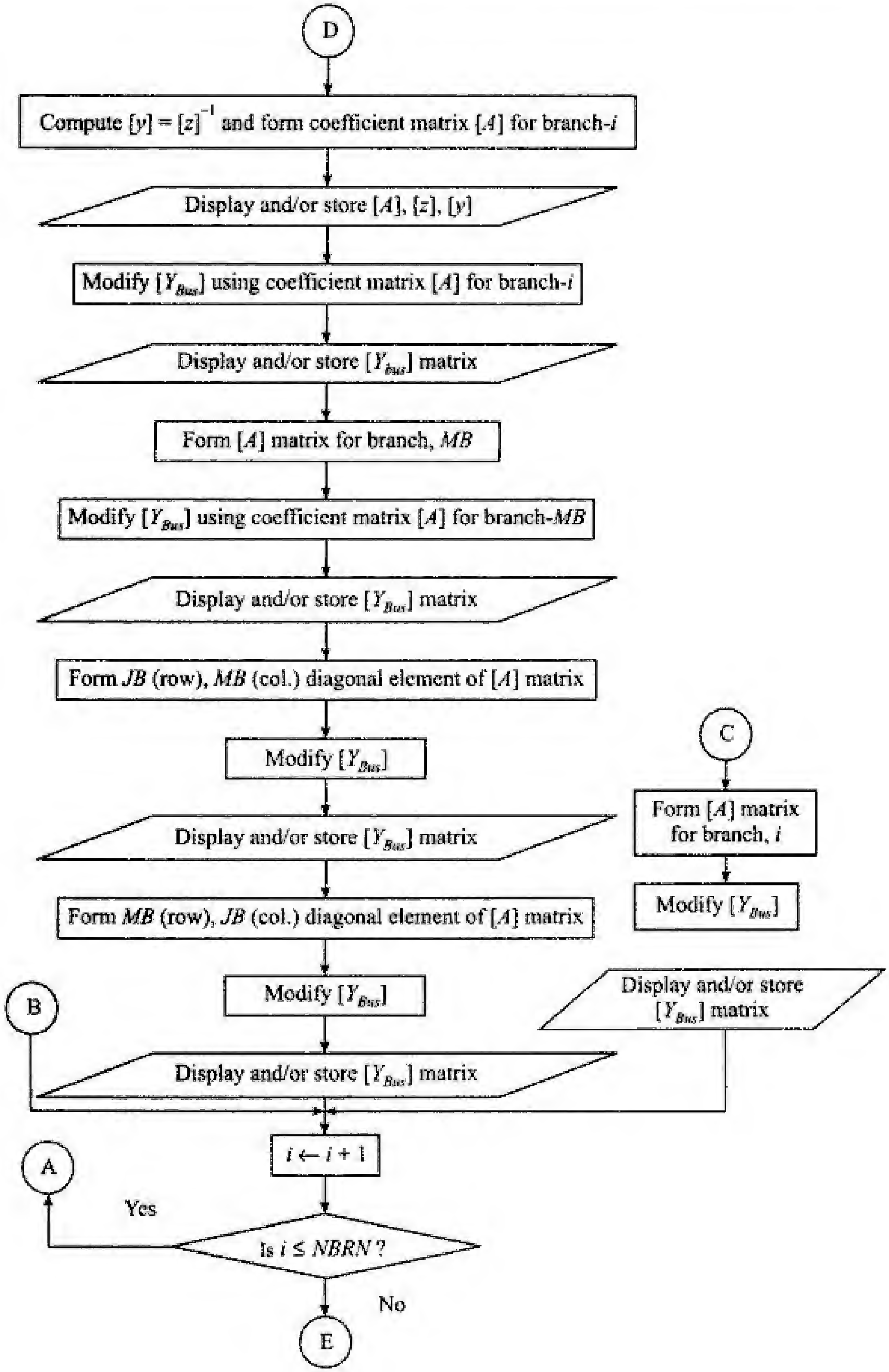
Fig. 3.12(b) Branch *ij*.

(Keep in mind that the columns and rows follow identical order.)

Step 3: Combine together the admittance elements of Step 2 for the matrices obtained, having identical row and column pointers. $[Y_{Bus}]$ is formed.

Figure 3.13 represents the complete flowchart for the development of $[Y_{Bus}]$ matrix using coefficient matrix.




D
Compute [y] = [z]^-1 and form coefficient matrix [A] for branch-i
Display and/or store [A], [z], [y]
Modify [Y_Bus] using coefficient matrix [A] for branch-i
Display and/or store [Y_bus] matrix
Form [A] matrix for branch, MB
Modify [Y_Bus] using coefficient matrix [A] for branch-MB
Display and/or store [Y_Bus] matrix
Form JB (row), MB (col.) diagonal element of [A] matrix
Modify [Y_Bus]
Display and/or store [Y_Bus] matrix
Form MB (row), JB (col.) diagonal element of [A] matrix
Modify [Y_Bus]
Display and/or store [Y_Bus] matrix
C
Form [A] matrix for branch, i
Modify [Y_Bus]
Display and/or store [Y_Bus] matrix
B
i ← i + 1
A
Yes
Is i ≤ NBRN ?
No
E

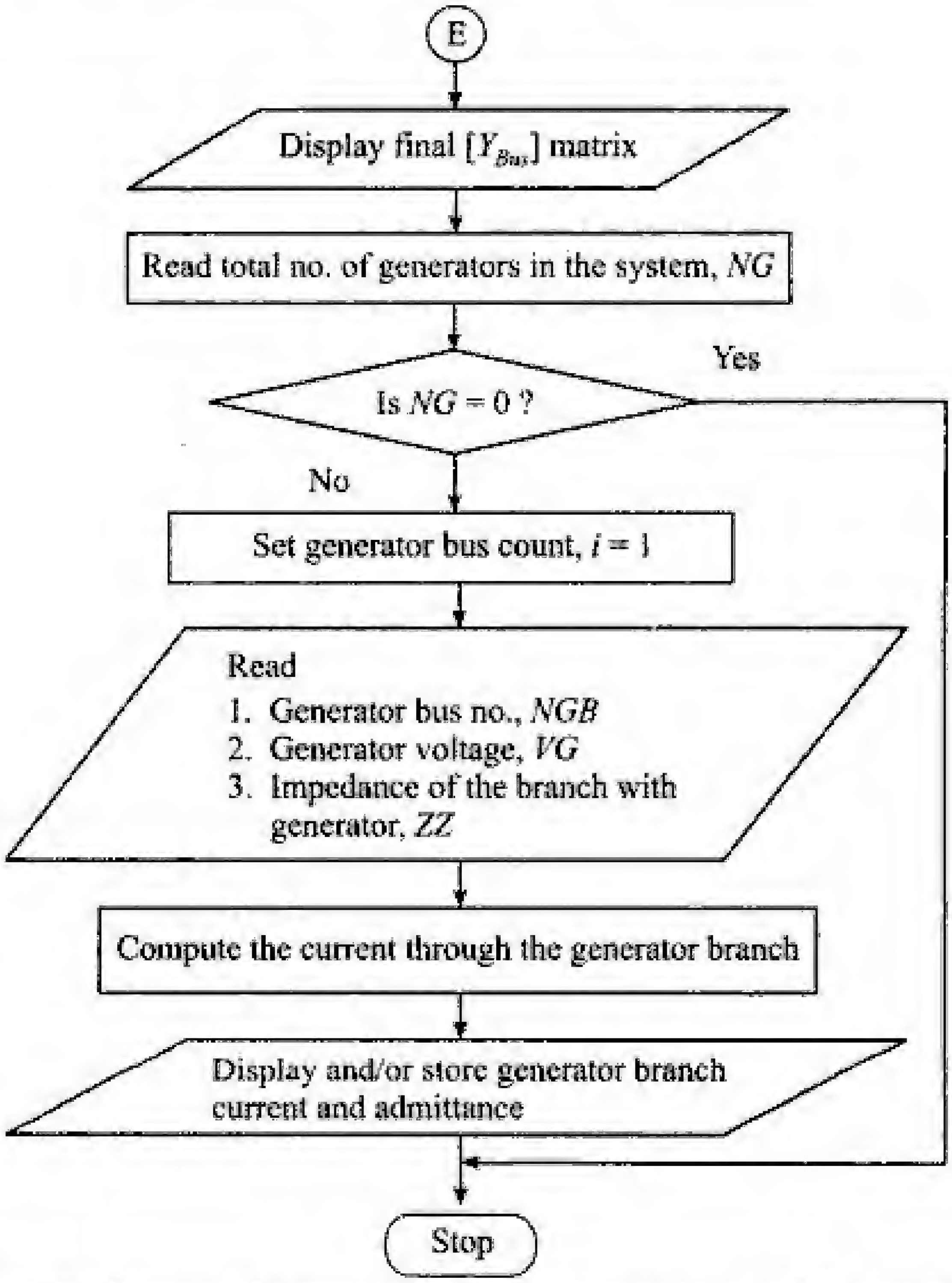


Fig. 3.13 Flowchart for development of $[Y_{Bus}]$ matrix using coefficient matrix.

Example 3.11: For the three-bus system shown in Fig. E3.6 develops the bus admittance matrix using the principle of $[Y_{Bus}]$ development using coefficient matrix. Also, write nodal admittance matrix equation relating bus voltages and currents.

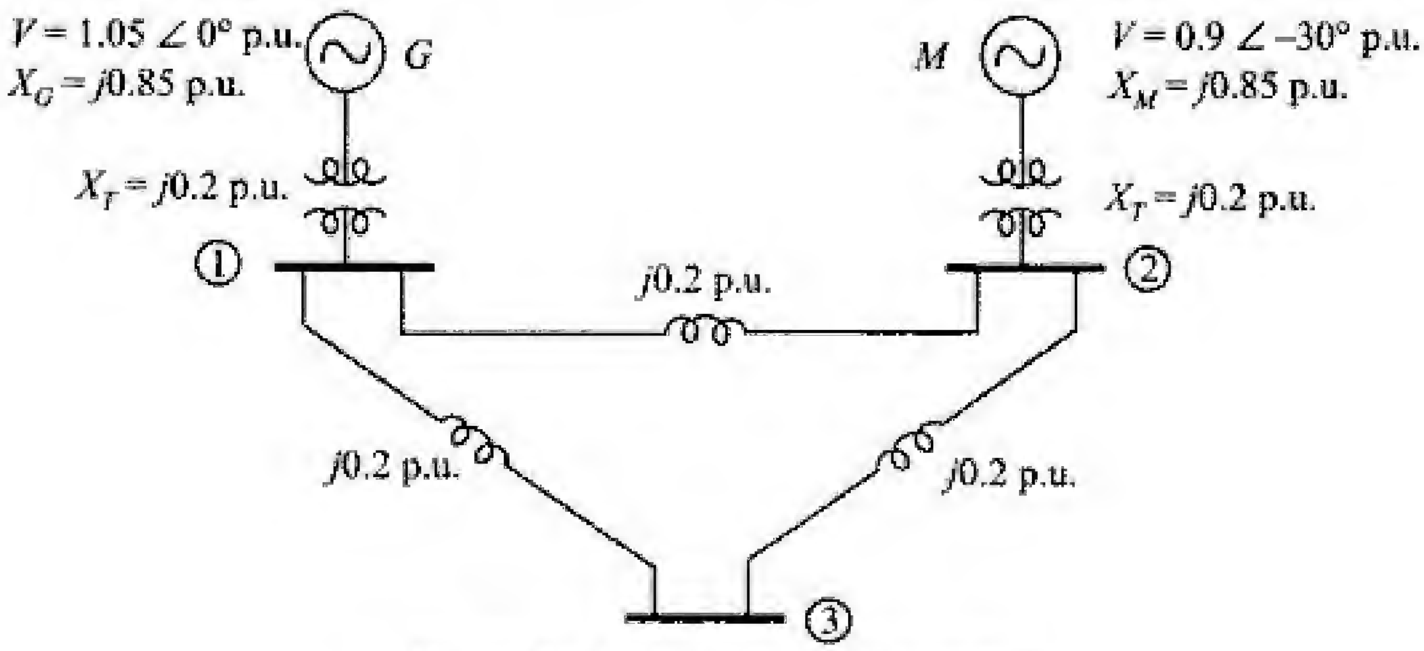


Fig. E3.6 A three-bus power system.

Execution of computer program, CYBUS.FOR, for $[Y_{Bus}]$ formulation using coefficient matrix method for Example 3.11

No. of buses = 3 ; no. of branches = 5

The branch numbers are configured as shown below (Fig. E3.8)

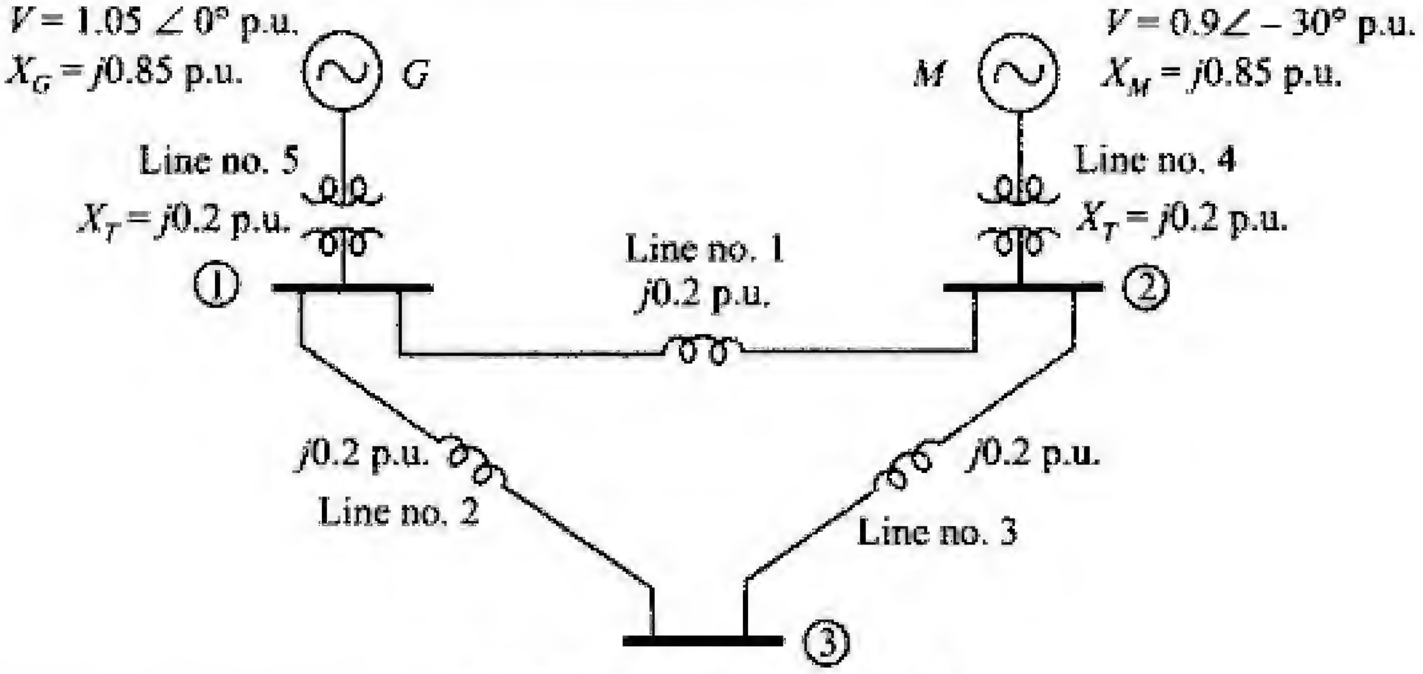


Fig. E3.8 Network of Ex. 3.11.

The inputs are shown in the table below:

Branch no.	*Impedance (p.u.)*	*From node*	*To node*	*Mutually coupled with*
1	$j0.2$	1	2	No branch
2	$j0.2$	1	3	No branch
3	$j0.2$	2	3	No branch
4	$j0.2 + j0.85 = j1.05$	2	0	No branch
5	$j0.2 + j0.85 = j1.05$	1	0	No branch

No. of branches with generator in the system = 2

For generator in bus-1

Generator bus no. = 1; generator voltage = 1.05

Branch impedance = $j1.05$; branch current = $\dfrac{1.05}{j1.05} = 1 \angle -90°$

For generator in bus-2

Generator bus no. = 4; generator voltage = $(0.779433 - j0.45)$

Branch impedance = $j1.05$; branch current $= \dfrac{(0.779433 - j0.45)}{j1.05}$

$= 0.857143 \angle -120°$ (approx)

All quantities are expressed in p.u. The angles are expressed in degree.

Output of CYBUS.FOR: CYBUS3.DAT

```
Coefficient matrix for branch - 1

A( 1,  1 ) = (0.000000E+00,  -5.000000)  A11
A( 1,  2 ) = (0.000000E+00,   5.000000)  A12
A( 2,  1 ) = (0.000000E+00,   5.000000)  A21
A( 2,  2 ) = (0.000000E+00,  -5.000000)  A22

Ybus matrix considering branch -  1

Ybus(  1,  1 ) = (.000000,  -5.000000 )  Y11
Ybus(  1,  2 ) = (.000000,   5.000000 )  Y12
Ybus(  1,  3 ) = (.000000,    .000000 )
Ybus(  2,  1 ) = (.000000,   5.000000 )
Ybus(  2,  2 ) = (.000000,  -5.000000 )
Ybus(  2,  3 ) = (.000000,    .000000 )
Ybus(  3,  1 ) = (.000000,    .000000 )
Ybus(  3,  2 ) = (.000000,    .000000 )
Ybus(  3,  3 ) = (.000000,    .000000 )  Y33

Coefficient matrix for branch -  2

A( 1,  1 ) = (0.000000E+00,  -5.000000)  A11
A( 1,  3 ) = (0.000000E+00,   5.000000)  A13
A( 3,  1 ) = (0.000000E+00,   5.000000)  A31
A( 3,  3 ) = (0.000000E+00,  -5.000000)  A33

Ybus matrix considering branch - 2

Ybus(  1,  1 ) = (.000000,  -10.000000 )  Y11
Ybus(  1,  2 ) = (.000000,    5.000000 )  Y12
Ybus(  1,  3 ) = (.000000,    5.000000 )
Ybus(  2,  1 ) = (.000000,    5.000000 )
Ybus(  2,  2 ) = (.000000,   -5.000000 )
Ybus(  2,  3 ) = (.000000,     .000000 )
Ybus(  3,  1 ) = (.000000,    5.000000 )
Ybus(  3,  2 ) = (.000000,     .000000 )
Ybus(  3,  3 ) = (.000000,   -5.000000 )  Y33

Coefficient matrix for branch -  3

A( 2,  2 ) = (0.000000E+00,   -5.000000)  A22
A( 2,  3 ) = (0.000000E+00,    5.000000)  A23
A( 3,  2 ) = (0.000000E+00,    5.000000)  A32
A( 3,  3 ) = (0.000000E+00,   -5.000000)  A33
```

```
Ybus matrix considering branch - 3

Ybus(  1,  1 ) = (    .000000, -10.000000 )   Y11
Ybus(  1,  2 ) = (    .000000,   5.000000 )   Y12
Ybus(  1,  3 ) = (    .000000,   5.000000 )
Ybus(  2,  1 ) = (    .000000,   5.000000 )
Ybus(  2,  2 ) = (    .000000, -10.000000 )
Ybus(  2,  3 ) = (    .000000,   5.000000 )
Ybus(  3,  1 ) = (    .000000,   5.000000 )
Ybus(  3,  2 ) = (    .000000,   5.000000 )
Ybus(  3,  3 ) = (    .000000, -10.000000 )   Y33

Coefficient matrix for branch -  4

A( 2,   2 ) = (0.000000E+00,  -9.523810E-01)  A22
A( 2,   0 ) = (0.000000E+00,   0.000000E+00)  A20
A( 0,   2 ) = (0.000000E+00,   0.000000E+00)  A02
A( 0,   0 ) = (0.000000E+00,   0.000000E+00)  A00

Ybus matrix considering branch - 4

Ybus(  1,  1 ) = (    .000000, -10.000000 )   Y11
Ybus(  1,  2 ) = (    .000000,   5.000000 )   Y12
Ybus(  1,  3 ) = (    .000000,   5.000000 )   Y13
Ybus(  2,  1 ) = (    .000000,   5.000000 )   Y21
Ybus(  2,  2 ) = (    .000000, -10.952380 )
Ybus(  2,  3 ) = (    .000000,   5.000000 )
Ybus(  3,  1 ) = (    .000000,   5.000000 )
Ybus(  3,  2 ) = (    .000000,   5.000000 )
Ybus(  3,  3 ) = (    .000000, -10.000000 )   Y33

Coefficient matrix for branch -  5

A( 1,   1 ) = (0.000000E+00,  -9.523810E-01)  A11
A( 1,   0 ) = (0.000000E+00,   0.000000E+00)  A10
A( 0,   1 ) = (0.000000E+00,   0.000000E+00)  A01
A( 0,   0 ) = (0.000000E+00,   0.000000E+00)  A00

Ybus matrix considering branch - 5

Ybus(  1,  1 ) = (    .000000, -10.952380 )   Y11
Ybus(  1,  2 ) = (    .000000,   5.000000 )   Y12
Ybus(  1,  3 ) = (    .000000,   5.000000 )
Ybus(  2,  1 ) = (    .000000,   5.000000 )
Ybus(  2,  2 ) = (    .000000, -10.952380 )
Ybus(  2,  3 ) = (    .000000,   5.000000 )
```

```
Ybus( 3,  1 ) = (   .000000,   5.000000 )
Ybus( 3,  2 ) = (   .000000,   5.000000 )
Ybus( 3,  3 ) = (   .000000, -10.000000 )  Y33

No. of buses =  3

Final Ybus matrix

Ybus( 1,  1 ) = (   .000000, -10.952380 )  Y11
Ybus( 1,  2 ) = (   .000000,   5.000000 )  Y12
Ybus( 1,  3 ) = (   .000000,   5.000000 )
Ybus( 2,  1 ) = (   .000000,   5.000000 )
Ybus( 2,  2 ) = (   .000000, -10.952380 )
Ybus( 2,  3 ) = (   .000000,   5.000000 )
Ybus( 3,  1 ) = (   .000000,   5.000000 )
Ybus( 3,  2 ) = (   .000000,   5.000000 )
Ybus( 3,  3 ) = (   .000000, -10.000000 )  Y33

For the generator in branch -  1
==================================
Magnitude of branch current = 1.000000
Angle of branch current (in Degree) = -90.000
Branch admittance = (   .000000,   -.952381)

For the generator in branch -  2
==================================
Magnitude of branch current = .857143
Angle of branch current (in Degree) = -120.000
Branch admittance = (   .000000,   -.952381)
```

Example 3.12: For the three-bus system shown in Fig. E3.9(a), develop the bus admittance matri using the principle of $[Y_{Bus}]$ development using coefficient matrix.

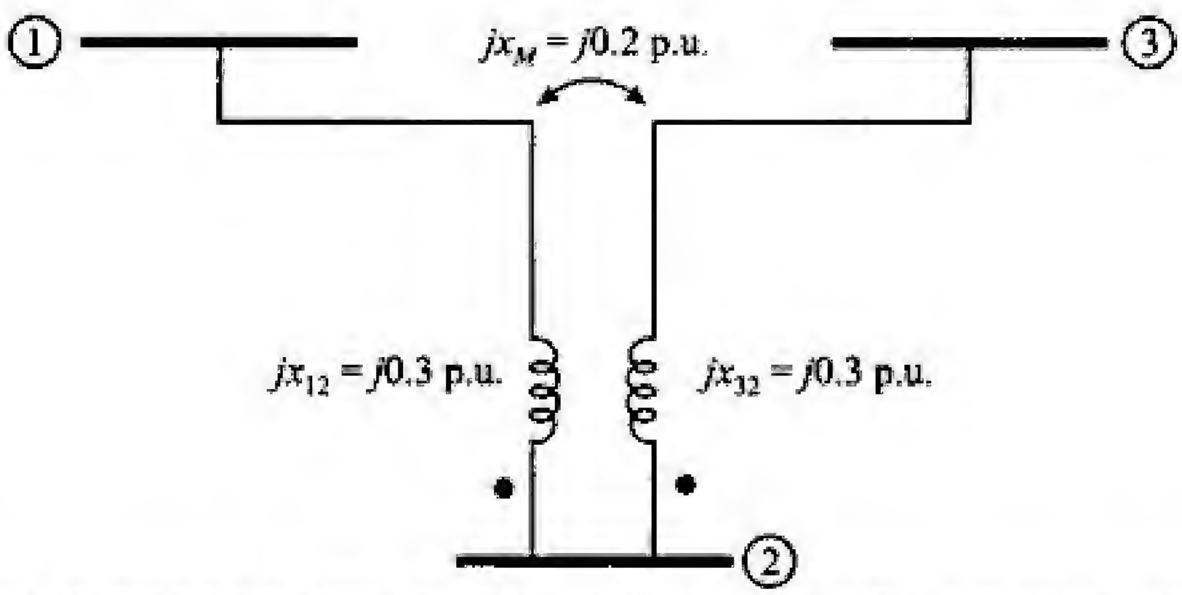


Fig. E3.9(a) A three-bus two-line power transmission system with a mutually coupled branch.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Coefficient matrix for branch - 1

A(2, 2) = (0.000000E+00, -6.000000) A_{11}
A(2, 1) = (0.000000E+00, 6.000000) A_{21}
A(1, 2) = (0.000000E+00, 6.000000) A_{12}
A(1, 1) = (0.000000E+00, -6.000000) A_{11}

Ybus matrix considering branch - 1

Ybus(1, 1) = (.000000, -6.000000) Y_{11}
Ybus(1, 2) = (.000000, 6.000000) Y_{12}
Ybus(1, 3) = (.000000, .000000)
Ybus(2, 1) = (.000000, 6.000000)
Ybus(2, 2) = (.000000, -6.000000)
Ybus(2, 3) = (.000000, .000000)
Ybus(3, 1) = (.000000, .000000)
Ybus(3, 2) = (.000000, .000000)
Ybus(3, 3) = (.000000, .000000) Y_{33}

Coefficient matrix for branch - 2

A(2, 2) = (0.000000E+00, -6.000000) A_{22}
A(2, 3) = (0.000000E+00, 6.000000) A_{23}
A(3, 2) = (0.000000E+00, 6.000000) A_{32}
A(3, 3) = (0.000000E+00, -6.000000) A_{33}

Ybus matrix considering branch - 2

Ybus(1, 1) = (.000000, -6.000000) Y_{11}
Ybus(1, 2) = (.000000, 6.000000) Y_{12}
Ybus(1, 3) = (.000000, .000000) Y_{13}
Ybus(2, 1) = (.000000, 6.000000) Y_{21}
Ybus(2, 2) = (.000000,-12.000000)
Ybus(2, 3) = (.000000, 6.000000)
Ybus(3, 1) = (.000000, .000000)
Ybus(3, 2) = (.000000, 6.000000)
Ybus(3, 3) = (.000000, -6.000000) Y_{33}

1 - 2 Diagonal element of coefficient matrix

A(2, 2) = (0.000000E+00, 4.000000) A_{22}
A(2, 3) = (0.000000E+00, -4.000000) A_{23}
A(1, 2) = (0.000000E+00, -4.000000) A_{12}
A(1, 3) = (0.000000E+00, 4.000000) A_{13}

Ybus matrix considering 1 - 2 Diagonal element of coefficient matrix

```
Ybus( 1, 1 ) = ( .000000, -6.000000 )  Y11
Ybus( 1, 2 ) = ( .000000,  2.000000 )  Y12
Ybus( 1, 3 ) = ( .000000,  4.000000 )
Ybus( 2, 1 ) = ( .000000,  6.000000 )
Ybus( 2, 2 ) = ( .000000, -8.000000 )
Ybus( 2, 3 ) = ( .000000,  2.000000 )
Ybus( 3, 1 ) = ( .000000,   .000000 )
Ybus( 3, 2 ) = ( .000000,  6.000000 )
Ybus( 3, 3 ) = ( .000000, -6.000000 )  Y33
```

2 - 1 Diagonal element of coefficient matrix

```
B( 2, 2 ) = (0.000000E+00,  4.000000)  B22
B( 2, 1 ) = (0.000000E+00, -4.000000)  B21
B( 3, 2 ) = (0.000000E+00, -4.000000)  B32
B( 3, 1 ) = (0.000000E+00,  4.000000)  B31
```

Ybus matrix considering 2 - 1 Diagonal element of coefficient matrix

```
Ybus( 1, 1 ) = ( .000000, -6.000000 )  Y11
Ybus( 1, 2 ) = ( .000000,  2.000000 )  Y12
Ybus( 1, 3 ) = ( .000000,  4.000000 )
Ybus( 2, 1 ) = ( .000000,  2.000000 )
Ybus( 2, 2 ) = ( .000000, -4.000000 )
Ybus( 2, 3 ) = ( .000000,  2.000000 )
Ybus( 3, 1 ) = ( .000000,  4.000000 )
Ybus( 3, 2 ) = ( .000000,  2.000000 )
Ybus( 3, 3 ) = ( .000000, -6.000000 )  Y33
```

No. of buses = 3

Final Ybus matrix

```
Ybus( 1, 1 ) = ( .000000, -6.000000 )  Y11
Ybus( 1, 2 ) = ( .000000,  2.000000 )  Y12
Ybus( 1, 3 ) = ( .000000,  4.000000 )
Ybus( 2, 1 ) = ( .000000,  2.000000 )
Ybus( 2, 2 ) = ( .000000, -4.000000 )
Ybus( 2, 3 ) = ( .000000,  2.000000 )
Ybus( 3, 1 ) = ( .000000,  4.000000 )
Ybus( 3, 2 ) = ( .000000,  2.000000 )
Ybus( 3, 3 ) = ( .000000, -6.000000 )  Y33
```

3.7 FORMULATION OF COMPLETE $[Y_{BUS}]$ FOR A GENERAL NETWORK

Procedure of formulation of a complete $[Y_{Bus}]$ for a general network having non-coupled as well as coupled branches is illustrated below, for a simple network model.

A three-bus power network is shown in Fig. 3.14(a). Branches x_{12} and x_{32} are mutually coupled with each other. Find $[Y_{Bus}]$.

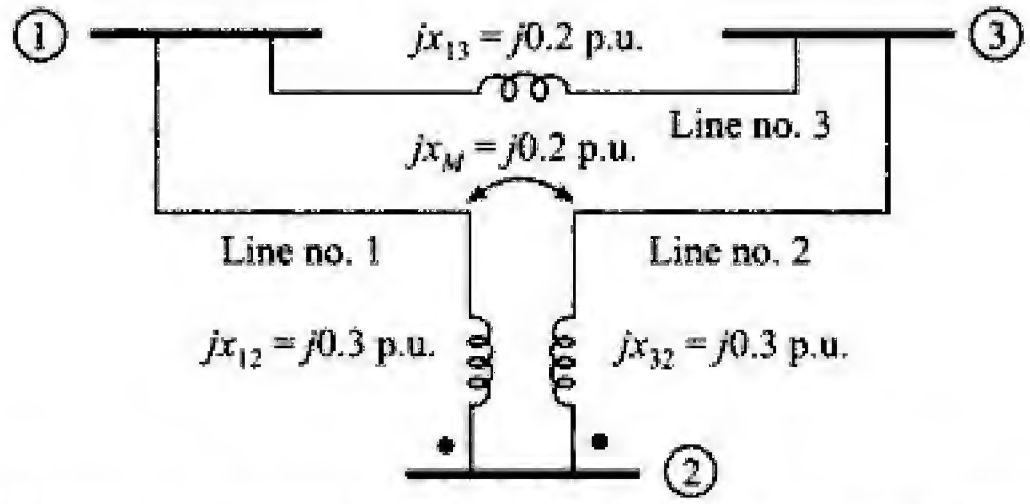


Fig. 3.14(a) A three bus power network with mutually coupled branches.

Let us first segregate the network in two parts for better understanding. The first part represents non-mutual part while the second part represents the mutual part (refer Fig. 3.14(b)) for non-mutual part and Fig. 3.14(c) for mutual part).

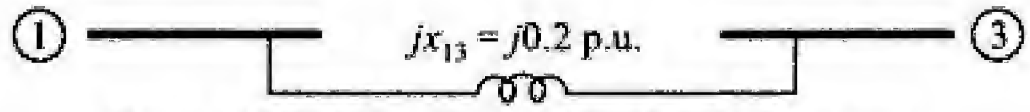


Fig. 3.14(b) Non-mutual part of the given network.

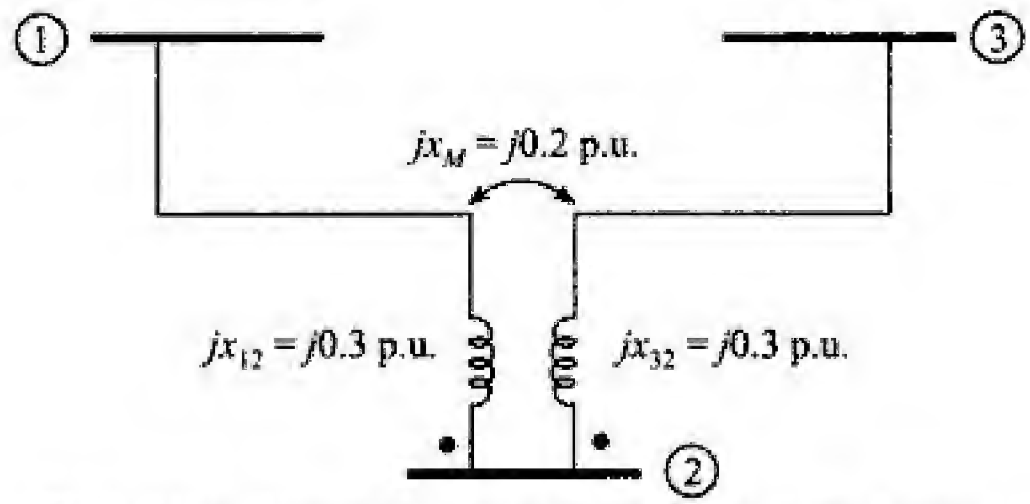


Fig. 3.14(c) Mutually coupled portion of the given network.

The bus admittance matrix for the mutual part has already been obtained in Example 3.10 (final $[Y_{Bus}]$ matrix) and is reproduced here

$$[Y_{Bus}]_{\text{for mutual portion}} = \begin{bmatrix} -j6 & j2 & j4 \\ j2 & -j4 & j2 \\ j4 & j2 & -j6 \end{bmatrix} \text{p.u.}$$

The equivalent circuit of $[Y_{Bus}]_{\text{for mutual portion}}$ is then drawn in Fig. 3.14(d) with branch admittances having respective suffixes. Since the line between buses 1 and 3 in Fig. 3.14(a) does not have any mutual

coupling with any other line, we can write for the admittance of this branch as $y'_{13} = 1/jx_{13} = -j5$ p.u. In the next step the equivalent circuit of Fig. 3.14(d) is modified by inserting y'_{13} in it (refer Fig. 3.14(e)).

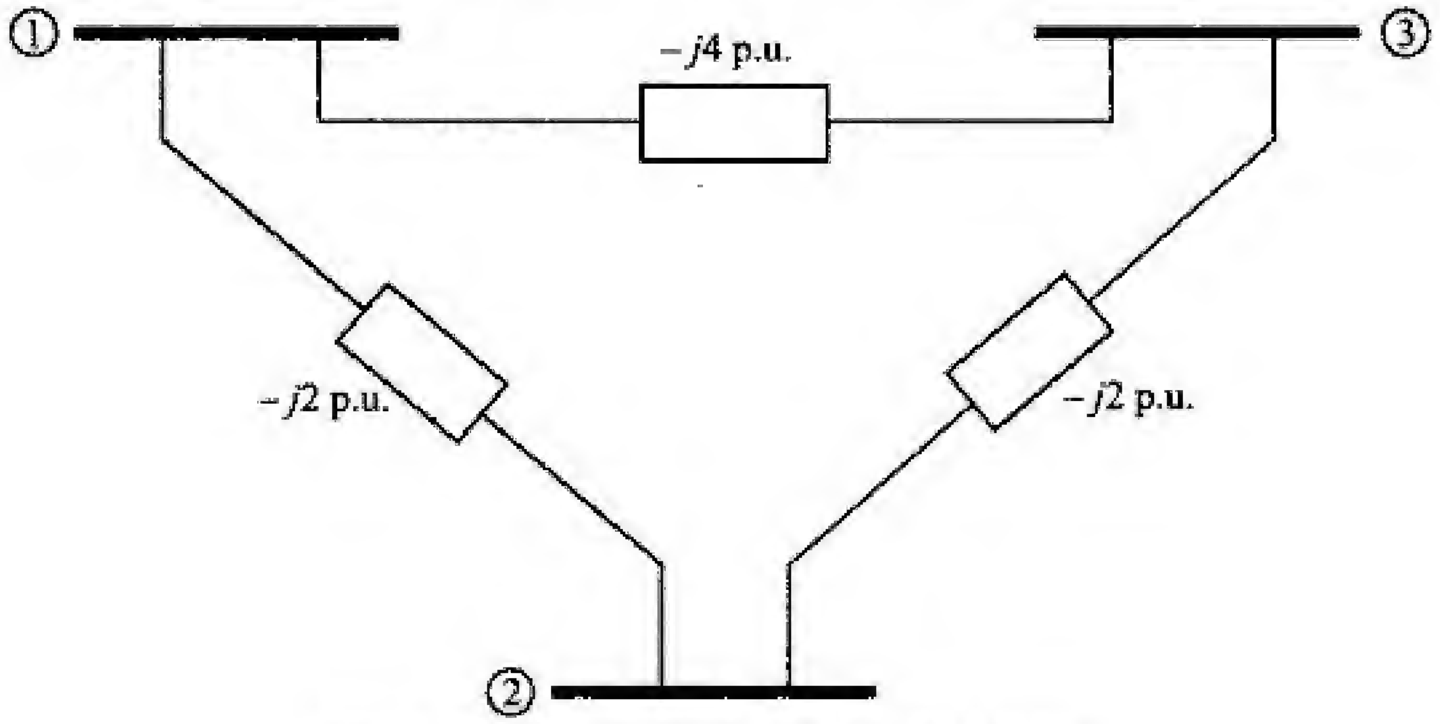


Fig. 3.14(d) Equivalent network of Fig. 3.14(a).

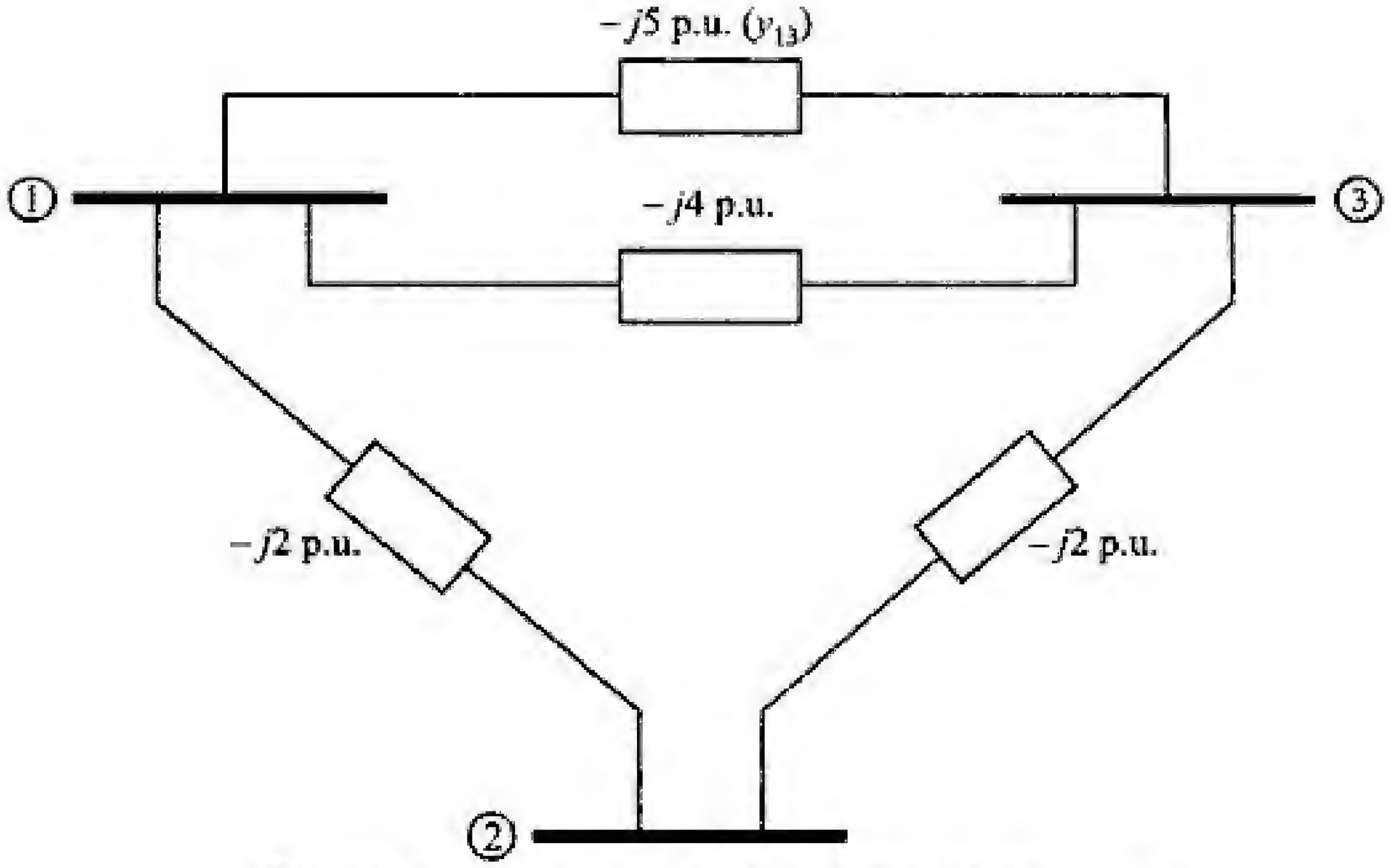


Fig. 3.14(e) Complete equivalent network of Fig. 3.14(a).

$[Y_{Bus}]$ now can be formed, where

$$Y_{11} = -j2 + (-j4) + (-j5) = -j11 \text{ p.u.}, Y_{12} = -(-j2) = j2 = Y_{21} \text{ p.u.},$$

$$Y_{13} = -(-j4 - j5) = j9 = Y_{31} \text{ p.u.},$$

$$Y_{22} = -j2 + (-j2) = -j4 \text{ p.u.}, Y_{23} = -(-j2) = j2 = Y_{32},$$

$$Y_{33} = -j5 + (-j4) + (-j2) = -j11 \text{ p.u.}$$

Execution of the computer program CYBUS.FOR for the given illustration

No. of buses = 3; no. of branches = 3

The branch numbers are configured in Fig. 3.14

The inputs are shown in the table below:

Branch no.	*Impedance (p.u.)*	*From node*	*To node*	*Mutually coupled with*	*Mutual impedance*
1	$j0.3$	2*	1	Branch-2	$j0.2$
2	$j0.3$	2	3	No branch**	
3	$j0.2$	1	3	No branch	

* Since branch-1 has a mutual coupling with branch-2 and dot mark is near to bus-2, hence bus-2 is considered 'from' node.

** Branch-2 has already been considered as mutually coupled branch. Hence, there is no more coupling to be considered with branch-1.

No. of branches with generator in the system = 0

Output of CYBUS.FOR: CYBUS4.DAT

```
Primitive [z] matrix for mutually coupled branch 1 - 2

z(  1,   1 ) = (    .000000,    .300000 )  z11
z(  1,   2 ) = (    .000000,    .200000 )  z12
z(  2,   1 ) = (    .000000,    .200000 )  z21
z(  2,   2 ) = (    .000000,    .300000 )  z22

Primitive [y] matrix

y(  1,   1 ) = (    .000000, -6.000000 )  y11
y(  1,   2 ) = (    .000000,  4.000000 )  y12
y(  2,   1 ) = (    .000000,  4.000000 )  y21
y(  2,   2 ) = (    .000000, -6.000000 )  y22

Coefficient matrix for branch - 1

A( 2,   2 ) = (0.000000E+00,  -6.000000)  A22
A( 2,   1 ) = (0.000000E+00,   6.000000)  A21
A( 1,   2 ) = (0.000000E+00,   6.000000)  A12
A( 1,   1 ) = (0.000000E+00,  -6.000000)  A11

Ybus matrix considering branch - 1

Ybus(  1,   1 ) = (    .000000,  -6.000000 )  Y11
Ybus(  1,   2 ) = (    .000000,   6.000000 )  Y12
Ybus(  1,   3 ) = (    .000000,    .000000 )  Y13
```

```
Ybus(  2,   1 ) = (    .000000,   6.000000 )
Ybus(  2,   2 ) = (    .000000,  -6.000000 )
Ybus(  2,   3 ) = (    .000000,    .000000 )
Ybus(  3,   1 ) = (    .000000,    .000000 )
Ybus(  3,   2 ) = (    .000000,    .000000 )
Ybus(  3,   3 ) = (    .000000,    .000000 )  Y33

Coefficient matrix for branch - 2

A( 2,   2 ) = (0.000000E+00,    -6.000000)   A22
A( 2,   3 ) = (0.000000E+00,     6.000000)   A23
A( 3,   2 ) = (0.000000E+00,     6.000000)   A32
A( 3,   3 ) = (0.000000E+00,    -6.000000)   A33

Ybus matrix considering branch - 2

Ybus(  1,   1 ) = (    .000000,  -6.000000 )  Y11
Ybus(  1,   2 ) = (    .000000,   6.000000 )  Y12
Ybus(  1,   3 ) = (    .000000,    .000000 )
Ybus(  2,   1 ) = (    .000000,   6.000000 )
Ybus(  2,   2 ) = (    .000000, -12.000000 )
Ybus(  2,   3 ) = (    .000000,   6.000000 )
Ybus(  3,   1 ) = (    .000000,    .000000 )
Ybus(  3,   2 ) = (    .000000,   6.000000 )
Ybus(  3,   3 ) = (    .000000,  -6.000000 )  Y33

1 - 2  Diagonal element of coefficient matrix

A( 2,   2 ) = (0.000000E+00,      4.000000)   A22
A( 2,   3 ) = (0.000000E+00,     -4.000000)   A23
A( 1,   2 ) = (0.000000E+00,     -4.000000)   A12
A( 1,   3 ) = (0.000000E+00,      4.000000)   A13

Ybus matrix considering 1 - 2 Diagonal element
of coefficient matrix

Ybus(  1,   1 ) = (    .000000,  -6.000000 )  Y11
Ybus(  1,   2 ) = (    .000000,   2.000000 )  Y12
Ybus(  1,   3 ) = (    .000000,   4.000000 )
Ybus(  2,   1 ) = (    .000000,   6.000000 )
Ybus(  2,   2 ) = (    .000000,  -8.000000 )
Ybus(  2,   3 ) = (    .000000,   2.000000 )
Ybus(  3,   1 ) = (    .000000,    .000000 )
Ybus(  3,   2 ) = (    .000000,   6.000000 )
Ybus(  3,   3 ) = (    .000000,  -6.000000 )  Y33
```

```
Ybus( 2, 1 ) = (    .000000,   2.000000 )
Ybus( 2, 2 ) = (    .000000,  -4.000000 )
Ybus( 2, 3 ) = (    .000000,   2.000000 )
Ybus( 3, 1 ) = (    .000000,   9.000000 )
Ybus( 3, 2 ) = (    .000000,   2.000000 )
Ybus( 3, 3 ) = (    .000000, -11.000000 )  Y33
```

Example 3.13: In the four-bus power network (Fig. E3.10) branches x_{43} and x_{23} are mutually coupled. Find $[Y_{Bus}]$ and write the nodal matrix equation.

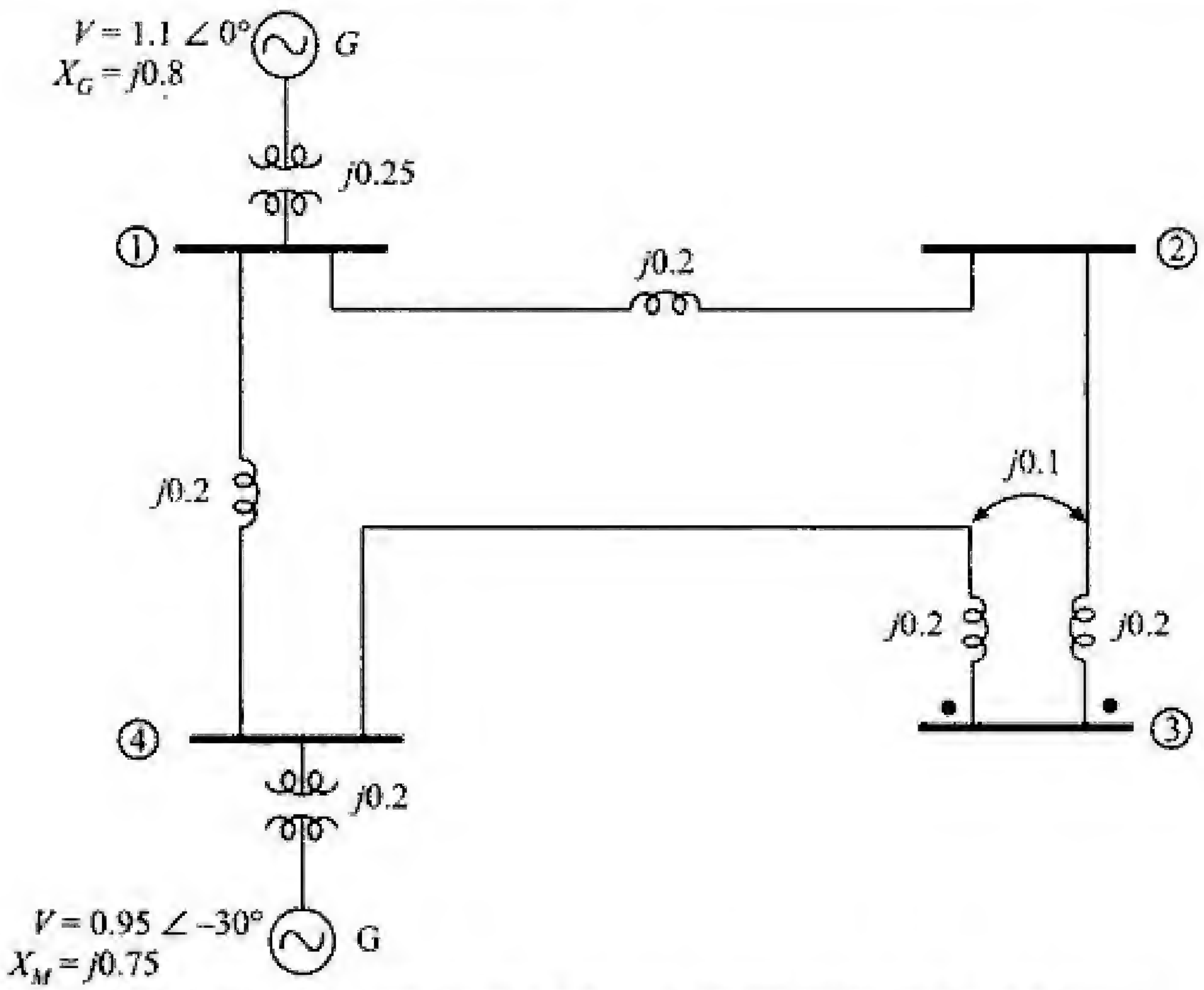


Fig. E3.10 A four-bus power system. (All values except angles are in p.u.).

Solution: Let us first compute the $[Y_{Bus}]$ for mutually connected part (shown below in Fig. E3.11).

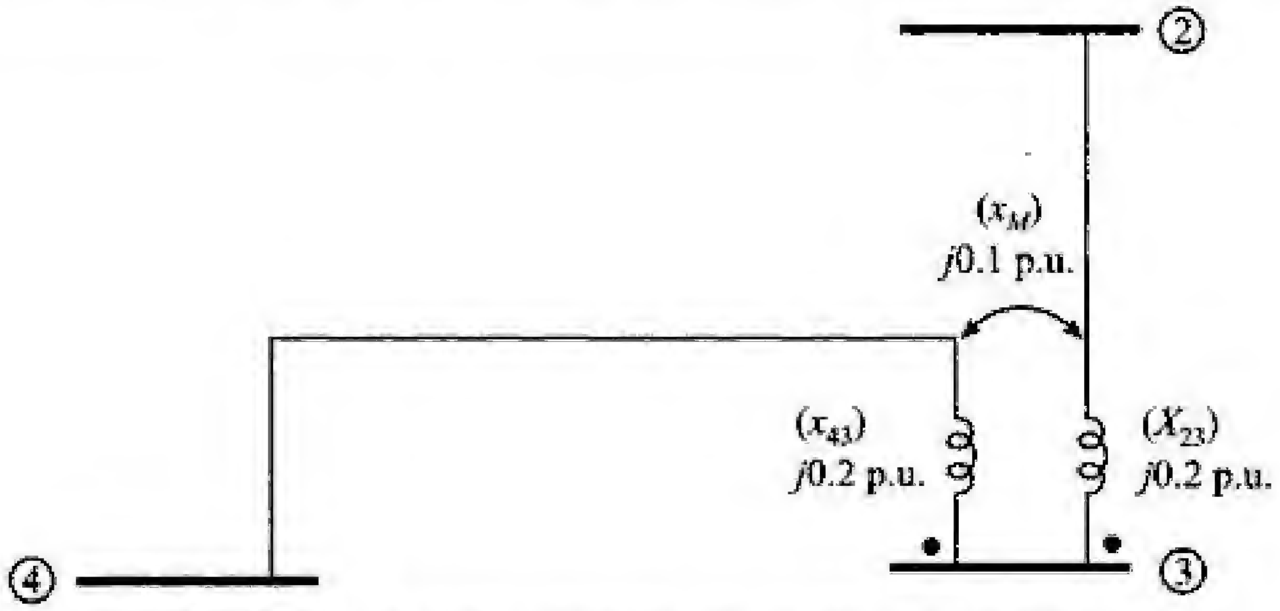


Fig. E3.11 Mutually connected part of the given network.

Execution of the computer program CYBUS.FOR for the problem shown in Fig. E3.13

No. of buses = 3; no. of branches = 6

The branch nos. are configured as shown in Fig. E3.10

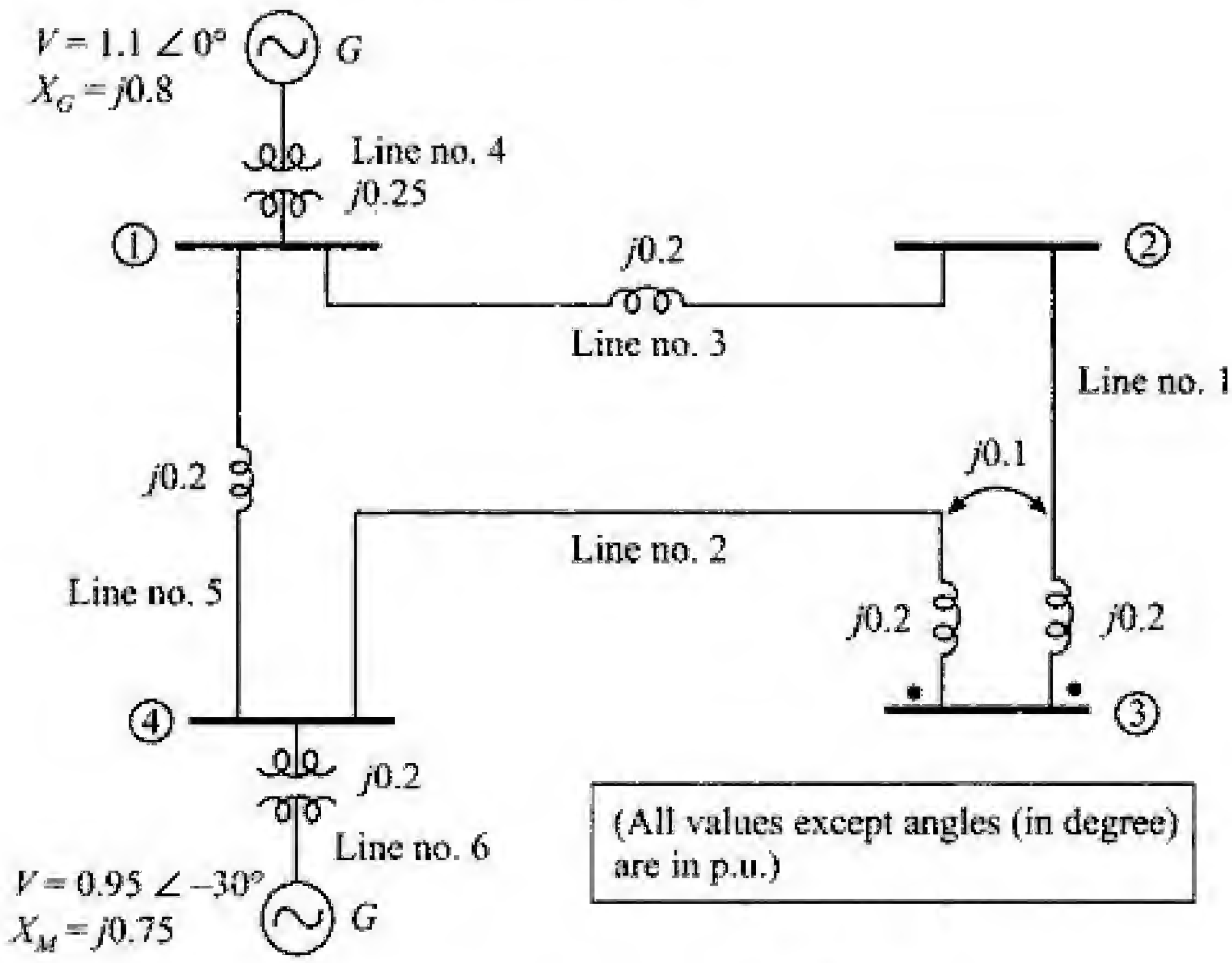


Fig. E3.13 Branch configuration of Fig. E3.10.

The inputs are shown in the table below:

Branch no.	*Impedance (p.u.)*	*From node*	*To node*	*Mutually coupled with*	*Mutual impedance (p.u.)*
1	$j0.2$	3*	2	Branch-2	$j0.1$
2	$j0.2$	3	4	No branch**	
3	$j0.2$	1	2	No branch	
4	$j0.25$	1	0	No branch	
5	$j0.2$	1	4	No branch	
6	$j0.95$	4	0	No branch	

* Since branch-1 has a mutual coupling with branch-2 and dot mark is near to bus-3, hence bus-2 is considered 'from' node.

** Branch-2 has already been considered as mutually coupled with branch-1.

No. of branches with generator in the system = 2

For generator in bus-1

Generator bus no. = 1; generator voltage = 1.1

Branch impedance = $j1.05$.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Similarly, considering all the branches final $[Y_{Bus}]$ matrix is obtained as given below:

```
No. of buses = 4

Final Ybus matrix

Ybus(  1,  1 ) = (   .000000,-10.952380 )  Y11
Ybus(  1,  2 ) = (   .000000,  5.000000 )  Y12
Ybus(  1,  3 ) = (   .000000,   .000000 )
Ybus(  1,  4 ) = (   .000000,  5.000000 )
Ybus(  2,  1 ) = (   .000000,  5.000000 )
Ybus(  2,  2 ) = (   .000000,-11.666670 )
Ybus(  2,  3 ) = (   .000000,  3.333333 )
Ybus(  2,  4 ) = (   .000000,  3.333333 )
Ybus(  3,  1 ) = (   .000000,   .000000 )
Ybus(  3,  2 ) = (   .000000,  3.333333 )
Ybus(  3,  3 ) = (   .000000, -6.666667 )
Ybus(  3,  4 ) = (   .000000,  3.333333 )
Ybus(  4,  1 ) = (   .000000,  5.000000 )
Ybus(  4,  2 ) = (   .000000,  3.333333 )
Ybus(  4,  3 ) = (   .000000,  3.333333 )
Ybus(  4,  4 ) = (   .000000,-12.719300 )  Y44

For the generator in bus -  1
================================
Magnitude of branch current =    1.047619
Angle of branch current (in Degree) =     -90.000
Branch admittance = (    .000000,   -.952381)

For the generator in bus -  4
================================
Magnitude of branch current =    1.000000
Angle of branch current (in Degree) =    -120.000
Branch admittance = (    .000000, -1.052632)
```

3.8 MODIFICATION OF $[Y_{BUS}]$ FOR BRANCH ADDITION/DELETION

It is much easier to modify $[Y_{Bus}]$ to account for branch additions or branch removals in the system network. In order to modify $[Y_{Bus}]$, to include a branch addition between node p and q, the admittance of this branch element y_b is to be added to elements Y_{pp} and Y_{qq} of $[Y_{Bus}]$, while y_b is to be subtracted from symmetrical off-diagonal elements Y_{pq} and Y_{qp}. Thus, the addition of a branch (of admittance y_b) between two-bus system would modify the old bus admittance matrix as indicated above and the new bus admittance matrix is then given by,

$$[Y_{Bus}] = \begin{array}{c} \\ p \\ q \end{array} \begin{array}{c} \begin{array}{cc} \quad p \quad & \quad q \quad \end{array} \\ \begin{bmatrix} Y_{pp_{old}} + y_b & Y_{pq_{old}} - y_b \\ Y_{qp_{old}} - y_b & Y_{qq_{old}} + y_b \end{bmatrix} \end{array} \tag{3.41}$$

Similarly, to remove a branch of admittance y_b already connected between bus p and q of the system, simply the branch admittance $(- y_b)$ is to be added between the same buses. This amounts to subtraction of elements of change matrix $[\Delta Y_{Bus}]$ from old $[Y_{Bus}]$.

3.8.1 Development of $[Y_{Bus}]$ by Step by Step $[y]$ Array Formation

Let us assume a simple three-bus power network in admittance form of representation (Fig. 3.15).

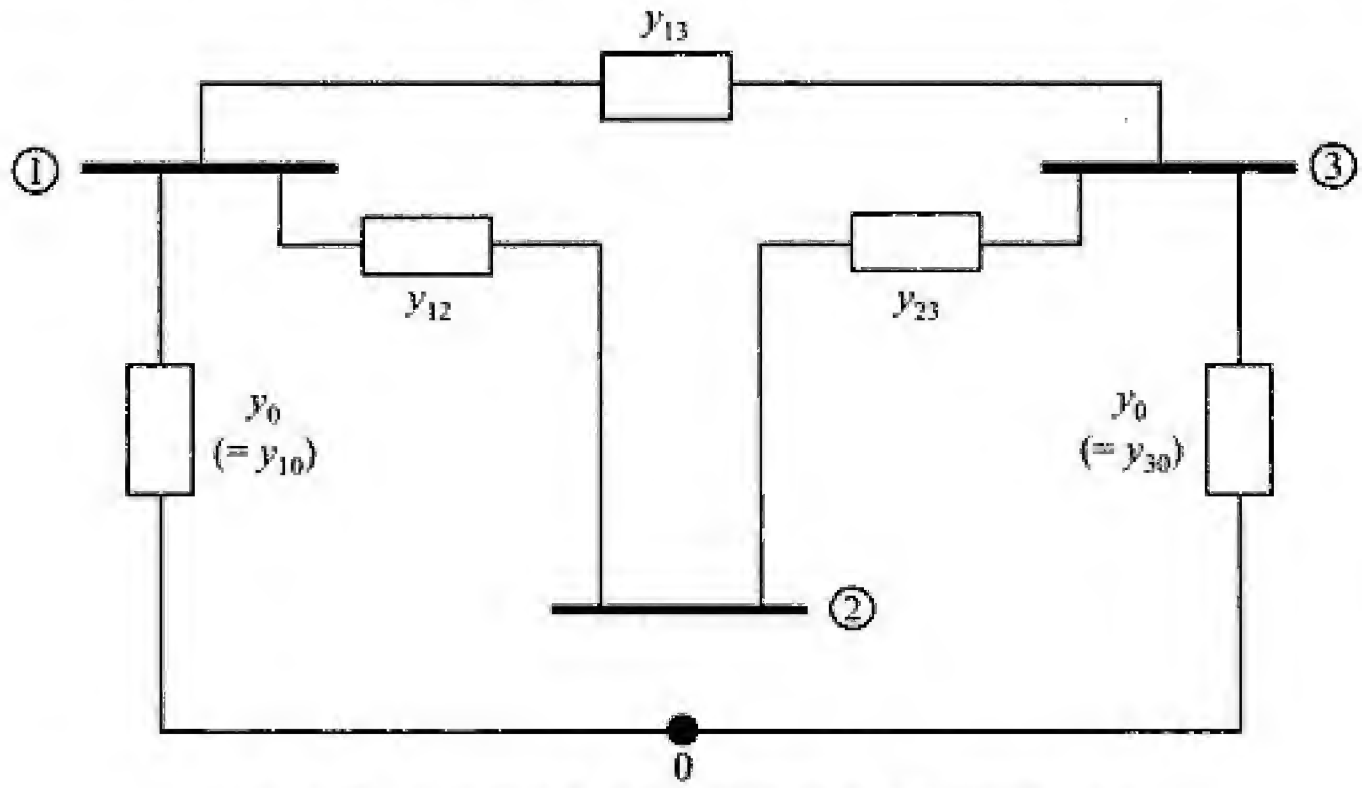


Fig. 3.15 A three-bus power network in admittance form.

We start with setting $[Y_{Bus}]$ matrix equal to zero.

$$\therefore \qquad [Y_{Bus}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(Since the number of buses are three, the $[Y_{Bus}]$ matrix is 3 × 3.)

Let us now start developing $[Y_{Bus}]$ matrix combining this matrix with each of the branches as described below:

Combination of branch of admittance y_{12}

This will affect the elements of $[Y_{Bus}]$ connected with bus 1 and 2. Thus we write

$$Y_{11} = 0 + y_{12} = y_{12}, \; Y_{12} = 0 - y_{12} = -y_{12}, \; Y_{22} = 0 + y_{12} = y_{12}$$

$$\therefore \qquad [Y_{Bus}] = \begin{bmatrix} y_{12} & -y_{12} & 0 \\ -y_{12} & y_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Combination of branch of admittance y_{13}

Since this will affect the elements of $[Y_{Bus}]$ connected with bus 1 and 3, we write, combining this branch with $[Y_{Bus}]$ just obtained,

$$Y_{11} = y_{12} + y_{13},\ Y_{13} = 0 - y_{13} = -y_{13},\ Y_{33} = 0 + y_{13} = y_{13}$$

$$\therefore \qquad [Y_{Bus}] = \begin{bmatrix} y_{12}+y_{13} & -y_{12} & -y_{13} \\ -y_{12} & y_{12} & 0 \\ -y_{13} & 0 & y_{13} \end{bmatrix}$$

Combination of branch of admittance y_{32}

Following the procedure outlined above,

$$Y_{33} = y_{13} + y_{32},\ Y_{32} = 0 - y_{32} = -y_{32} = Y_{23},\ Y_{22} = y_{12} + y_{32}$$

$$\therefore \qquad [Y_{Bus}] = \begin{bmatrix} y_{12}+y_{13} & -y_{12} & -y_{13} \\ -y_{12} & y_{12}+y_{32} & -y_{32} \\ -y_{13} & -y_{32} & y_{13}+y_{32} \end{bmatrix}$$

Combination of branch of admittance y_{10}

This will affect the element of Y_{11} only;

i.e. $$Y_{11} = y_{12} + y_{13} + y_{10} = y_{12} + y_{13} + y_0$$

Combination of branch of admittance y_{30}

This will affect the elements of Y_{33} only;

i.e. $$Y_{33} = y_{13} + y_{32} + y_{30} = y_{13} + y_{32} + y_0$$

$\therefore$ Finally,

$$[Y_{Bus}] = \begin{bmatrix} (y_{12}+y_{13}+y_0) & -y_{12} & -y_{13} \\ -y_{12} & (y_{12}+y_{32}) & -y_{32} \\ -y_{13} & -y_{32} & (y_{13}+y_{32}+y_0) \end{bmatrix} \qquad (3.42)$$

The above method is very simple and is amenable to programming methods. Moreover, it extends to any number of nodes and elements. It is also easy to modify $[Y_{Bus}]$ once it is framed. To add a branch, just process the element as usual and to remove the branch, simply $-y_{branch}$ is to be added (since y_{branch} paralleled with $-y_{branch}$ gives network zero admittance, i.e. infinite impedance, i.e. open circuit).

An Illustration

Let us illustrate the addition of a line (shown in Fig. 3.16).

In Fig. 3.16, addition of a line between bus 1 and 2 is represented by series and shunt elements in the admittance diagram. Here,

$$Y_{11_{(new)}} = Y_{11_{(old)}} + y'_{12} + y'_{10},\ Y_{22_{(new)}} = Y_{22_{(old)}} + y'_{12} + y'_{20}$$

and $$Y_{12_{(new)}} = Y_{12_{(new)}} = Y_{12_{(old)}} - y'_{12}$$

sequence of arithmetic computations, rounding errors due to working with a finite number of decimal places accumulate and the computational errors introduced are likely to become large with respect to the actual solution. By incorporating proper values of shunt admittances, this problem can be reduced. Incorporation of proper values of shunt admittances ensures proper electrical connection between the network bus bars with reference node too. Role of reference node is important because in its absence, in the equation $[I] = [Y]\,[V]$, infinite number of voltage solutions would satisfy the given injected current values. Moreover, the bus admittance matrix becomes singular and has no inverse. This time solution of $[V] = (= [Y]^{-1}\,[I])$ is not possible.

EXERCISES

1. (a) For the network shown in Fig. P3.1, form $[Y_{Bus}]$ matrix by (i) Nodal method and (ii) Singular transformation method.

   (b) Connect a new bus (bus no. 4) with bus no. 2 through a new transmission line of impedance $(0.04 + j0.3)$ p.u. and form $[Y_{Bus}]$ for the new system.

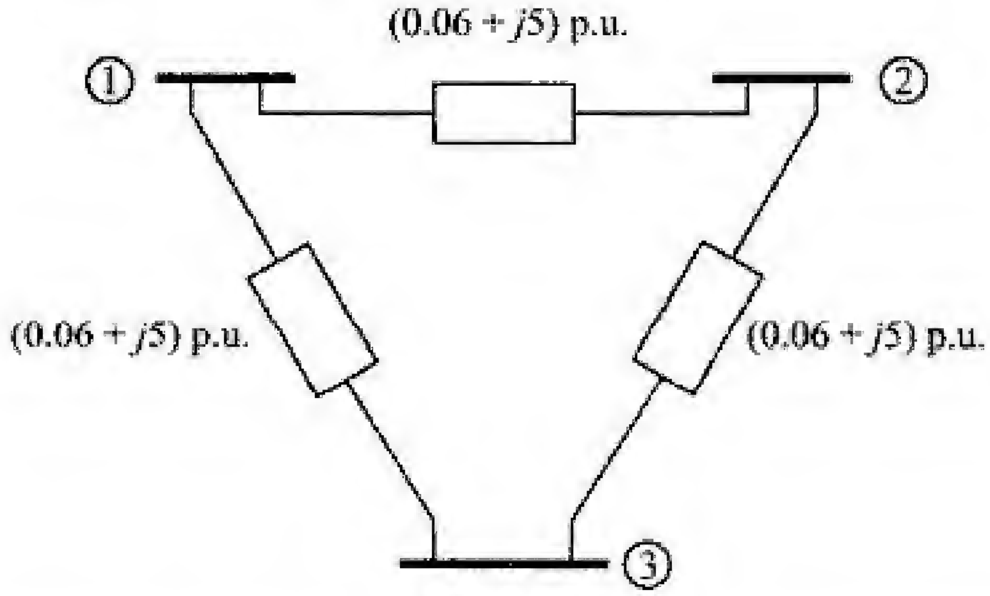


Fig. P3.1

2. For the power system, shown in Fig. P3.2, form network matrices as follows:

   (a) $[Y_{Bus}]$ using coefficient matrix,

   (b) $[Y_{Bus}]$ by nodal method.

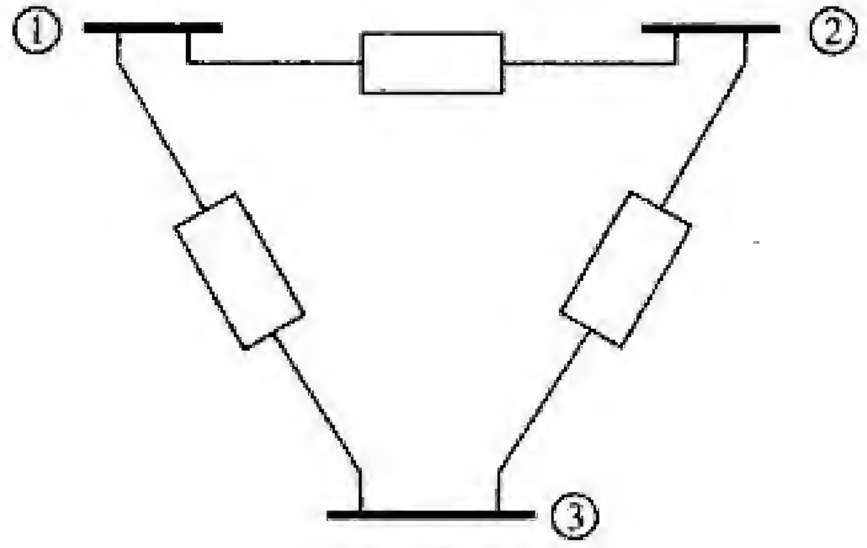


Fig. P3.2

Line data of the system are given below:

Line no.	*Bus code*	*Line impedance (in p.u.)*	*Half line charging (in p.u.)*
1	1-2	$0.08 + j0.20$	$j0.04$
2	2-3	$0.08 + j0.20$	$j0.04$
3	1-3	$0.08 + j0.20$	$j0.04$

3. For the network shown in Fig. P3.3, perform the following computation with the aid of computer.
   (a) Form $[Y_{Bus}]$ matrix of the system by nodal method. Assume line model at non-unity side of tap ratio.
   (b) Compute $[Z_{Bus}]$ matrix by inversion of $[Y_{Bus}]$ matrix.
   (c) Form $[Y_{Bus}]$ matrix of the system excluding lines 1-3 and 4-5. Inverse the $[Y_{Bus}]$ matrix and get $[Z_{Bus}]$ matrix of the system excluding lines 1-3 and 4-5.

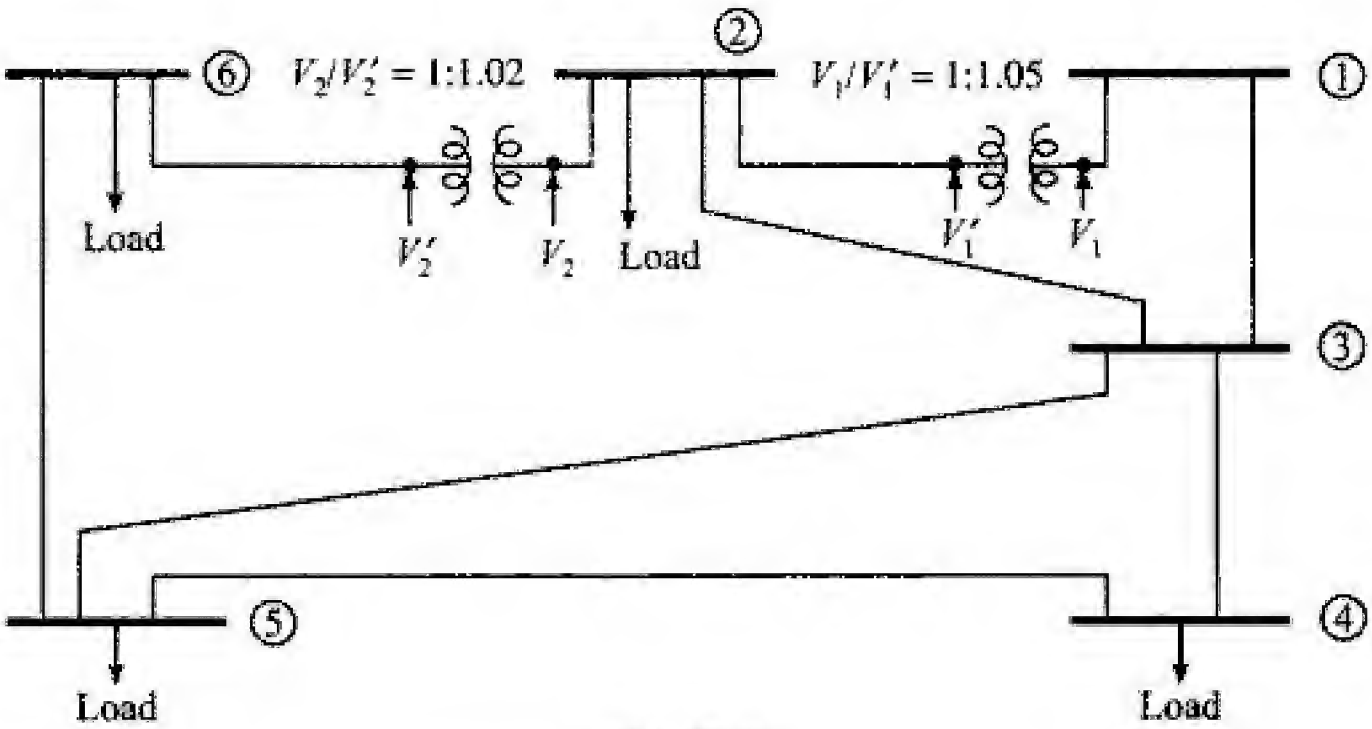


Fig. P3.3

Line data of the system are given below:

Line no.	*Bus code*	*Line impedance (in p.u.)*	*Half line charging (in p.u.)*	*Off-nominal turn ratio of transformer*
1	1-3	$0.04 + j0.30$	$j0.01$	
2	2-3	$0.03 + j0.20$	$j0.01$	
3	3-4	$0.04 + j0.20$	$j0.01$	
4	3-5	$0.04 + j0.20$	$j0.01$	
5	4-5	$0.03 + j0.15$	$j0.01$	
6	5-6	$0.06 + j0.3$	$j0.01$	
7	1-2	$j0.4$		1.05
8	2-6	$j0.2$		1.02

```
Ybus (  5,  3 ) = (   -.96154,   4.80769 )
Ybus (  5,  4 ) = (  -1.28205,   6.41026 )
Ybus (  5,  5 ) = (   2.88462, -14.39308 )
Ybus (  5,  6 ) = (   -.64103,   3.20513 )
Ybus (  6,  1 ) = (    .00000,    .00000 )
Ybus (  6,  2 ) = (    .00000,   5.10000 )
Ybus (  6,  3 ) = (    .00000,    .00000 )
Ybus (  6,  4 ) = (    .00000,    .00000 )
Ybus (  6,  5 ) = (   -.64103,   3.20513 )
Ybus (  6,  6 ) = (    .64103,  -8.19513 )   Y66
```

Result of exercise 3(b)

```
No. of buses =  6
Zbus matrix is

Zbus (  1,  1 ) = (    .06033,  -8.08088 )   Z11
Zbus (  1,  2 ) = (    .03770,  -8.42353 )   Z12
Zbus (  1,  3 ) = (    .03713,  -8.40766 )
Zbus (  1,  4 ) = (    .02930,  -8.46615 )
Zbus (  1,  5 ) = (    .02843,  -8.48460 )
Zbus (  1,  6 ) = (    .02864,  -8.56050 )
Zbus (  2,  1 ) = (    .03770,  -8.42353 )
Zbus (  2,  2 ) = (    .02709,  -8.56562 )
Zbus (  2,  3 ) = (    .02150,  -8.61933 )
Zbus (  2,  4 ) = (    .01450,  -8.67091 )
Zbus (  2,  5 ) = (    .01441,  -8.68357 )
Zbus (  2,  6 ) = (    .01909,  -8.72709 )
Zbus (  3,  1 ) = (    .03713,  -8.40767 )
Zbus (  3,  2 ) = (    .02150,  -8.61933 )
Zbus (  3,  3 ) = (    .03223,  -8.54863 )
Zbus (  3,  4 ) = (    .02247,  -8.61456 )
Zbus (  3,  5 ) = (    .02025,  -8.63814 )
Zbus (  3,  6 ) = (    .01319,  -8.74183 )
Zbus (  4,  1 ) = (    .02930,  -8.46615 )
Zbus (  4,  2 ) = (    .01450,  -8.67091 )
Zbus (  4,  3 ) = (    .02247,  -8.61455 )
Zbus (  4,  4 ) = (    .03639,  -8.55996 )
Zbus (  4,  5 ) = (    .02186,  -8.64332 )
Zbus (  4,  6 ) = (    .00724,  -8.77536 )
Zbus (  5,  1 ) = (    .02843,  -8.48460 )
Zbus (  5,  2 ) = (    .01441,  -8.68357 )
Zbus (  5,  3 ) = (    .02025,  -8.63814 )
Zbus (  5,  4 ) = (    .02186,  -8.64332 )
Zbus (  5,  5 ) = (    .02819,  -8.62126 )
Zbus (  5,  6 ) = (    .00803,  -8.77418 )
```

```
Zbus (  6,  1 ) = (     .02864,  -8.56050 )
Zbus (  6,  2 ) = (     .01909,  -8.72709 )
Zbus (  6,  3 ) = (     .01319,  -8.74183 )
Zbus (  6,  4 ) = (     .00724,  -8.77536 )
Zbus (  6,  5 ) = (     .00803,  -8.77418 )
Zbus (  6,  6 ) = (     .01758,  -8.74138 )   Z66
```

Result of exercise 3(c)

E3CYBUS.DAT

```
No. of buses =  6

Ybus (  1,  1 ) = (     .00000,   -2.75625 ) Y11
Ybus (  1,  2 ) = (     .00000,    2.62500 ) Y12
Ybus (  1,  3 ) = (     .00000,     .00000 )
Ybus (  1,  4 ) = (     .00000,     .00000 )
Ybus (  1,  5 ) = (     .00000,     .00000 )
Ybus (  1,  6 ) = (     .00000,     .00000 )
Ybus (  2,  1 ) = (     .00000,    2.62500 )
Ybus (  2,  2 ) = (     .73350,  -12.58197 )
Ybus (  2,  3 ) = (    -.73350,    4.88998 )
Ybus (  2,  4 ) = (     .00000,     .00000 )
Ybus (  2,  5 ) = (     .00000,     .00000 )
Ybus (  2,  6 ) = (     .00000,    5.10000 )
Ybus (  3,  1 ) = (     .00000,     .00000 )
Ybus (  3,  2 ) = (    -.73350,    4.88998 )
Ybus (  3,  3 ) = (    2.65657,  -14.47536 )
Ybus (  3,  4 ) = (    -.96154,    4.80769 )
Ybus (  3,  5 ) = (    -.96154,    4.80769 )
Ybus (  3,  6 ) = (     .00000,     .00000 )
Ybus (  4,  1 ) = (     .00000,     .00000 )
Ybus (  4,  2 ) = (     .00000,     .00000 )
Ybus (  4,  3 ) = (    -.96154,    4.80769 )
Ybus (  4,  4 ) = (     .96154,   -4.79769 )
Ybus (  4,  5 ) = (     .00000,     .00000 )
Ybus (  4,  6 ) = (     .00000,     .00000 )
Ybus (  5,  1 ) = (     .00000,     .00000 )
Ybus (  5,  2 ) = (     .00000,     .00000 )
Ybus (  5,  3 ) = (    -.96154,    4.80769 )
Ybus (  5,  4 ) = (     .00000,     .00000 )
Ybus (  5,  5 ) = (    1.60256,   -7.99282 )
Ybus (  5,  6 ) = (    -.64103,    3.20513 )
Ybus (  6,  1 ) = (     .00000,     .00000 )
Ybus (  6,  2 ) = (     .00000,    5.10000 )
Ybus (  6,  3 ) = (     .00000,     .00000 )
```

Chapter 4

Complex Power Flows

4.1 INTRODUCTION

Complex power flow (or *load flow*) techniques provide basic calculation procedure in order to determine the characteristics of power system under *steady state* operating mode. Complex power flow studies are usually conducted for planning purposes, or to obtain system behaviour in order to predict the loading of lines and equipment (as well as voltages and currents) of the entire network. Usually, these techniques are off-line and involve computers for solving *steady state loadflow equations* (SLFE).

In load flow operations, active power generations are normally specified and the generator bus voltage magnitudes are normally maintained at a specified level (by AVR and machine excitation). The loads are usually specified as constant power type with real and reactive components being given. The loads are assumed to be unaffected by small variations of voltage and frequency expected during normal steady state operation.

The types of buses and corresponding parameters being specified are as under:

1. *Load bus* (or *PQ bus*): The total injected power ($P_i + jQ_i$) is specified and normally the loads are constant power type and remains unaffected by small variations in bus voltages.
2. *Slack bus* (or *swing bus*): This type of bus arises because the system losses are not known in advance before the load flow calculation. Any generating bus may be chosen as slack bus, which is assumed to supply line losses. Usually a generator bus with maximum capacity is considered as slack bus. The slack bus voltage is assigned as system reference; its complex bus voltage ($V + j0$) is specified.
3. *Voltage controlled bus* (or *PV bus*): The total injected power (active) P_i is specified while the voltage magnitude is maintained at $|V|$ (using reactive power injection). This type of bus usually corresponds to a generator bus or may even a load bus where the bus voltage is fixed by supplying reactive power from reactive power compensators.

The power flow problem is formulated assuming the power system network to be linear, bilateral and balanced and having lumped parameters. However, the power and voltage constraints impose non-linearity in the power flow formulation and this invites the help of *iterative techniques* for solution. This

Equation (4.3) represents the polar form of the power flow also and is known as steady state load flow equation (SLFE). It provides the calculated values of the network real and reactive power injected at any bus *i*.

If P_{g_i} denotes the scheduled active power being generated at the *i*-th bus and P_{d_i} scheduled power demand of the load at that bus, then

$$P_{i_{\text{scheduled}}} = P_{g_i} - P_{d_i}$$

Therefore, active power mismatch is given by

$$\Delta P_i = P_{i_{\text{scheduled}}} - P_{i_{\text{calculated}}} = \left(P_{g_i} - P_{d_i}\right) - P_{i_{\text{calculated}}} \; (= \theta) \tag{4.4a}$$

Similarly, for reactive power mismatch at the *i*-th bus,

$$\Delta Q_i = Q_{i_{\text{scheduled}}} - Q_{i_{\text{calculated}}} = \left(Q_{g_i} - Q_{d_i}\right) - Q_{i_{\text{calculated}}} \; (= \psi) \tag{4.4b}$$

In solving power flow problems, mismatches do appear where P_i and Q_i (calculated values) do not coincide with scheduled values. The mismatches approach zero when the calculated value of *P* and *Q* coincides with the scheduled values.

Each bus *i* has two equations involving *P* and *Q* (equation 4.3). The power flow problem requires solution of equation (4.3) for values of the unknown bus voltages which cause equation (4.3) to be numerically satisfied at each bus. In case there is no scheduled value $(P_{i_{\text{calculated}}})$ at any bus *i*, then the mismatch (ΔP_i) cannot be defined and there is no need to satisfy equation (4.3). Similarly, once (ΔQ_i) is not specified, equation (4.3) need not be solved *(for PV buses ΔQ_i is not specified)*.

In solving the power flow problems, the bus type is first identified and at each bus, two of the four quantities $|V_i|$, δ_i, P_i and Q_i are specified and the remaining two are to be calculated. *For a PQ bus, P_i and Q_i are known and* $|V_i|$ and δ_i *are to be determined. In a PV bus, P_i and $|V_i|$ being known it is required to obtain Q_i and δ_i.*

The real and reactive power system losses can be obtained as follows:

Real power loss

$$(P_L) = \underbrace{\sum_{i=1}^{N} P_{g_i}}_{\substack{\text{Total active}\\ \text{generation}}} - \underbrace{\sum_{i=1}^{N} P_{d_i}}_{\substack{\text{Total active}\\ \text{demand}}} \tag{4.5a}$$

[= summation of real power flows $= \sum_{i=1}^{N} \sum_{k=1}^{N} (P_{ik} + P_{ki})$]

Reactive power loss

$$(Q_L) = \underbrace{\sum_{i=1}^{N} Q_{g_i}}_{\substack{\text{Total reactive}\\ \text{generation}}} - \underbrace{\sum_{i=1}^{N} Q_{d_i}}_{\substack{\text{Total reactive}\\ \text{demand}}} \tag{4.5b}$$

[= summation of reactive power flows $= \sum_{i=1}^{N} \sum_{k=1}^{N} (Q_{ik} + Q_{ki})$]

4.3 GAUSS-SEIDAL (G-S) METHOD OF POWER FLOW

Let us rewrite first power flow equation (4.2):

$$S_i^* = P_i - jQ_i = V_i^* \sum_{k=1}^{N} Y_{ik} V_k$$

This equation may be expressed in the following form also

$$P_i - jQ_i = V_i^* Y_{ii} V_i + V_i^* \sum_{\substack{k=1 \\ k \neq i}}^{N} Y_{ik} V_k, \quad \text{for } i = 2, 3, \ldots, N$$

or,

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{N} Y_{ik} V_k \right], \quad \text{for } i = 2, 3, \ldots, N \tag{4.6}$$

In *G-S algorithm*, equation (4.6) is utilised to find the final bus voltages using successive steps of iterations, where

$$V_i^{p+1} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{\left(V_i^p\right)^*} - \sum_{\substack{k=1 \\ k \neq i}}^{N} Y_{ik} V_k^p \right], \quad \text{for } i = 2, 3, \ldots, N \tag{4.7}$$

i.e.,

$$V_i^{p+1} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{(V_i^p)^*} - \sum_{k=1}^{i-1} Y_{ik} V_k^{p+1} - \sum_{k=i+1}^{N} Y_{ik} V_k^p \right] \tag{4.7a}$$

As V_1 is known *a priori*, computations are to be performed for buses 2, 3, …, N.

G-S algorithm convergence is slower and it is conventional to use *acceleration factor* for speeding up the convergence process. However, if it increases too much, the system may diverge. The acceleration factor is introduced in the algorithm in the iteration test as follows:

$x^{p+1} = x^p + \alpha \Delta x$, α being the acceleration factor.

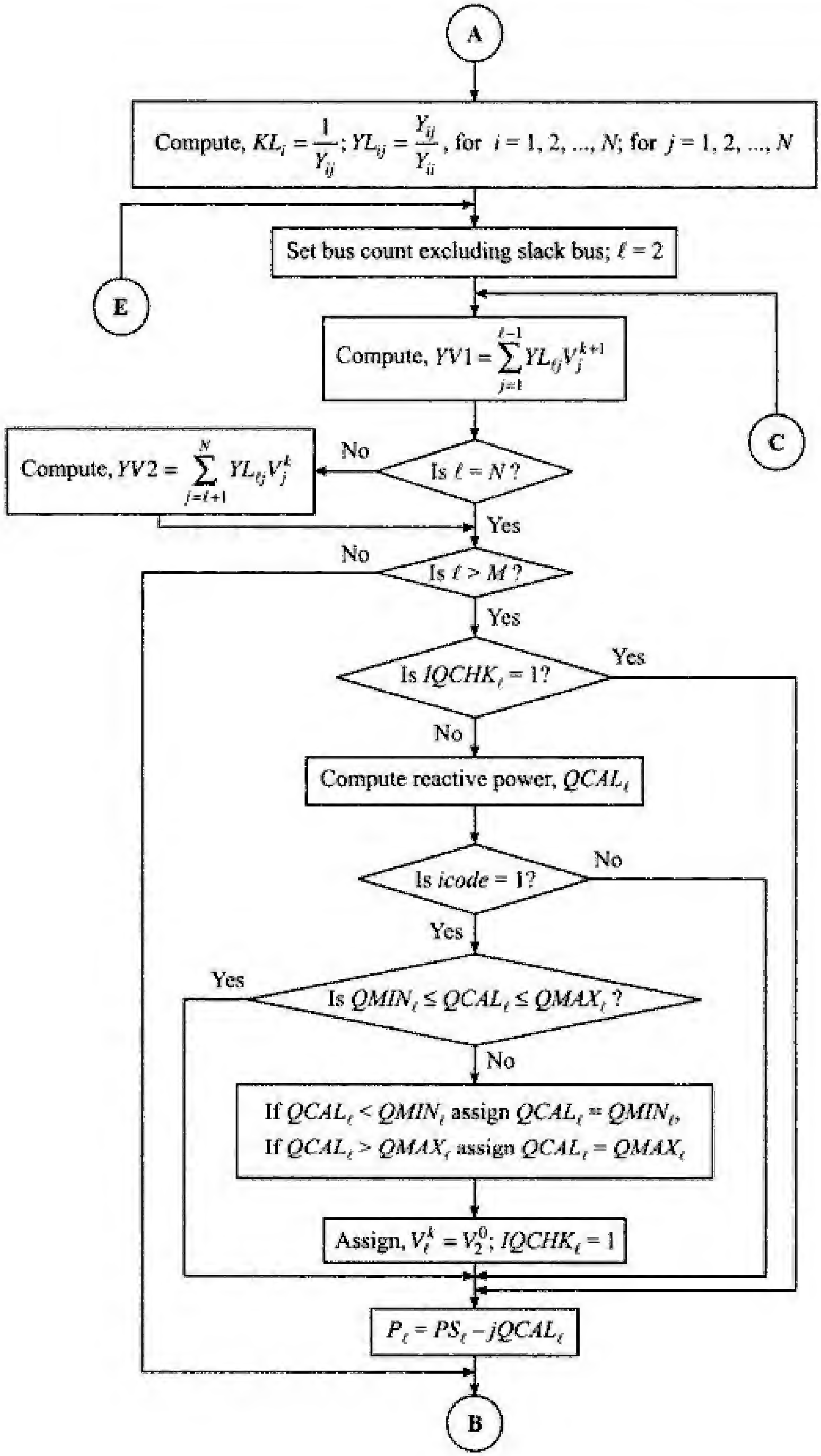
[Usually, the choice of acceleration factor of 1.4 or 1.6 gives best convergence.]

Advantages of G-S method

- It is one of the simplest iterative methods in power flow studies since early days of digital power flow analysis.
- Because of its simplicity, G-S method does have a definite tutorial value, particularly for the beginners.
- G-S method can be conveniently used for load flow studies in small power systems.
- G-S method may be used for even large systems to obtain first approximate solution, which can then be used as "initial solution" for Newton-Raphson method.

The flowchart of G-S method of power flow is shown in Fig. 4.1. However, G-S method convergence becomes increasingly slower as the system size grows and hence it is not very much common to use for practical load flow studies or for general research studies involving power flows in complex networks.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.


A
Compute, $KL_i = \frac{1}{Y_{ij}}$; $YL_{ij} = \frac{Y_{ij}}{Y_{ii}}$, for $i = 1, 2, ..., N$; for $j = 1, 2, ..., N$
Set bus count excluding slack bus; $\ell = 2$
E
Compute, $YV1 = \sum_{j=1}^{\ell-1} YL_{\ell j} V_j^{k+1}$
C
Compute, $YV2 = \sum_{j=\ell+1}^{N} YL_{\ell j} V_j^k$
No
Is $\ell = N$?
Yes
No
Is $\ell > M$?
Yes
Is $IQCHK_\ell = 1$?
Yes
No
Compute reactive power, $QCAL_\ell$
Is $icode = 1$?
No
Yes
Yes
Is $QMIN_\ell \le QCAL_\ell \le QMAX_\ell$?
No
If $QCAL_\ell < QMIN_\ell$ assign $QCAL_\ell = QMIN_\ell$,
If $QCAL_\ell > QMAX_\ell$ assign $QCAL_\ell = QMAX_\ell$
Assign, $V_\ell^k = V_2^0$; $IQCHK_\ell = 1$
$P_\ell = PS_\ell - jQCAL_\ell$
B

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Also, $V_1^{(0)} = (1.06 + j0.0)$, $V_2^{(0)} = (1.0 + j0.0)$, $V_3^{(0)} = (1.0 + j0.0)$.

Here bus-4 being PV bus, $P_4 = 1.2$, $V_4^0 = (1.02 + j0.0)$, $Q_{\min} = 0.1$ and

$Q_{\max} = 1$ [all quantities are expressed in p.u.]

Reactive power at bus-4 is given by,

$$Q_4 = -\operatorname{Im}\left[V_4^{0^*} \left\{ Y_{41}V_1^0 + Y_{42}V_2^0 + Y_{43}V_3^0 + Y_{44}V_4^0 \right\} \right]$$

$$= -\operatorname{Im}\left[1.02 \left\{ (-0.8219178 + j2.191281) \times 1.06 + 0 \right.\right.$$
$$\left.\left. + (-1.038576 + j3.709199) \times 1.0 + (1.860493 - j5.830979) \times 1.02 \right\} \right]$$

$$= -0.086586 \text{ p.u.}$$

Since this Q_4 is less than minimum value of Q_4 given in the problem, hence this PV bus will no longer be a PV bus. It becomes a PQ bus with Q-generation fixed at 0.1 p.u.

$\therefore \qquad P_4 - jQ_4 = (1.2 - j0.1)$ p.u. and $V_4^0 = (1.0 + j0.0)$ p.u.

[flat start for voltage of load buses]

[If this calculated Q_4 had been within Q-limit, then it would have been considered PV bus and then $P_4 - jQ_4 = (1.2 - jQ_4)$ p.u. Also, in this problem, Q_4 is negative; it indicates that generator is supplying capacitive reactive power, which is not normally possible.]

We know in G-S method, the bus voltage at i-th bus at $(p+1)$-th iteration is given by

$$V_i^{(p+1)} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{\left(V_i^{(p)}\right)^*} - \sum_{j=1}^{i-1} Y_{ij}V_j^{(p+1)} - \sum_{j=i+1}^{N} Y_{ij}V_j^{(p)} \right]$$

Let us assume, $\quad L_{ii} = \dfrac{1}{Y_{ii}}$, $(P_i - jQ_i)L_{ii} = KL_i$, $Y_{ij}L_{ii} = YL_{ij}$

Now for bus-2,

$$KL_2 = (-0.6 + j0.3) \times \frac{1}{(1.309588 - j4.536731)} = (-0.096281 - j0.104461)$$

Similarly,

$$KL_3 = -0.067704 - j0.051814, \; KL_4 = -0.075161 + j0.181815$$

$$YL_{21} = (-0.3921568 + j1.568627) \times \frac{1}{(1.309588 - j4.536731)}$$
$$= (-0.342199 - j0.012340)$$

Also,

$$YL_{23} = -0.676113 - j0.110487, \; YL_{24} = 0 + j0 = YL_{42}$$

$$YL_{31} = -0.313313 - j0.007654, \; YL_{32} = -0.316422 + j0.001857$$

$$YL_{34} = -0.381636 + j0.009282, \; YL_{41} = -0.381974 - j0.019081$$

$$YL_{43} = -0.628922 + j0.022557$$

$$\therefore \quad V_2^{(1)} = \frac{KL_2}{\left(V_2^{(0)}\right)^*} - YL_{21}V_1 - YL_{23}V_3^{(0)} - YL_{24}V_4^{(0)}$$

$$= \frac{-0.096281 - j0.104461}{1 - j0} - (-0.342199 + j0.012340) \times (1.06)$$

$$- (-0.676113 - j0.007054) \times 1.0 + 0$$

$$= (0.942563 - j0110487) \text{ p.u.}$$

$\therefore$ The change in voltage is,

$$\Delta V_2^{(1)} = V_2^{(1)} - V_2^{(0)} = (0.942563 - j0.110487) - (1.0 + j0.0)$$

$$= (-0.057437 - j0.110487) \text{ p.u.}$$

$$\therefore \quad V_{2_{(\text{accelerated})}}^{(1)} = V_2^{(0)} + \alpha\Delta V_2^{(1)} = 1.0 + 1.4\,(-0.057437 - j0.110487)$$

$$= 0.919588 - j0.154682 = 0.932507 \angle -9.548222° \text{ p.u.}$$

This value of voltage of bus-2 will be used in subsequent calculations of voltage for the remaining buses. For this reason, in load flow data file two columns of bus voltages are required**.

Again,

$$V_3^{(1)} = \frac{KL_3}{\left(V_3^{(0)}\right)^*} - YL_{31}V_1 - YL_{32}V_2^{(1)} - YL_{34}V_4^{(0)}$$

$$= \frac{-0.067704 - j0.051814}{1 - j0} - (-0.313313 + j0.07654) \times (1.06)$$

$$- (-0.316422 + j0.01857) \times (0919588 - j0.154682)$$

$$- (0.381636 + j0.009282) \times 1.0$$

$$= (0.936734 - j0.103635) \text{ p.u.}$$

$$\therefore \quad \Delta V_3^{(1)} = V_3^{(1)} - V_3^{(0)} = (0.9367343 - j0.103635) - (1.0 + j0.0)$$

$$= (-0.063266 - j0.103635) \text{ p.u.}$$

$$\therefore \quad V_{3_{(\text{accelerated})}}^{(1)} = V_3^{(1)} + \alpha\Delta V_3^{(1)} = 1.0 + 1.4\,(-0.063266 - j0.103635)$$

$$= (0.911428 - j0.145089) \text{ p.u.}$$

$$V_4^{(1)} = \frac{KL_4}{\left(V_4^{(0)}\right)^*} - YL_{41}V_1 - YL_{42}V_2^{(1)} - YL_{43}V_3^{(1)}$$

**Two columns of bus voltages are required at the start of computer execution for the ease of algorithm, the first column being for first iteration while the second column for second iteration, i.e. to calculate voltage of bus-4 at iteration count $k = 3$, bus voltages of bus-1, bus-2, bus-3 of 3rd iteration are required. Here power injection is considered in $(P + jQ)$ form, i.e. *inductive reactive power generation is assumed to be positive* as well as real *power generation as positive.*

**Second column of initial bus voltages for all buses except slack bus is $(0 + j0)$, because for these buses, voltages in 2nd iteration are yet to be calculated.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

$$\therefore \quad \left.\begin{aligned} P_{ij} &= \text{Real}\left[V_i^*\left(V_i - V_j\right)y_{ij} + V_i^* V_i y_{ij_0}\right] \\ Q_{ij} &= -\text{Im}\left[V_i^*\left(V_i - V_j\right)y_{ij} + V_i^* V_i y_{ij_0}\right] \end{aligned}\right\} \left[\because y_{ij} = \frac{1}{z_{ij}}\right] \tag{4.8a}$$

Similarly,

$$\left.\begin{aligned} P_{ji} &= \text{Real}\left[V_j^*\left(V_j - V_i\right)y_{ij} + V_j^* V_j y_{ij_0}\right] \\ Q_{ji} &= -\text{Im}\left[V_j^*\left(V_j - V_i\right)y_{ij} + V_j^* V_j y_{ij_0}\right] \end{aligned}\right\} \tag{4.8b}$$

[where y_{ij_0} is the line charging and z_{ij} is the line impedance of transmission line between i and j. Note that here positive reactive power flow indicates inductive VAR flow].

For the transmission lines with transformer, the line should be transformed to π-equivalent representation of the line with transformer (explained in Chapter 3). For example, let us assume that there is a transmission line with transformer between bus-i and bus-j and the transformer is near to bus-i. Let series admittance of the line, shunt admittance near to from bus (bus-i) and shunt admittance near to to bus (bus-j) be yy_{ij}, $yc1_{ij}$ and $yc2_{ij}$, respectively.

Therefore

$$\therefore \quad \left.\begin{aligned} P_{ij} &= \text{Real}\left[V_i^*\left(V_i - V_j\right)yy_{ij} + V_i^* V_i yc1_{ij}\right] \\ Q_{ij} &= -\text{Im}\left[V_i^*\left(V_i - V_j\right)yy_{ij} + V_i^* V_i yc1_{ij}\right] \end{aligned}\right\} \tag{4.8c}$$

and

$$\left.\begin{aligned} P_{ji} &= \text{Real}\left[V_j^*\left(V_j - V_i\right)yy_{ij} + V_j^* V_j yc2_{ij}\right] \\ Q_{ji} &= -\text{Im}\left[V_j^*\left(V_j - V_i\right)yy_{ij} + V_j^* V_j yc2_{ij}\right] \end{aligned}\right\} \tag{4.8d}$$

Also,

SBP (Slack bus power) = Summation of power flows terminated to slack bus + load demand on slack bus (if any)

$$= \sum_{i=1}^{NL}\left(P_{1i} + jQ_{1i}\right) \text{ [where } NL = \text{ total number of lines]} \tag{4.8e}$$

For the lines without transformer, the line flow from bus-i to bus-j is not equal to the line flow from bus-j to bus-i. The reason is explained below:
for power flow in line i–j,

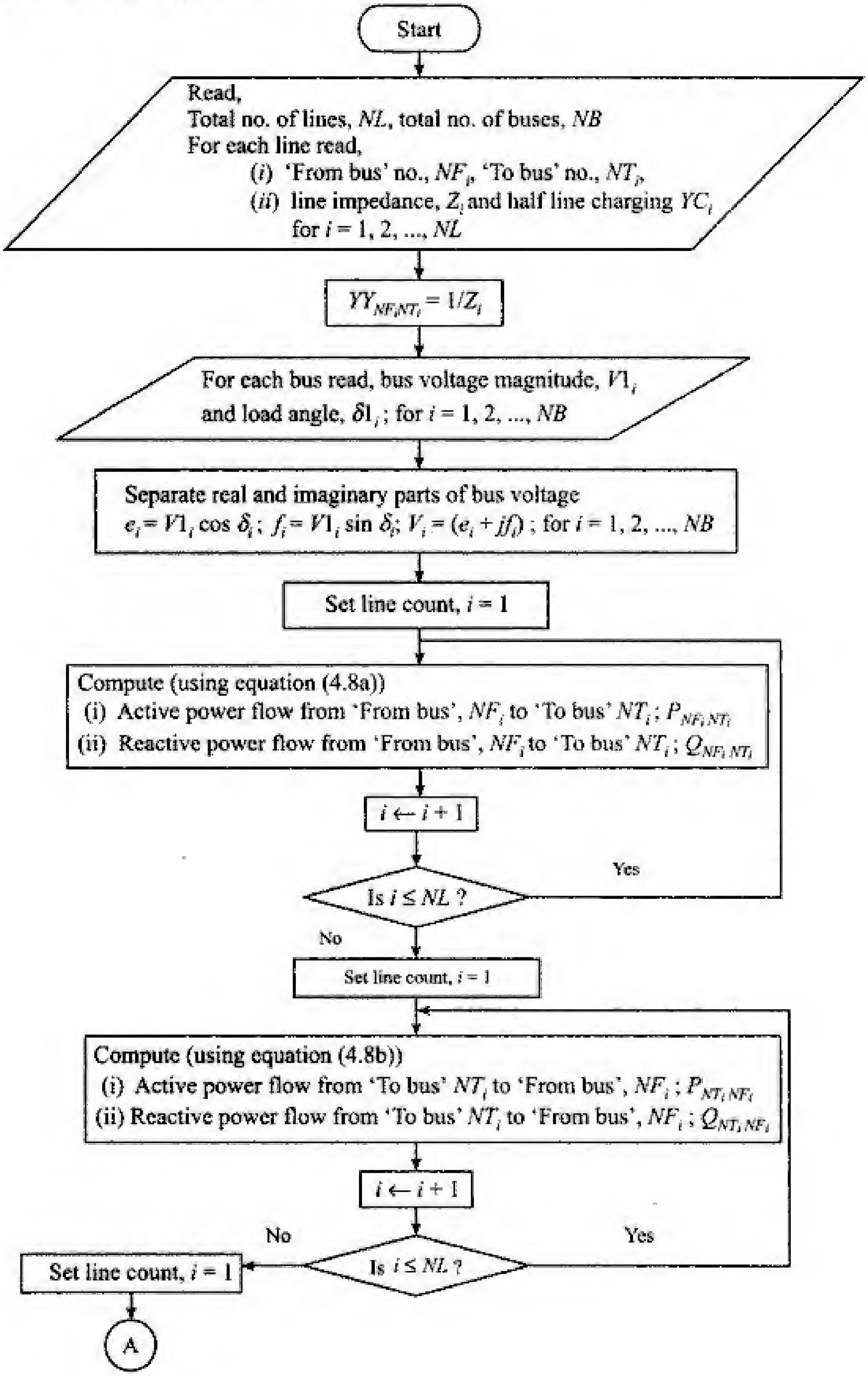
$S_{ij} = V_i^*\left\{\left(V_i - V_j\right)y_{ij} + V_i y_{ij_0}\right\}$ and $S_{ji} = V_j^*\left\{\left(V_j - V_i\right)y_{ji} + V_j y_{ji_0}\right\}$, whereas $z_{ij} = z_{ji}$, $y_{ij_0} = y_{ji_0}$ and $V_i \neq V_j$, therefore $S_{ij} \neq S_{ji}$. Note that also for the lines with transformer, $y_{ij_0} \neq y_{ji_0}$.

The total line loss being the summation of all power flows, we can write,

$$\text{Line loss} = \sum_{i=1}^{NB}\sum_{j=1}^{NB}\left\{\left(P_{ij} + jQ_{ij}\right) + \left(P_{ji} + jQ_{ji}\right)\right\} \tag{4.8f}$$

[where NB = total number of buses]

The flowchart of line flow calculation *without* and *with transformer* has been furnished in Figs. 4.2(a) and 4.2(b), respectively.



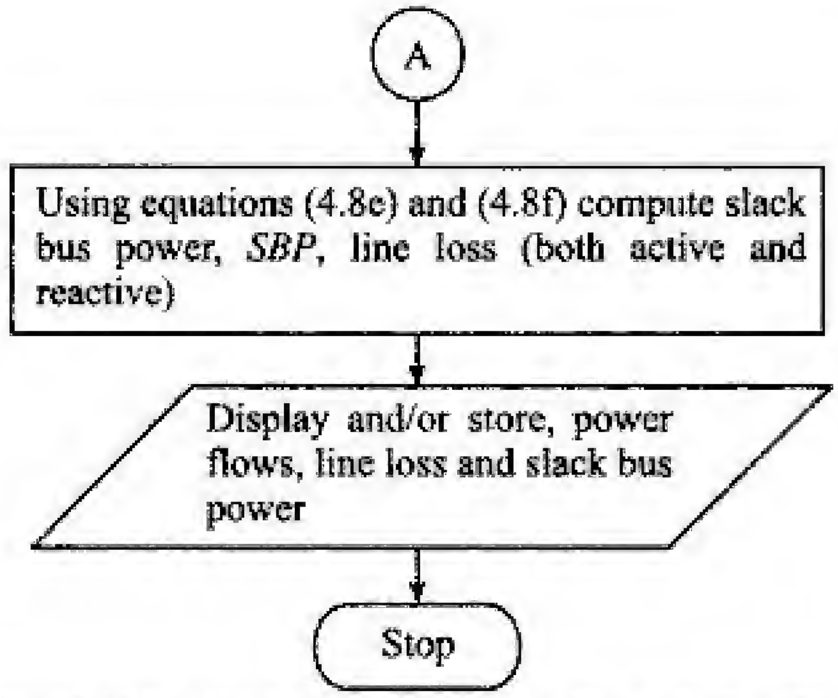
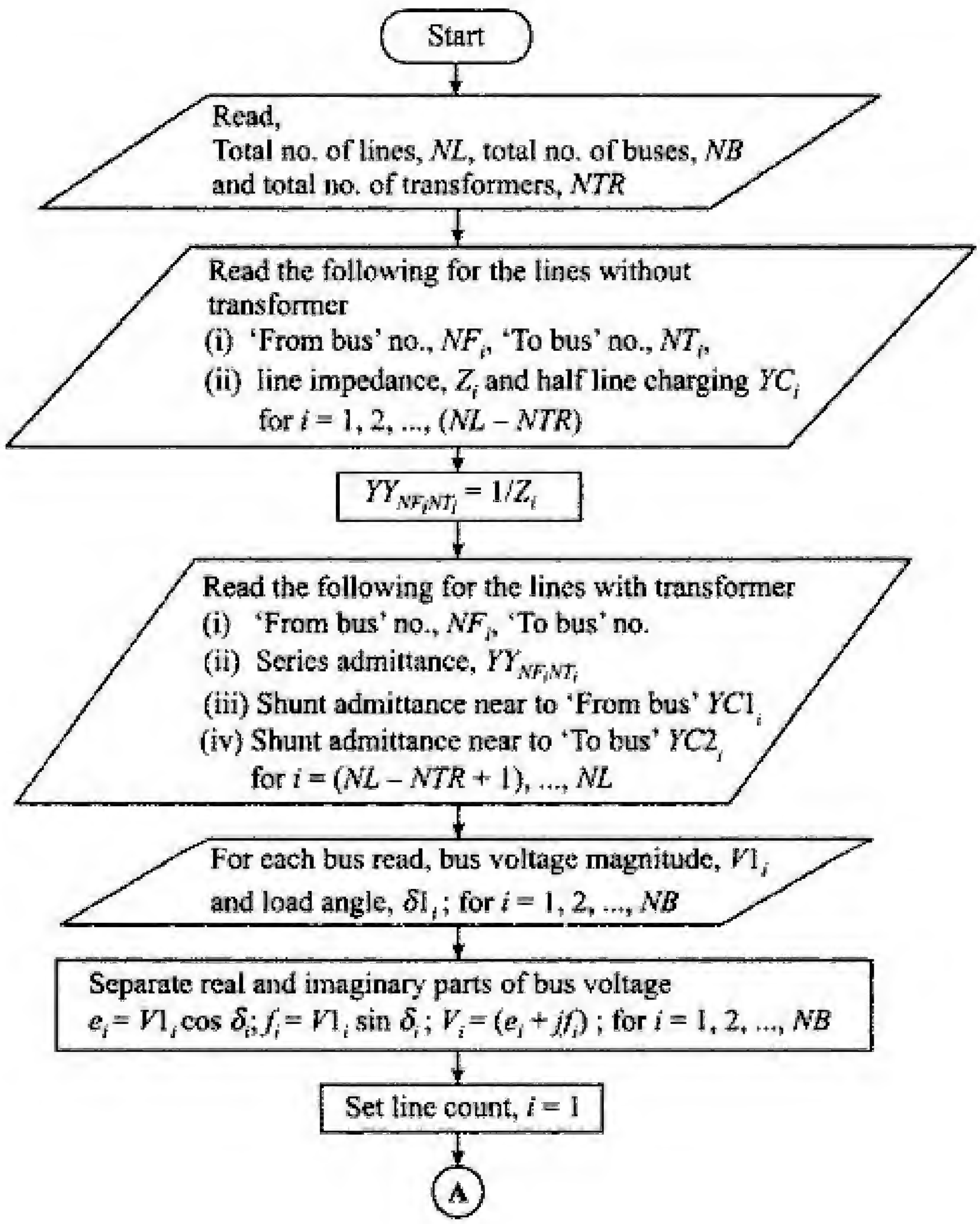


Fig. 4.2(a) Flowchart to calculate power flow for the system without transformer.



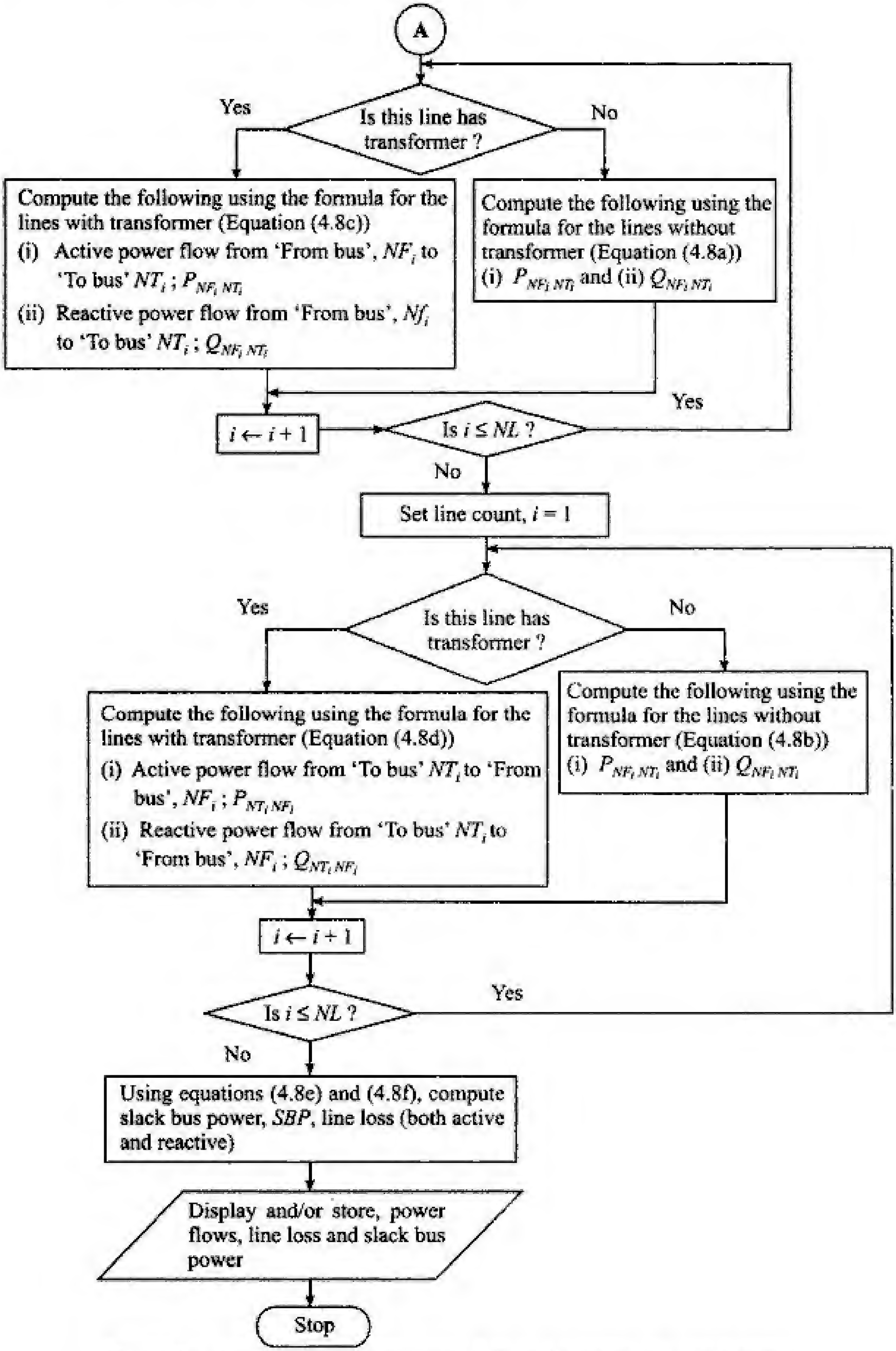


Fig. 4.2(b) Flowchart to calculate power flow for the system with transformer.

Example 4.2: A six-bus, eight-line system has the following line data [schematic of the system is shown in Fig. E4.2]

Line no.	*From bus*	*To bus*	*Line impedance (p.u.)*	*B/2 (p.u.)*	*Off-nominal tap ratio in case of transformer*
1	1	3	$(0.04 + j0.3)$	$j0.01$	–
2	2	3	$(0.03 + j0.2)$	$j0.01$	–
3	3	4	$(0.04 + j0.2)$	$j0.01$	–
4	3	5	$(0.04 + j0.2)$	$j0.01$	–
5	4	5	$(0.03 + j0.15)$	$j0.01$	–
6	5	6	$(0.06 + j0.3)$	$j0.01$	–
7	1	2	$j0.4$	–	1.05
8	2	6	$j0.2$	–	1.02

The scheduled bus powers and initial bus voltages (in p.u.) are as under:

Bus no.	P_d	Q_d	P_g	Q_g	V	*Bus type*
1	0	0	?	?	$1.04\angle 0^\circ$	Slack bus
2	1.0	0.1	0	0	$1.0\angle 0^\circ$	PQ bus
3	0	0	1.5	0.75	$1.0\angle 0^\circ$	PQ bus
4	0.45	0.25	0	0	$1.0\angle 0^\circ$	PQ bus
5	0.40	0.25	0	0	$1.0\angle 0^\circ$	PQ bus
6	0.35	0.1	0	0	$1.0\angle 0^\circ$	PQ bus

Find final bus voltages, line flows, line loss, slack bus power by G-S method [all data are referred on 100 MVA base and in per unit]

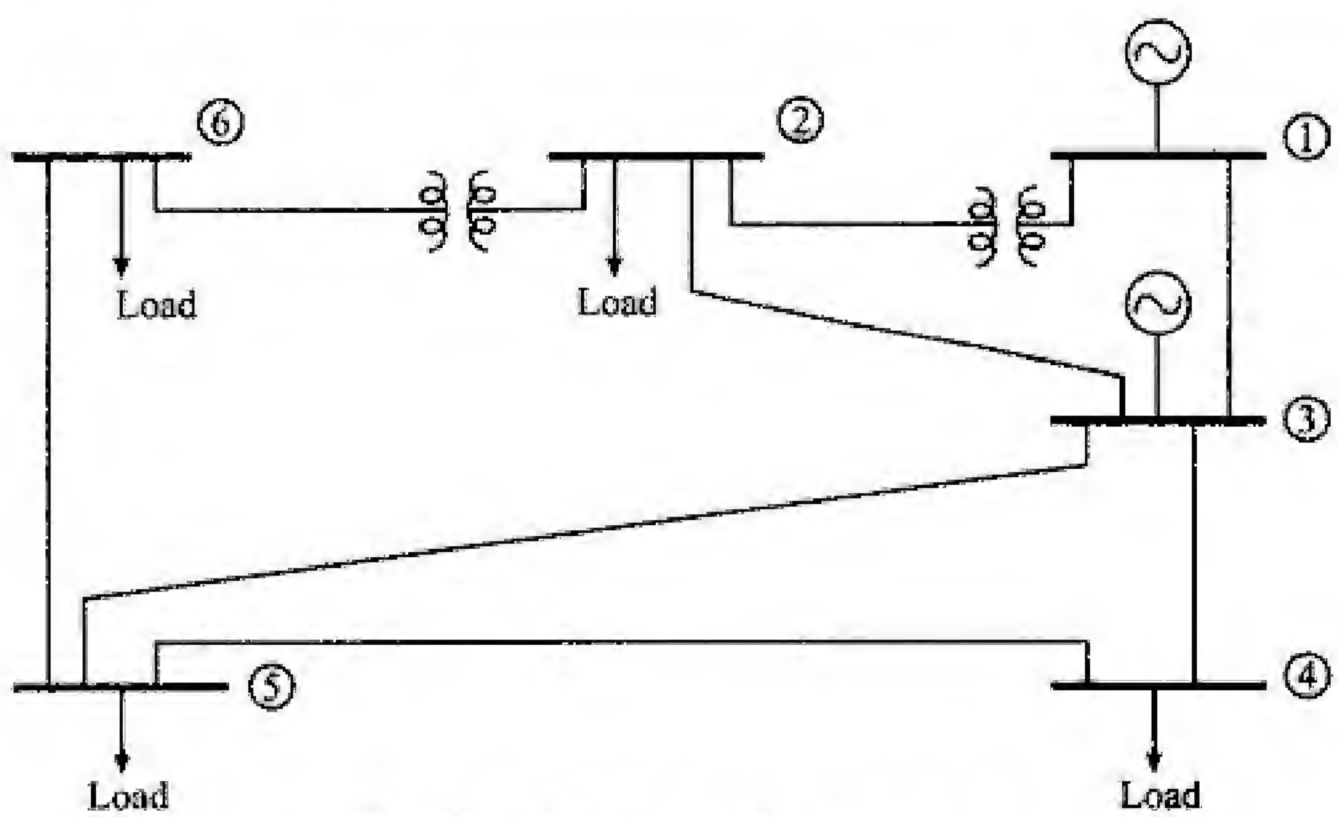


Fig. E4.2 A six-bus, eight-line system.

```
LINE LOSS
=========

4.224439 MW,        23.486810 MVAR

SLACK BUS POWER (IN P.U)
 (7.341943E-01,1.774251E-01)
```

4.4 NEWTON-RAPHSON (N-R) METHOD

4.4.1 Review of Newton-Raphson Method

Newton-Raphson (N-R) method is basically an iterative process for solving a set of simultaneous non-linear equation with an equal number of unknowns.

Let the set of simultaneous non-linear equations be written as

$$f_m(x_n) = 0 \quad \text{for } m = 1, 2, 3, \ldots, N \text{ and } n = 1, 2, 3, \ldots, N$$

The basic philosophy of N-R method of solution is that at each step of the iteration process, the non-linear problem is approximated by linear matrix equation. Let us show the linearising approximation in case of a single variable problem.

Let x^p be an approximation to the solution of the single variable equation while it possesses an error Δx^p at the p-th process of iteration (Fig. 4.3). We can then write

$$f(x^p + \Delta x^p) = 0 \tag{4.9}$$

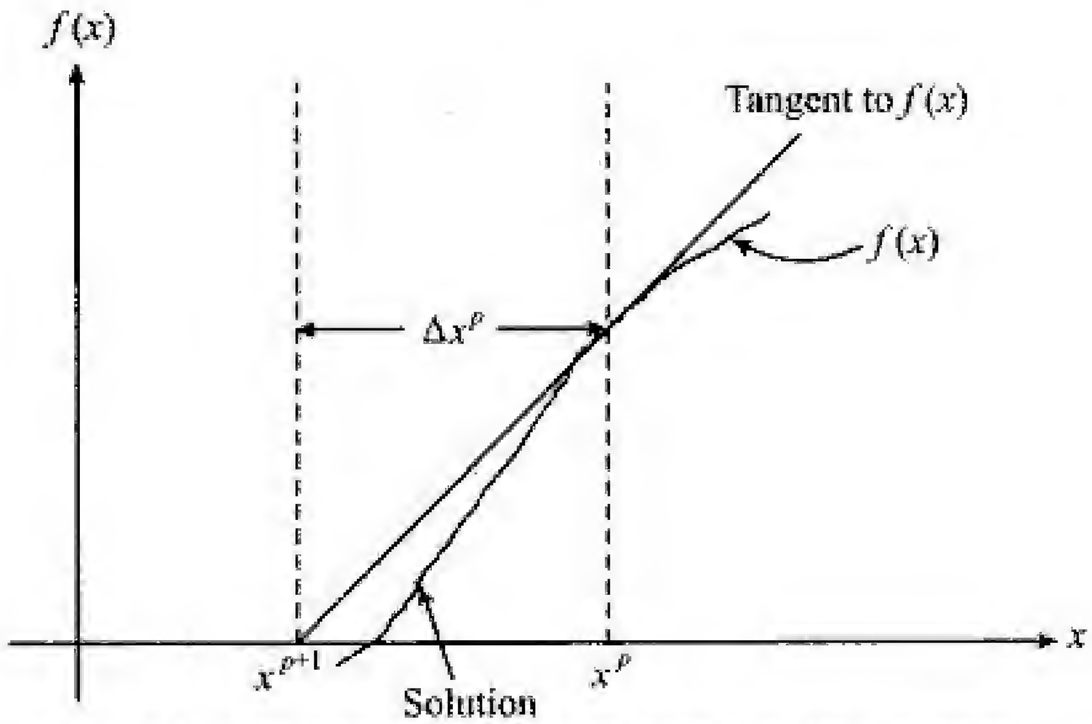


Fig. 4.3 Linear approximation of a single variable equation.

Expanding in Taylor series,

$$f(x^p + \Delta x^p) = 0 = f(x^p) + \Delta x^p f'(x^p) + \frac{(\Delta x^p)^2}{2!} f''(x^p) + \cdots$$

The first derivative $f'(x^p)$ is called *Jacobian* while the second derivative $f''(x^p)$ is known as *Hessian*. Provided the initial estimate of the variable x^p approaches the solution value, Δx^p will be very small and the higher power of Δx^p can easily be neglected.

This gives

$$f(x^p) + \Delta x^p f'(x^p) = 0 \tag{4.10}$$

$$f_{21}(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n) = 0$$
$$f_{22}(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n) = 0$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$f_{2i}(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n) = 0$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$f_{2n}(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n) = 0$$

At p^{th} stage of iteration,

$$\left.\begin{aligned} f_{1i}(\tilde{x}^p, \tilde{y}^p) &= 0 \\ f_{2i}(\tilde{x}^p, \tilde{y}^p) &= 0 \end{aligned}\right\} \tag{4.20}$$

where,
$$\tilde{x}^p = \begin{bmatrix} x_1^p \\ x_2^p \\ \vdots \\ x_i^p \\ \vdots \\ x_n^p \end{bmatrix}; \; \tilde{y}^p = \begin{bmatrix} y_1^p \\ y_2^p \\ \vdots \\ y_i^p \\ \vdots \\ y_n^p \end{bmatrix}; \; \tilde{f}_1 = \begin{bmatrix} f_{11}^p \\ f_{12}^p \\ \vdots \\ f_{1i}^p \\ \vdots \\ f_{1n}^p \end{bmatrix}; \; \tilde{f}_2 = \begin{bmatrix} f_{21}^p \\ f_{22}^p \\ \vdots \\ f_{2i}^p \\ \vdots \\ f_{2n}^p \end{bmatrix}$$

Also,
$$\left[J^p\right] = \begin{bmatrix} \left[\dfrac{\partial f_1^p}{\partial x}\right] & \left[\dfrac{\partial f_1^p}{\partial y}\right] \\ \left[\dfrac{\partial f_2^p}{\partial x}\right] & \left[\dfrac{\partial f_2^p}{\partial y}\right] \end{bmatrix} = \begin{bmatrix} \text{Quad I} & \text{Quad II} \\ \text{Quad III} & \text{Quad IV} \end{bmatrix}$$

$[J^p]$ is $2n \times 2n$ matrix when each quadrant gives a $n \times n$ matrix as shown below:

$$\text{Quad I} = \left[\frac{\partial f_1^p}{\partial x}\right]; \quad \text{Quad II} = \left[\frac{\partial f_1^p}{\partial y}\right]$$
$$\text{Quad III} = \left[\frac{\partial f_2^p}{\partial x}\right]; \quad \text{Quad IV} = \left[\frac{\partial f_2^p}{\partial y}\right]$$

The Jacobian matrix thus becomes an array of all first order partial derivatives.

Also,
$$\Delta\tilde{x} = \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_i \\ \vdots \\ \Delta x_n \end{bmatrix} \text{ and } \Delta\tilde{y} = \begin{bmatrix} \Delta y_1 \\ \Delta y_2 \\ \vdots \\ \Delta y_i \\ \vdots \\ \Delta y_n \end{bmatrix}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

while *diagonal elements* of $[J_2]$ are

$$\frac{\partial P_i}{\partial f_i} = -e_i B_{ii} + 2 f_i G_{ii} + e_i B_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{N} (f_k G_{ik} + e_k B_{ik}) \tag{4.37}$$

Combining equations (4.34) and (4.37), we obtain

$$\frac{\partial P_i}{\partial f_i} = -e_i B_{ii} + f_i G_{ii} + d_i \tag{4.38}$$

Again, the reactive power equation for bus-i is given as

$$\begin{aligned} Q_i &= f_i (e_i G_{ii} - f_i B_{ii}) - e_i (f_i G_{ii} + e_i B_{ii}) \\ &\quad + \sum_{\substack{k=1 \\ k \neq 1}}^{N} \{ f_i (e_k G_{ik} - f_k B_{ik}) - e_i (f_k G_{ik} + e_k B_{ik}) \} \end{aligned} \tag{4.39}$$

$$\left[i = 1, 2, 3, \ldots, (N-1) \right]$$

By differentiation, *off-diagonal elements* of $[J_3]$ are obtained as

$$\frac{\partial Q_i}{\partial e_k} = -e_i B_{ik} + f_i G_{ik}, \; k \neq i; \tag{4.40}$$

while *diagonal elements* of $[J_3]$ are

$$\frac{\partial Q_i}{\partial e_i} = f_i G_{ii} - f_i G_{ii} - 2 e_i B_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{N} (f_k G_{ik} + e_k B_{ik}) \tag{4.41}$$

Combining equations (4.34) and (4.41), we obtain

$$\frac{\partial Q_i}{\partial e_i} = -e_i B_{ii} + f_i G_{ii} - d_i \tag{4.42}$$

The *off-diagonal elements* of $[J_4]$ can be obtained from equation (4.39) as

$$\frac{\partial Q_i}{\partial f_k} = -e_i G_{ik} - f_i B_{ik}, \; i \neq k; \tag{4.43}$$

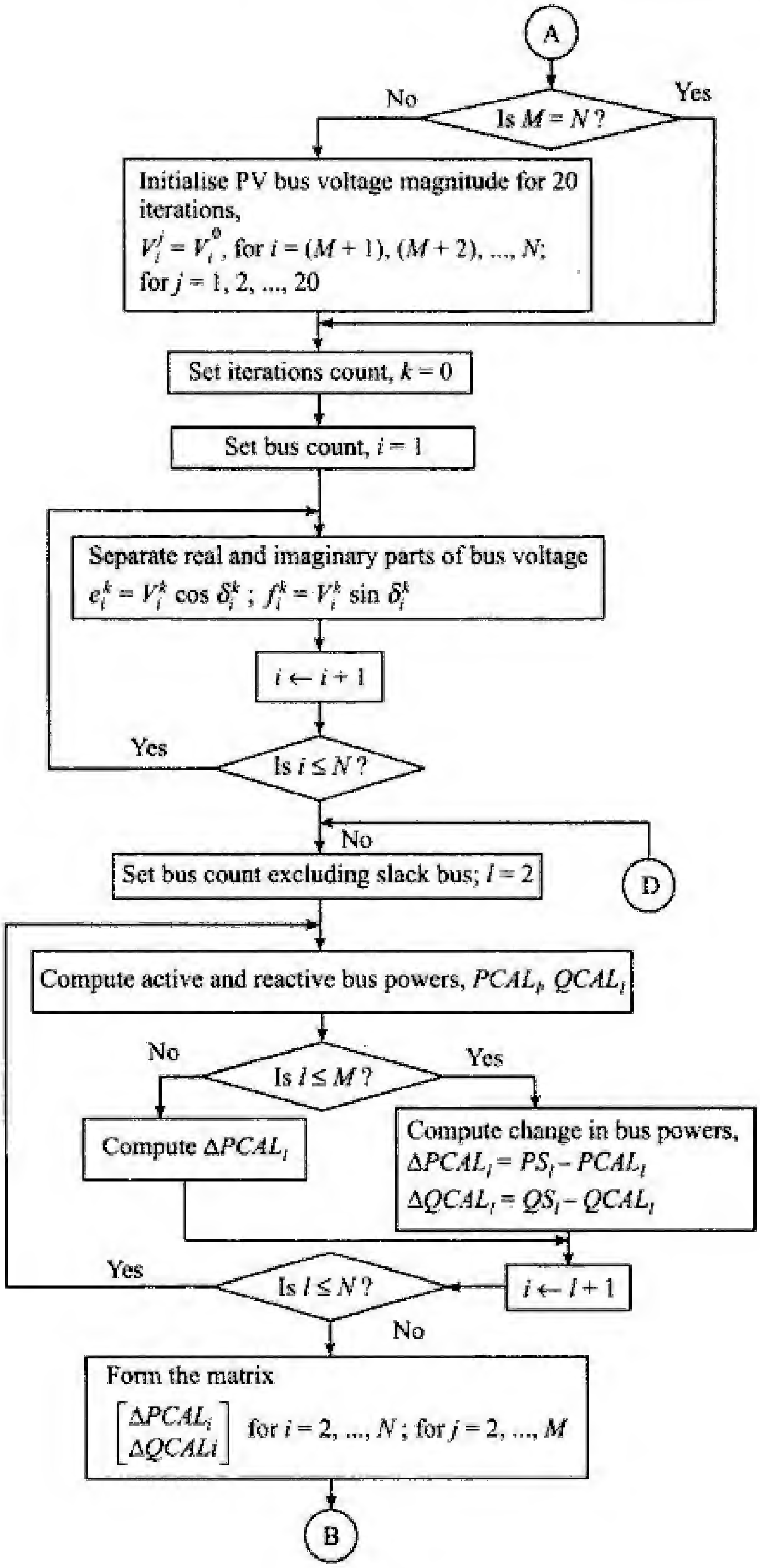
while the *diagonal elements* of $[J_4]$ can also be obtained as

$$\frac{\partial Q_i}{\partial f_i} = e_i G_{ii} - 2 f_i B_{ii} - e_i G_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{N} (e_k G_{ik} - f_k B_{ik}) \tag{4.44}$$

Combining equations (4.33) and (4.44), we obtain

$$\frac{\partial Q_i}{\partial f_i} = -e_i G_{ii} - f_i B_{ii} + c_i \tag{4.45}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.


A
Is $M = N$?
No
Yes
Initialise PV bus voltage magnitude for 20 iterations,
$V_i^j = V_i^0$, for $i = (M + 1), (M + 2), \ldots, N$;
for $j = 1, 2, \ldots, 20$
Set iterations count, $k = 0$
Set bus count, $i = 1$
Separate real and imaginary parts of bus voltage
$e_i^k = V_i^k \cos \delta_i^k$; $f_i^k = V_i^k \sin \delta_i^k$
$i \leftarrow i + 1$
Is $i \leq N$?
Yes
No
D
Set bus count excluding slack bus; $l = 2$
Compute active and reactive bus powers, $PCAL_l$, $QCAL_l$
Is $l \leq M$?
No
Yes
Compute $\Delta PCAL_l$
Compute change in bus powers,
$\Delta PCAL_l = PS_l - PCAL_l$
$\Delta QCAL_l = QS_l - QCAL_l$
Is $l \leq N$?
Yes
No
$i \leftarrow l + 1$
Form the matrix
$\begin{bmatrix} \Delta PCAL_i \\ \Delta QCALi \end{bmatrix}$ for $i = 2, \ldots, N$; for $j = 2, \ldots, M$
B

It may be noted that equations (4.48) and (4.3) are identical. The elements of the Jacobian matrix are then obtained as follows:

For quadrant-1 [J_1]

Diagonal elements:
$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i V_k Y_{ik}| \sin(\phi_{ik} + \delta_k - \delta_i)$$

Off-diagonal elements:
$$\frac{\partial P_i}{\partial \delta_k} = -|V_i V_k Y_{ik}| \sin(\phi_{ik} + \delta_k - \delta_i); \; i \neq k$$

For quadrant-2 [J_2]

Diagonal elements:
$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i Y_{ii}| \cos\phi_{ii} + \sum_{\substack{k=1 \\ k\neq i}}^{N} |V_k Y_{ik}| \cos(\phi_{ik} + \delta_k - \delta_i)$$

Off-diagonal elements:
$$\frac{\partial P_i}{\partial |V_k|} = |V_i Y_{ik}| \cos(\phi_{ik} + \delta_k - \delta_i); \; i \neq k$$

For quadrant-3 [J_3]

Diagonal elements:
$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k\neq i}}^{N} |V_i V_k Y_{ik}| \cos(\phi_{ik} + \delta_k - \delta_i)$$

Off-diagonal elements:
$$\frac{\partial Q_i}{\partial \delta_k} = -|V_i V_k Y_{ik}| \cos(\phi_{ik} + \delta_k - \delta_i); \; i \neq k$$

For quadrant-4 [J_4]

Diagonal elements:
$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i Y_{ii}| \sin\phi_{ii} + \sum_{\substack{k=1 \\ k\neq i}}^{N} |V_k Y_{ik}| \sin(\phi_{ik} + \delta_k - \delta_i)$$

Off-diagonal elements:
$$\frac{\partial Q_i}{\partial |V_k|} = -|V_i Y_{ik}| \sin(\phi_{ik} + \delta_k - \delta_i); \; i \neq k$$

Again, rewriting the power equations

$$P_i = \sum_{k=1}^{N} |V_i V_k Y_{ik}| \cos(\phi_{ik} + \delta_k - \delta_i)$$

and
$$Q_i = -\sum_{k=1}^{N} |V_i V_k Y_{ik}| \sin(\phi_{ik} + \delta_k - \delta_i); \; i = 1, 2, 3, \ldots, (N-1)$$

as
$$P_i = |V_i|^2 G_{ii} + \sum_{\substack{k=1 \\ k\neq i}}^{N} |V_i V_k Y_{ik}| \cos(\phi_{ik} + \delta_k - \delta_i) \tag{4.49}$$

and
$$Q_i = -|V_i|^2 B_{ii} - \sum_{\substack{k=1 \\ k\neq i}}^{N} |V_i V_k Y_{ik}| \sin(\phi_{ik} + \delta_k - \delta_i) \tag{4.50}$$

the elements of the Jacobian can also be shown to have the following form of representations:

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

$$\Delta P_i = \frac{\partial P_i}{\partial \delta_2}\Delta\delta_2 + \frac{\partial P_i}{\partial \delta_3}\Delta\delta_3 + \frac{\partial P_i}{\partial \delta_4}\Delta\delta_4$$
$$+|V_2|\frac{\partial P_i}{\partial |V_2|}\frac{\Delta|V_2|}{|V_2|} + |V_3|\frac{\partial P_i}{\partial |V_3|}\frac{\Delta|V_3|}{|V_3|} + |V_4|\frac{\partial P_i}{\partial |V_4|}\frac{\Delta|V_4|}{|V_4|} \tag{4.64}$$

[Since $|V_i|\frac{\partial P_i}{\partial |V_i|}\times\frac{\Delta|V_i|}{|V_i|} \equiv \frac{\partial P_i}{\partial |V_i|}\Delta|V_i|$ hence it is evident that equations (4.63) and (4.64) do have same dimension and values.] Similarly,

$$\Delta Q_i = \frac{\partial Q_i}{\partial \delta_2}\Delta\delta_2 + \frac{\partial Q_i}{\partial \delta_3}\Delta\delta_3 + \frac{\partial Q_i}{\partial \delta_4}\Delta\delta_4$$
$$+|V_2|\frac{\partial Q_i}{\partial |V_2|}\frac{\Delta|V_2|}{|V_2|} + |V_3|\frac{\partial Q_i}{\partial |V_3|}\frac{\Delta|V_3|}{|V_3|} + |V_4|\frac{\partial Q_i}{\partial |V_4|}\frac{\Delta|V_4|}{|V_4|}$$

Each load bus thus has two mismatch equations. Combining these equations for the four buses, in matrix form we have

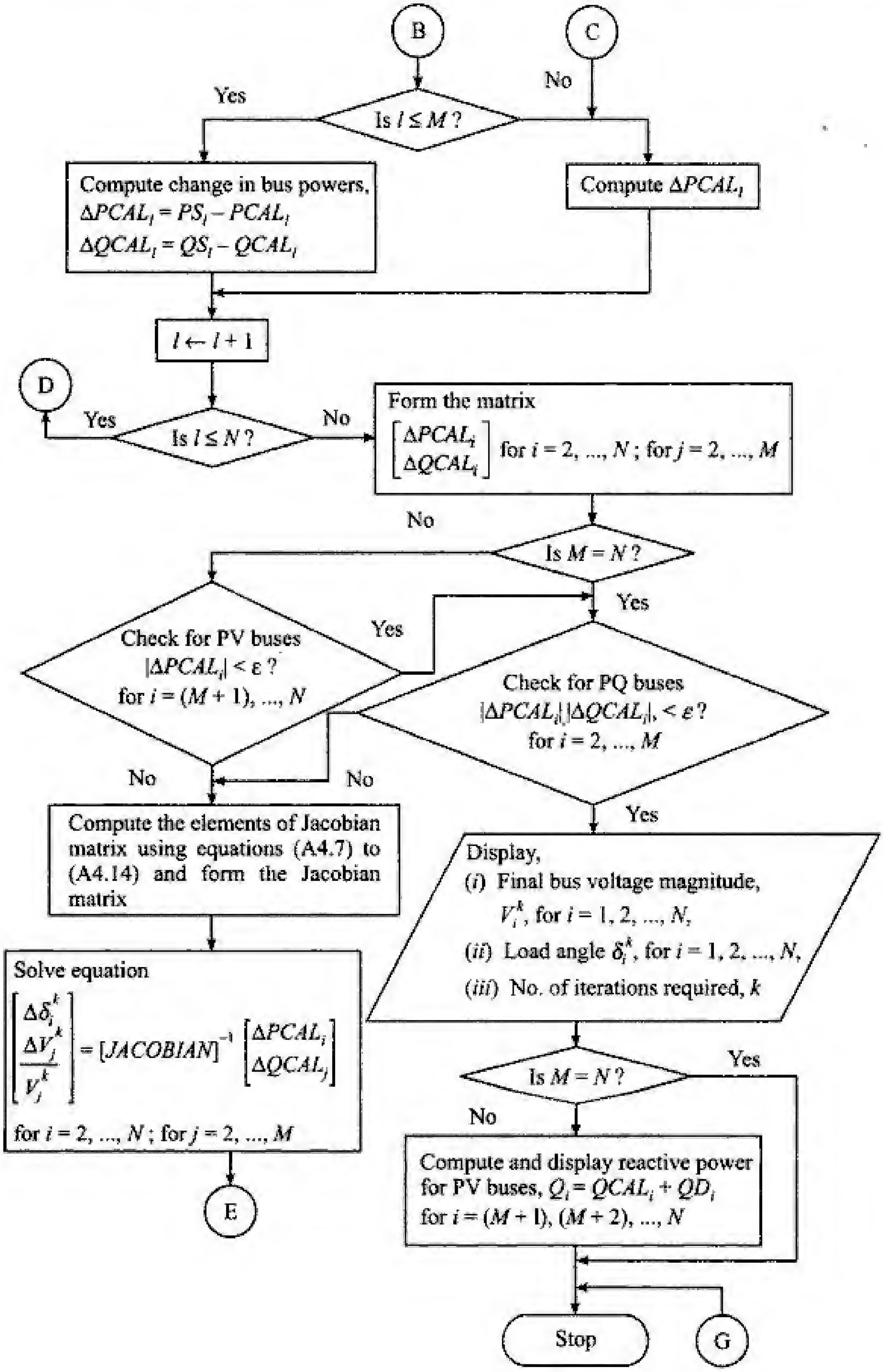
$$\left[\begin{array}{ccc|ccc}
\frac{\partial P_2}{\partial \delta_2} & \cdots & \frac{\partial P_2}{\partial \delta_4} & |V_2|\frac{\partial P_2}{\partial |V_2|} & \cdots & |V_4|\frac{\partial P_2}{\partial |V_4|} \\
\vdots & & & \vdots & & \\
\frac{\partial P_4}{\partial \delta_2} & \cdots & \frac{\partial P_4}{\partial \delta_4} & |V_2|\frac{\partial P_4}{\partial |V_2|} & \cdots & |V_4|\frac{\partial P_4}{\partial |V_4|} \\
\hline
\frac{\partial Q_2}{\partial \delta_2} & \cdots & \frac{\partial Q_2}{\partial \delta_4} & |V_2|\frac{\partial Q_2}{\partial |V_2|} & \cdots & |V_4|\frac{\partial Q_2}{\partial |V_4|} \\
\vdots & & & \vdots & & \\
\frac{\partial Q_4}{\partial \delta_2} & \cdots & \frac{\partial Q_4}{\partial \delta_4} & |V_2|\frac{\partial Q_4}{\partial |V_2|} & \cdots & |V_4|\frac{\partial Q_4}{\partial |V_4|}
\end{array}\right]
\left[\begin{array}{c}
\Delta\delta_2 \\ \vdots \\ \Delta\delta_4 \\ \hline \frac{\Delta|V_2|}{|V_2|} \\ \vdots \\ \frac{\Delta|V_4|}{|V_4|}
\end{array}\right]
=
\left[\begin{array}{c}
\Delta P_2 \\ \vdots \\ \Delta P_4 \\ \hline \Delta Q_2 \\ \vdots \\ \Delta Q_4
\end{array}\right]$$

or,
$$\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}\begin{bmatrix} \Delta\delta_i \\ \Delta|V_i|/|V_i| \end{bmatrix} = \begin{bmatrix} \Delta P_i \\ \Delta Q_i \end{bmatrix} \tag{4.65}$$

Next equation (4.65) is to be solved iteratively to find $\Delta\delta_i^p$ and $\Delta|V_i|^p$. The variables can then be updated as

$$\left.\begin{aligned}
\delta_i^{p+1} &= \delta_i^p + \Delta\delta_i^p \\
|V_i|^{p+1} &= |V_i|^p + \Delta|V_i|^p = |V_i|^p\left(1 + \frac{\Delta\ |V_i|^p}{|V_i|^p}\right)
\end{aligned}\right\} \tag{4.66}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.


B
C
Is $l \le M$?
Yes
No
Compute change in bus powers,
$\Delta PCAL_l = PS_l - PCAL_l$
$\Delta QCAL_l = QS_l - QCAL_l$
Compute $\Delta PCAL_l$
$l \leftarrow l + 1$
D
Is $l \le N$?
Yes
No
Form the matrix
$\begin{bmatrix} \Delta PCAL_i \\ \Delta QCAL_i \end{bmatrix}$ for $i = 2, ..., N$; for $j = 2, ..., M$
Is $M = N$?
No
Yes
Check for PV buses
$|\Delta PCAL_i| < \varepsilon$?
for $i = (M + 1), ..., N$
Yes
Check for PQ buses
$|\Delta PCAL_i|, |\Delta QCAL_i|, < \varepsilon$?
for $i = 2, ..., M$
No
No
Yes
Compute the elements of Jacobian matrix using equations (A4.7) to (A4.14) and form the Jacobian matrix
Display,
(i) Final bus voltage magnitude,
V_i^k, for $i = 1, 2, ..., N$,
(ii) Load angle δ_i^k, for $i = 1, 2, ..., N$,
(iii) No. of iterations required, k
Solve equation
$\begin{bmatrix} \Delta\delta_i^k \\ \dfrac{\Delta V_j^k}{V_j^k} \end{bmatrix} = [JACOBIAN]^{-1} \begin{bmatrix} \Delta PCAL_i \\ \Delta QCAL_j \end{bmatrix}$
for $i = 2, ..., N$; for $j = 2, ..., M$
Is $M = N$?
Yes
No
Compute and display reactive power for PV buses, $Q_i = QCAL_i + QD_i$
for $i = (M + 1), (M + 2), ..., N$
E
Stop
G

$\therefore \quad \Delta P_2^0 = -0.5-(-0.01525) = -0.48475$ p.u.

and $\quad \Delta Q_2^0 = -0.25-(-0.164) = -0.086$ p.u.

As changes in bus powers (both active and reactive) are more than tolerance ($\varepsilon = 0.01$), we have to find the elements of Jacobian matrix and then voltages in next iteration. If the changes in bus power remain within the tolerance then bus voltage at this iteration (here, first iteration) would be the final value of bus voltages. Otherwise the iterative process will continue.

Formation of Jacobian matrix

The Jacobian is given by

$$[J] = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}$$

In this problem since there are only two buses in the system out of which one of them is a slack bus, there is no off-diagonal element of J_1, J_2, J_3 and J_4. The problem thus gets simpler with diagonal elements being given by

$$J_{1_{22}} = -Q_2^0 - B_{22}\,(|V_2|^0)^2 = -(-0.164)-(-1.936)(1.0)^2 = 2.1$$

$$J_{2_{22}} = P_2^0 + G_{22}\,(|V_2|^0)^2 = (-0.01525)+0.305\,(1.0)^2 = 0.28975$$

$$J_{3_{22}} = P_2^0 - G_{22}\,(|V_2|^0)^2 = (-0.01525)-0.305\,(1.0)^2 = -0.32025$$

$$J_{4_{22}} = Q_2^0 - B_{22}\,(|V_2|^0)^2 = -0.164-(-1.936)(1.0)^2 = 1.772$$

Thus, Jacobian matrix becomes a 2×2 square matrix and is as follows:

$$[J] = \begin{bmatrix} 2.1 & 0.28975 \\ -0.32025 & 1.772 \end{bmatrix}$$

This being the Jacobian matrix for first iteration, i.e. 0^{th} iteration, the mismatches can be written as

$$\therefore \quad \begin{bmatrix} \Delta\delta_2^0 \\ \dfrac{\Delta|V_2|^0}{|V_2|^0} \end{bmatrix} = \begin{bmatrix} 2.1 & 0.28975 \\ -0.32025 & 1.772 \end{bmatrix}^{-1} \begin{bmatrix} -0.48475 \\ -0.086 \end{bmatrix}$$

$$= \begin{bmatrix} 0.464605 & -0.075970 \\ 0.083967 & 0.550604 \end{bmatrix} \begin{bmatrix} -0.48475 \\ -0.086 \end{bmatrix} = \begin{bmatrix} -0.218684 \\ -0.088055 \end{bmatrix}$$

Next, new bus voltage magnitude and load angle are obtained using the following equations

$$\delta_i^{p+1} = \delta_i^p + \Delta\delta_i^p\,; \quad |V_i|^{p+1} = |V_i|^p \left(1 + \frac{\Delta|V_i|^p}{|V_i|^p}\right)$$

Here, $\quad \delta_2^1 = \delta_2^0 + \Delta\delta_2^0 = 0.0+(-0.218684) = -0.218684$ rad

$$= 1.05\left[\{1.05-(0.8964831-j0.1104118)\}/(0.08+j0.2)+1.05(j0.04)\right]$$

$$= (0.77762710-j0.45081288) \text{ p.u.}$$

$$[= (77.762710-j45.081288) \text{ MVA, assuming base MVA} = 100]$$

$\therefore$ P_{12} = Active power flow from bus-1 to bus-2 = 77.762710 MW

Q_{12} = Reactive power flow from bus-1 to bus-2 = 45.081288 MVAR

Execution of N-R method algorithm for Example 4.4

LINE DATA (p.u.): NZBUS2.DAT

3,3 [No. of lines, no. of buses]

1,2,(0.08,0.20),(0.0,0.04) [from bus, to bus, (R, X), (G/2, B/2)]

1,3,(0.08,0.2),(0.0,0.04) [Do]

2,3,(0.08,0.2),(0.0,0.04) [Do]

[Y_{Bus}] matrix (p.u.): NYBUS2.DAT

3 [No. of buses]

(3.448276,-8.540689) Y_{11} (-1.724138,4.310345) Y_{12}

(-1.724138,4.310345) Y_{13}

(-1.724138,4.310345) Y_{21} (3.448276,-8.540689) Y_{22}

(-1.724138,4.310345) Y_{23}

(-1.724138,4.310345) Y_{31} (-1.724138,4.310345) Y_{32}

(3.448276,-8.540689) Y_{33}

LOAD FLOW DATA (p.u.): NLF2.DAT

2,3 [No. of PQ buses, Total no. of buses]

1.05,0.0,0.0,0.0 [V_i^0, δ_i^0, PS1, QS1]

1.0,0.0,-0.5,-1.0 [Do]

1.02,0.0,-1.2,+0.5 [V_i^0, δ_i^0, PS3, QD3]

OUTPUT: NRVOLT2.DAT [Bus voltages and reactive power (p.u.) obtained]

```
Iteration reqd.,k =  3

OUTPUT - NRVOLT2.DAT
--------------------
     Bus-code            VOLTAGE            VOLTAGE ANGLE
   -----------         -----------           -----------
        1              1.050000              0.000000E+00
        2              9.032567E-01         -1.225439E-01
        3              1.020000             -2.212110E-01
```

4.10 APPLICATION ASPECT OF N-R METHOD IN MULTI-BUS SYSTEM

The system equations to be solved in complex power flow problem can be written in the form:

$$P_i = F_1\,(\delta, V)$$
$$Q_i = F_2\,(\delta, V)$$

where δ is vector of components δ_i and V is vector of components V_i. If (δ, V) denotes a solution, then $(\delta_{+\gamma e}, V)$ also denotes a solution provided γ is an arbitrary constant and e an N-dimensional vector with N components equal to 1. Then it is possible to select one of the nodes as the reference of phase angles (i.e. δ_i can be set as zero). This node then acts as the *reference node*.

The Jacobian matrix of the system can then be written as

$$[J] = \begin{bmatrix} \dfrac{\partial F_1}{\partial \delta} & \dfrac{\partial F_1}{\partial V} \\ \dfrac{\partial F_2}{\partial \delta} & \dfrac{\partial F_2}{\partial V} \end{bmatrix}$$

where $[J]$ is a $(2N \times 2N)$ square matrix. Without including slack bus, $[J]$ will have dimension of $(2N - 2) \times (2N - 2)$.

In power systems $[J]$ is a *sparse matrix* such that the magnitude of

$$\frac{\partial F_{1_i}}{\partial \delta_j} = \left(\frac{\partial F_{1_i}}{\partial V_j} = \frac{\partial F_{2_i}}{\partial \delta_j} = \frac{\partial F_{2_i}}{\partial V_j} \right) = 0, \text{ if } j \neq i\,;$$

also, the non-zero elements are located symmetrically with respect to the diagonal elements. Ordinarily, corresponding to the stable state of system operation we can write

$$\sin\left(\delta_j - \delta_i\right) \approx \left(\delta_j - \delta_i\right);\ \ V_i \approx V_j \approx V_{\text{nominal}}.$$

Since the resistance r_{ij} of an EHV transmission line is negligibly small in comparison to the inductive reactance x_{ij} (obviously, not for cables), further simplification can be obtained by setting shunt admittance (or line charging) y_{ij_0} to be zero. The non-zero elements of the submatrices $(\partial F_1/\partial V)$ and $(\partial F_2/\partial \delta)$ become very small in comparison to those of submatrices $(\partial F_1/\partial \delta)$ and $(\partial F_2/\partial V)$. We can then write for the power mismatches ΔP and ΔQ,

$$\Delta P = \left(\frac{\partial F_1}{\partial \delta}\right)\Delta\delta;\ \Delta Q = \left(\frac{\partial F_2}{\partial V}\right)\Delta V$$

where $\Delta\delta$ and ΔV are the mismatches of δ and V with respect to the solution.

In the Jacobian, first N column are not linearly dependent and for $i = 1, 2, 3, \ldots, N$,

$$\sum_{j=1}^{N} \frac{\partial F_{1_i}}{\partial \delta_j} = 0;\ \sum_{j=1}^{N} \frac{\partial F_{2_i}}{\partial \delta_j} = 0$$

Then the matrix $[J]$ approaches singularity irrespective of the values of δ and V under these special sets of assumptions. For the usual values of δ and V, during stable operation of the system, $[J]$ has rank $(2N - 1)$. Rank of $[J]$ becomes less when the system approaches instability.

In the presence of tap changer transformer, $[J]$ is no longer a topologically symmetric matrix but it contains more zero elements. If V_1 and V_2 be the primary and secondary voltages while tap changer is

Thus, the decoupled matrix representation of the load flow equations [(4.69) and (4.70)] can be further simplified with off-diagonal elements having terms given by $\{-|V_iV_k|\,B_{ik}\}$ and diagonal elements given by $\{-|V_i|^2\,B_{ii}\}$.

$\therefore$ For an N-bus system with bus no. 1 as swing (or slack) bus, we obtain the decoupled equation as

$$\begin{bmatrix}\Delta P_2\\ \vdots\\ \Delta P_N\end{bmatrix}=\begin{bmatrix}-|V_2V_2|B_{22} & -|V_2V_3|B_{23} & \cdots & -|V_2V_N|B_{2N}\\ \vdots & \vdots & & \vdots\\ -|V_2V_N|B_{N2} & -|V_3V_N|B_{N3} & \cdots & -|V_NV_N|B_{NN}\end{bmatrix}\begin{bmatrix}\Delta\delta_2\\ \vdots\\ \Delta\delta_N\end{bmatrix} \tag{4.73}$$

$$\begin{bmatrix}\Delta Q_2\\ \vdots\\ \Delta Q_N\end{bmatrix}=\begin{bmatrix}-|V_2V_2|B_{22} & -|V_2V_3|B_{23} & \cdots & -|V_2V_N|B_{2N}\\ \vdots & \vdots & & \vdots\\ -|V_2V_N|B_{N2} & -|V_3V_N|B_{N3} & \cdots & -|V_NV_N|B_{NN}\end{bmatrix}\begin{bmatrix}\dfrac{\Delta|V_2|}{|V_2|}\\ \vdots\\ \dfrac{\Delta|V_N|}{|V_N|}\end{bmatrix} \tag{4.74}$$

It has been found that the second set of equations [equation (4.74)] become *relatively unstable* at some distance from the exact solution (due to non-linear defining functions). An improvement is possible by *removing* the voltage from the entries of the Jacobian matrix. This is possible by multiplying each row of the second set of equations in the decoupled method by *correction vector* and then dividing the resultant equation by $|V_i|$ to obtain

$$\begin{aligned}\frac{\Delta Q_2}{|V_2|} &= -B_{22}\Delta|V_2| - B_{23}\Delta|V_3| - \cdots - B_{2N}\Delta|V_N|\\ &\;\vdots \\ \frac{\Delta Q_N}{|V_N|} &= -B_{N2}\Delta|V_2| - B_{N3}\Delta|V_3| - \cdots - B_{NN}\Delta|V_N|\end{aligned} \tag{4.74a}$$

It is thus evident that the coefficients in equation (4.74a) are equal to the negative of the susceptances in the row of $[Y_{Bus}]$ corresponding to the respective bus. All the entries in the coefficient matrix of equation (4.74a) become constants given by the known susceptances of $[Y_{Bus}]$.

Next, we modify the first set of the decoupled equations by multiplying each row by the *vector of angle correction* and division by $|V_i|$.

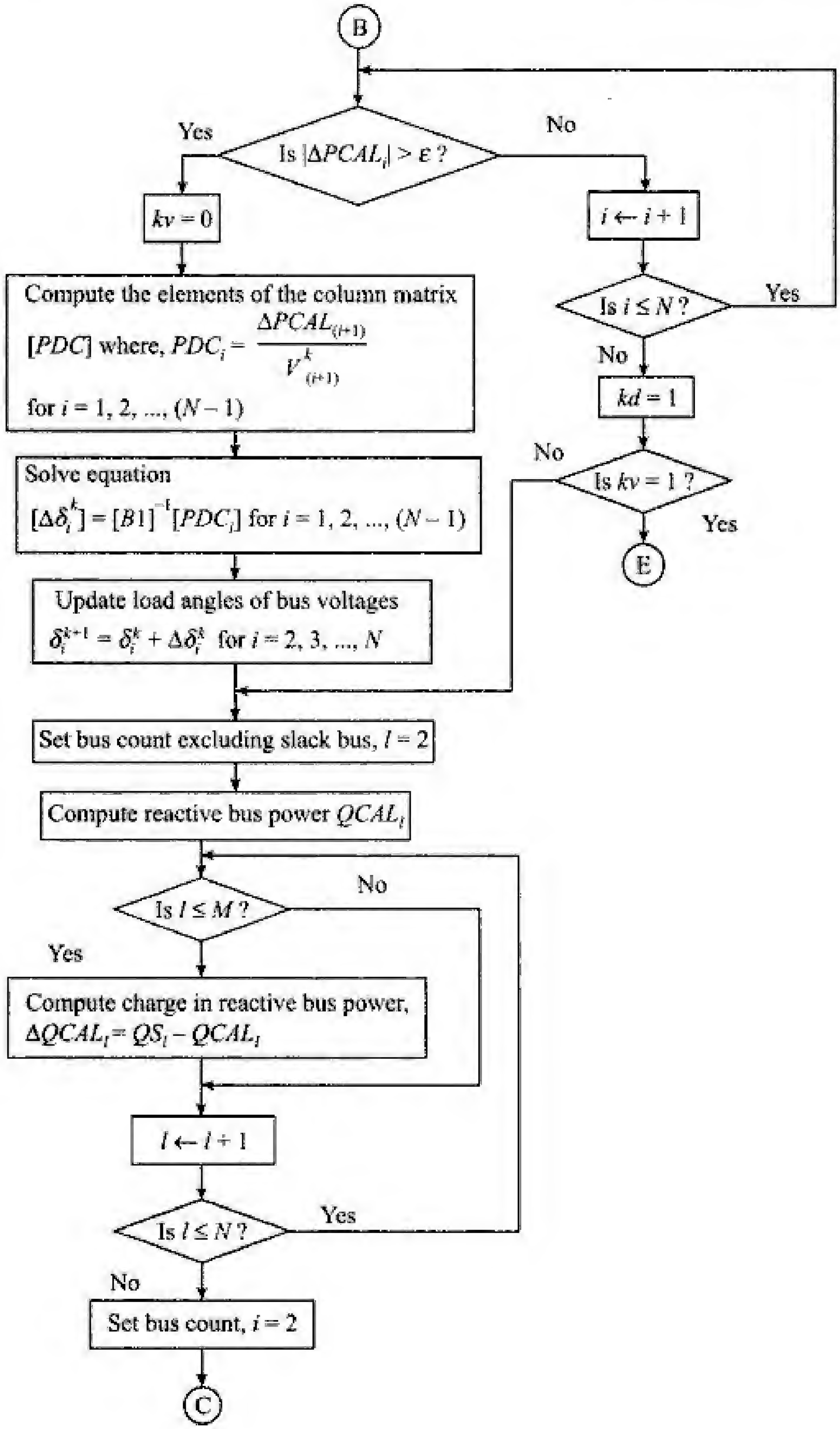
We then obtain

$$\begin{aligned}\frac{\Delta P_2}{|V_2|} &= -B_{22}\Delta\delta_2 - B_{23}\Delta\delta_3 - \cdots - B_{2N}\Delta\delta_N\\ &\;\vdots \\ \frac{\Delta P_N}{|V_N|} &= -B_{N2}\Delta\delta_2 - B_{N3}\Delta\delta_3 - \cdots - B_{NN}\Delta\delta_N\end{aligned} \tag{4.74b}$$

The coefficients in the above equation can be made the same as those of equation (4.74a) by setting the magnitudes of V_2, V_3, ..., V_N equal to 1.0 p.u. in the left-hand side of equation (4.74b). The decoupled equations can now be arranged as follows [equations (4.75) and (4.76)]:

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.


B
Is $|\Delta PCAL_i| > \varepsilon$?
Yes
No
$kv = 0$
$i \leftarrow i + 1$
Is $i \leq N$?
Yes
No
$kd = 1$
Compute the elements of the column matrix [PDC] where, $PDC_i = \dfrac{\Delta PCAL_{(i+1)}}{V^k_{(i+1)}}$ for $i = 1, 2, \ldots, (N - 1)$
Is $kv = 1$?
No
Yes
E
Solve equation $[\Delta\delta_i^k] = [B1]^{-1}[PDC_i]$ for $i = 1, 2, \ldots, (N - 1)$
Update load angles of bus voltages $\delta_i^{k+1} = \delta_i^k + \Delta\delta_i^k$ for $i = 2, 3, \ldots, N$
Set bus count excluding slack bus, $l = 2$
Compute reactive bus power $QCAL_l$
Is $l \leq M$?
No
Yes
Compute charge in reactive bus power, $\Delta QCAL_l = QS_l - QCAL_l$
$l \leftarrow l + 1$
Is $l \leq N$?
Yes
No
Set bus count, $i = 2$
C

$$|V_3|^0 = 1.0 \text{ p.u.}, \qquad \delta_3^0 = 0.0$$

$$\begin{aligned}
P_2^0 &= (|V_2|^0)^2 G_{22} + |V_2|^0 |V_1|^0 |Y_{21}| \cos(\theta_{21} + \delta_1 - \delta_2) \\
&\quad + |V_2|^0 |V_3|^0 |Y_{23}| \cos(\theta_{23} + \delta_3 - \delta_2) \\
&= 1.0^2 \times 3.846154 + 1.0 \times 1.03 \times 9.805806 \cos(1.768192 - 0 + 0) \\
&\quad + 1.0 \times 1.0 \times 9.805806 \cos(1.768192 - 0 + 0) \\
&= -0.057694 \text{ p.u.}
\end{aligned}$$

$$\begin{aligned}
P_3^0 &= (|V_3|^0)^2 G_{33} + |V_3|^0 |V_1|^0 |Y_{31}| \cos(\theta_{31} + \delta_1 - \delta_3) \\
&\quad + |V_3|^0 |V_2|^0 |Y_{32}| \cos(\theta_{32} + \delta_2 - \delta_3) \\
&= 1.0^2 \times 3.846154 + 1.0 \times 1.03 \times 9.805806 \cos(1.768192 - 0 + 0) \\
&\quad + 1.0 \times 1.0 \times 9.805806 \cos(1.768192 - 0 + 0) \\
&= -0.057694 \text{ p.u.}
\end{aligned}$$

$$\begin{aligned}
\Delta P_2^0 &= P_{2_{\text{scheduled}}} - P_2^0 = -1.5 - (-0.057694) \\
&= -1.442306 \text{ p.u.}
\end{aligned}$$

Similarly,

$$\begin{aligned}
\Delta P_3^0 &= P_{3_{\text{scheduled}}} - P_3^0 = -0.5 - (-0.057694) \\
&= -0.442306 \text{ p.u.}
\end{aligned}$$

Since both of these changes in bus power are more than given tolerance ($\varepsilon = 0.01$), load angles are required to be calculated.

$$\frac{\Delta P_2^0}{|V_2|^0} = -1.442306; \quad \frac{\Delta P_3^0}{|V_3|^0} = -0.442306$$

$\therefore$

$$\begin{bmatrix} \Delta\delta_2^0 \\ \Delta\delta_3^0 \end{bmatrix} = \begin{bmatrix} 19.150770 & -9.615384 \\ -9.615384 & 19.150770 \end{bmatrix}^{-1} \begin{bmatrix} -1.442306 \\ -0.442306 \end{bmatrix}$$

or,

$$\begin{bmatrix} \Delta\delta_2^0 \\ \Delta\delta_3^0 \end{bmatrix} = \begin{bmatrix} 0.0698178 & 0.0350547 \\ 0.0350547 & 0.0698178 \end{bmatrix} \begin{bmatrix} -1.442306 \\ -0.442306 \end{bmatrix}$$

$$\Delta\delta_2^0 = -0.116204 \text{ rad} \quad \text{and} \quad \Delta\delta_3^0 = 0.081441 \text{ rad.}$$

$\therefore$

$$\delta_2^1 = \delta_2^0 + \Delta\delta_2^0 = 0.0 - 0.116204 = -0.116204 \text{ rad.}$$

and

$$\delta_3^0 = -0.081441 \text{ rad.}$$

Next Q_2^0 will be calculated by using updated value of δ_2 and δ_3.

$$\begin{aligned}
Q_2^0 &= (|V_2|^0)^2 B_{22} + |V_2|^0 |V_1|^0 |Y_{21}| \sin(\theta_{21} + \delta_1 - \delta_2^1) \\
&\quad + |V_2|^0 |V_1|^0 |Y_{21}| \sin(\theta_{21} + \delta_1 - \delta_2^1)
\end{aligned}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Bus-code	MW	MVAR
1 - 3	23.105630	-8.621950
3 - 1	-22.887170	8.065471
2 - 3	-65.749360	-10.377880
3 - 2	67.048160	16.904390
3 - 4	50.954470	25.570250
4 - 3	-49.765870	-21.718790
3 - 5	54.882740	24.180310
5 - 3	-53.570710	-19.715480
4 - 5	4.738870	-3.137927
5 - 4	-4.730553	1.219366
5 - 6	18.273050	-6.359457
6 - 5	-18.051350	5.490314
1 - 2	51.253480	27.374790
2 - 1	-51.253480	-16.049330
2 - 6	16.975960	16.582570
6 - 2	-16.975960	-15.519910

```
LINE LOSS
=========

      4.247910 MW ,       23.886730 MVAR

SLACK BUS POWER (IN P.U.)

      (7.435911E-01,1.875284E-01)
```

Example 4.9: Find final bus voltages by FDLF method for the four-bus, five-line system given in Example 4.1 when bus-2 has a Q-limit of $0.1 \leq Q_2 \leq 1.0$ p.u.

Solution: The flowchart for the FDLF method for the calculation of bus voltages with Q-limit at PV buses is furnished in Fig. 4.8.

Execution of FDLF algorithm (with Q-limit at PV bus) for Example 4.9

LINE DATA - GZBUS1.DAT

Ybus Matrix - GYBUS1.DAT

For all of the above data, refer Example 4.1 (G-S method).

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

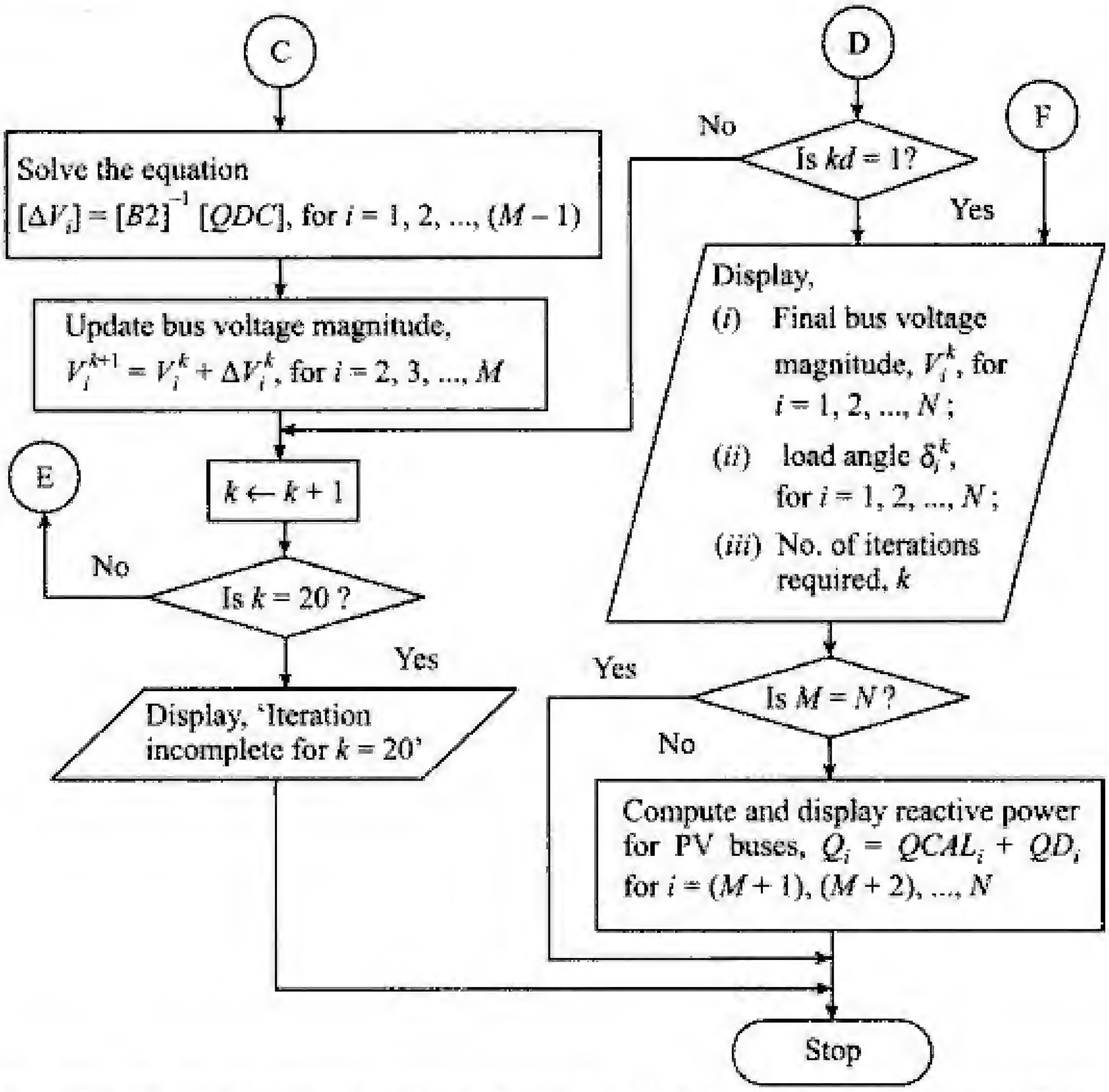


Fig. 4.8 Flowchart to calculate bus voltages by FDLF method for the system with Q-limit at PV buses.

LOAD FLOW DATA - FDLF3.DAT

3,4 [no. of PQ buses, total no. of buses]

1.06,0.0,0.0,0.0 [$V_1^{(0)}$,$\delta_1^{(0)}$,$PS1$,$QS1$]

1.0,0.0,-0.6,-0.3

1.0,0.0,-0.7,-0.5

1.02,0.0,1.2,0.0,0.1,1 [V_4^{0},δ_4^{0},$PS4$,$QD4$, $Q_{\min 4}$, $Q_{\max 4}$]

This data file is same as GLDFLO1.DAT; only orientation of data is different to match with algorithm.

OUTPUT OF FDLFQ.FOR - FDVOLT3.DAT [Final bus voltages]

```
No. of Iterations reqd. k =  5

FINAL BUS VOLTAGES (in p.u.) ARE
```

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Example 4.11: A 14 bus 20 line power system has the following line data:

Line no.	*From bus*	*To bus*	*Line impedance (p.u.)*	*B/2 (p.u.)*
1	1	2	$(0.02 + j0.26)$	$j0.06$
2	1	3	$(0.08 + j0.24)$	$j0.05$
3	2	3	$0.06 + j0.15)$	$j0.04$
4	2	4	$(0.05 + j0.14)$	$j0.04$
5	2	5	$(0.04 + j0.15)$	$j0.03$
6	2	6	$(0.06 + j0.15)$	$j0.05$
7	3	3	$(0.02 + j0.20)$	$j0.02$
8	4	5	$(0.08 + j0.25)$	$j0.05$
9	4	7	$(0.06 + j0.18)$	$j0.04$
10	5	10	$(0.02 + j0.25)$	$j0.025$
11	6	8	$(0.05 + j0.12)$	$j0.01$
12	6	12	$(0.04 + j0.35)$	$j0.005$
13	7	10	$(0.05 + j0.435)$	$j0.015$
14	7	12	$(0.056 + j0.315)$	$j0.025$
15	9	11	$(0.03 + j0.15)$	$j0.015$
16	9	14	$(0.04 + j0.2)$	$j0.02$
17	10	12	$(0.02 + j0.55)$	$j0.013$
18	10	14	$(0.05 + j0.95)$	$j0.02$
19	11	13	$(0.05 + j0.1)$	$j0.02$
20	12	14	$(0.015 + j0.35)$	$j0.03$

The scheduled bus powers and initial bus voltages (in p.u.) are as under:

Bus no.	P_d	Q_d	P_g	Q_g	V	*Bus type*
1	0	0	?	?	$1.04\angle 0°$	Slack bus
2	0.4	0.2	0	0	$1.0\angle 0°$	PQ bus
3	0.45	0.15	1.5	0	$1.0\angle 0°$	PQ bus
4	0.4	0.05	0	0	$1.0\angle 0°$	PQ bus
5	0.6	0.1	0	0	$1.0\angle 0°$	PQ bus
6	0.5	0.2	0	0	$1.0\angle 0°$	PQ bus
7	0.15	0.2	0	0	$1.0\angle 0°$	PQ bus
8	0.1	0.1	0	0	$1.0\angle 0°$	PQ bus
9	0.15	0.3	0	0	$1.0\angle 0°$	PQ bus
10	0.15	0.25	0	0	$1.0\angle 0°$	PQ bus
11	0.25	0.25	0	0	$1.0\angle 0°$	PQ bus
12	0	0.1	0.25	?	$1.01\angle 0°$	PV bus
13	0	0.15	0.25	?	$1.02\angle 0°$	PV bus
14	0	0.25	0.25	?	$1.025\angle 0°$	PV bus

Find final bus voltages, line flows, line loss, slack bus power by N-R method. Verify the results by Fast decoupled and DC load flow methods (all data are referred on 100 MVA base and in per unit). For DC load flow, calculations are to be done for real parts only.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Solution:

LINE DATA – S2ZBUS.DAT

```
30,22,3 [No. of lines, no. of buses, total no. of transformers]
1,4,(0.03,0.21),(0.0,0.04)        [from bus, to bus , (R,X), (G/2, B/2)]
1,5,(0.05,0.15),(0.0,0.01)
1,9,(0.02,0.2),(0.0,0.01)
2,3,(0.021,0.18),(0.0,0.028)
2,10,(0.04,0.15),(0.0,0.021)
2,20,(0.024,0.2),(0.0,0.026)
2,21,(0.01,0.136),(0.0,0.01)
3,7,(0.025,0.24),(0.0,0.06)
3,12,(0.015,0.125),(0.0,0.013)
5,13,(0.028,0.25),(0.0,0.02)
5,18,(0.05,0.15),(0.0,0.021)
6,11,(0.02,0.14),(0.0,0.025)
6,15,(0.04,0.2),(0.0,0.0)
6,20,(0.03,0.12),(0.0,0.02)
6,22,(0.01,0.16),(0.0,0.02)
7,19,(0.035,0.15),(0.0,0.03)
7,21,(0.01,0.136),(0.0,0.01)
8,20,(0.02,0.16),(0.0,0.05)
9,16,(0.025,0.16),(0.0,0.06)
10,15,(0.021,0.25),(0.0,0.02)
10,20,(0.02,0.1),(0.0,0.03)
11,18,(0.01,0.15),(0.0,0.028)
13,17,(0.04,0.12),(0.0,0.024)
14,19,(0.024,0.14),(0.0,0.018)
15,18,(0.026,0.156),(0.0,0.015)
15,20,(0.027,0.164),(0.0,0.005)
17,20,(0.01,0.14),(0.0,0.02)
1,8,(0.0,0.15),(1.01,0)           [from bus, to bus, (Rtr,Xtr), off-nominal tap ratio)]
8,13,(0.0,0.25),(1.02,0)
8,22,(0.0,0.2),(1.02,0.0)
```

LINE DATA USED IN CALCULATION OF POWER FLOW – S2PZBUS.DAT

```
30,22,3 [no. of lines, no. of buses, total no. of transformers]
1,4,(0.03,0.21),(0.0,0.04) [from bus, to bus , (R,X), (G/2, B/2)]
1,5,(0.05,0.15),(0.0,0.01)
2,3,(0.021,0.18),(0.0,0.028)
2,10,(0.04,0.15),(0.0,0.021)
3,7,(0.025,0.24),(0.0,0.06)
3,12,(0.015,0.125),(0.0,0.013)
5,18,(0.05,0.15),(0.0,0.021)
5,13,(0.028,0.25),(0.0,0.02)
6,15,(0.04,0.2),(0.0,0.0)
6,20,(0.03,0.12),(0.0,0.02)
7,19,(0.035,0.15),(0.0,0.03)
```

21	1.025000	-.321445
22	1.050000	-.221435

REACTIVE POWER (in p.u.) AT VOLTAGE CONTROLLED (PV) BUSES

bus no.	Reactive power
19	.765928
20	2.488045
21	.921138
22	1.122220

OUTPUT OF TPFLO.FOR: S2NPOW.DAT [Line power flows, Line loss, Slack Bus Power]

Base MVA = 100

************ LINEFLOWS ************

Bus-code	MW	MVAR
1 - 4	25.854890	25.556100
4 - 1	-25.399990	-29.999980
1 - 5	101.103800	43.409730
5 - 1	-95.125990	-27.311890
2 - 3	21.836630	29.627280
3 - 2	-21.499920	-31.709570
2 - 10	-13.199150	-1.612252
10 - 2	13.272780	-2.110882
3 - 7	-2.597765	-30.021420
7 - 3	2.789592	21.180870
3 - 12	12.097620	18.731080
12 - 3	-11.999940	-19.999920
5 - 18	52.623650	-1.791034
18 - 5	-50.925960	3.550787
5 - 13	27.501870	-15.896860
13 - 5	-27.172340	15.449930
6 - 15	7.914066	11.642060
15 - 6	-7.827249	-11.207970
6 - 20	-12.229850	-50.102580
20 - 6	13.044770	49.455380
7 - 19	-2.087165	-27.539410
19 - 7	2.312415	22.554500
8 - 20	55.113370	-36.526340
20 - 8	-54.261010	33.394830
10 - 15	-1.315661	17.934420
15 - 10	1.402480	-20.537220

20 - 2	23.774420	18.384000
2 - 21	1.253957	-37.591720
21 - 2	-1.112239	37.519880
7 - 21	-25.906570	-33.407040
21 - 7	26.087350	33.861310
10 - 20	-28.180420	-37.439510
20 - 10	28.595720	33.521290
6 - 22	-36.639980	-54.810950
22 - 6	37.094020	58.042600
6 - 11	6.881374	36.166180
11 - 6	-6.547477	-38.131000
8 - 22	22.209440	-27.129250
22 - 8	-22.209440	29.615070
1 - 8	118.309300	41.317690
8 - 1	-118.309300	-18.680470
8 - 13	18.436240	22.793980
13 - 8	-18.436240	-20.621450

```
LINE LOSS
=========

     17.160820 MW              8.540939 MVAR

SLACK BUS POWER (IN P.U)

              (3.021395,1.602857)
```

Execution of DC load flow algorithm for Example 4.12

OUTPUT OF DC.FOR – S2DVOLT.DAT [Final bus voltages (in p.u.) and angles are in radian.]

```
No. of Iterations reqd. k =  2

 FINAL BUS VOLTAGES ARE
```

Bus-code	VOLTAGE	VOLTAGE ANGLE
1	1.010000	.000000
2	1.000000	-.361085
3	1.000000	-.411755
4	1.000000	-.054899
5	1.000000	-.179328
6	1.000000	-.308538
7	1.000000	-.409467
8	1.000000	-.196650
9	1.000000	-.114759

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

3. Figure P4.2 represents a five-bus power system feeding constant power loads.The line data and load schedule are shown in Tables P4.3 and P4.4 (line data has been furnished on 100 MVA basis).

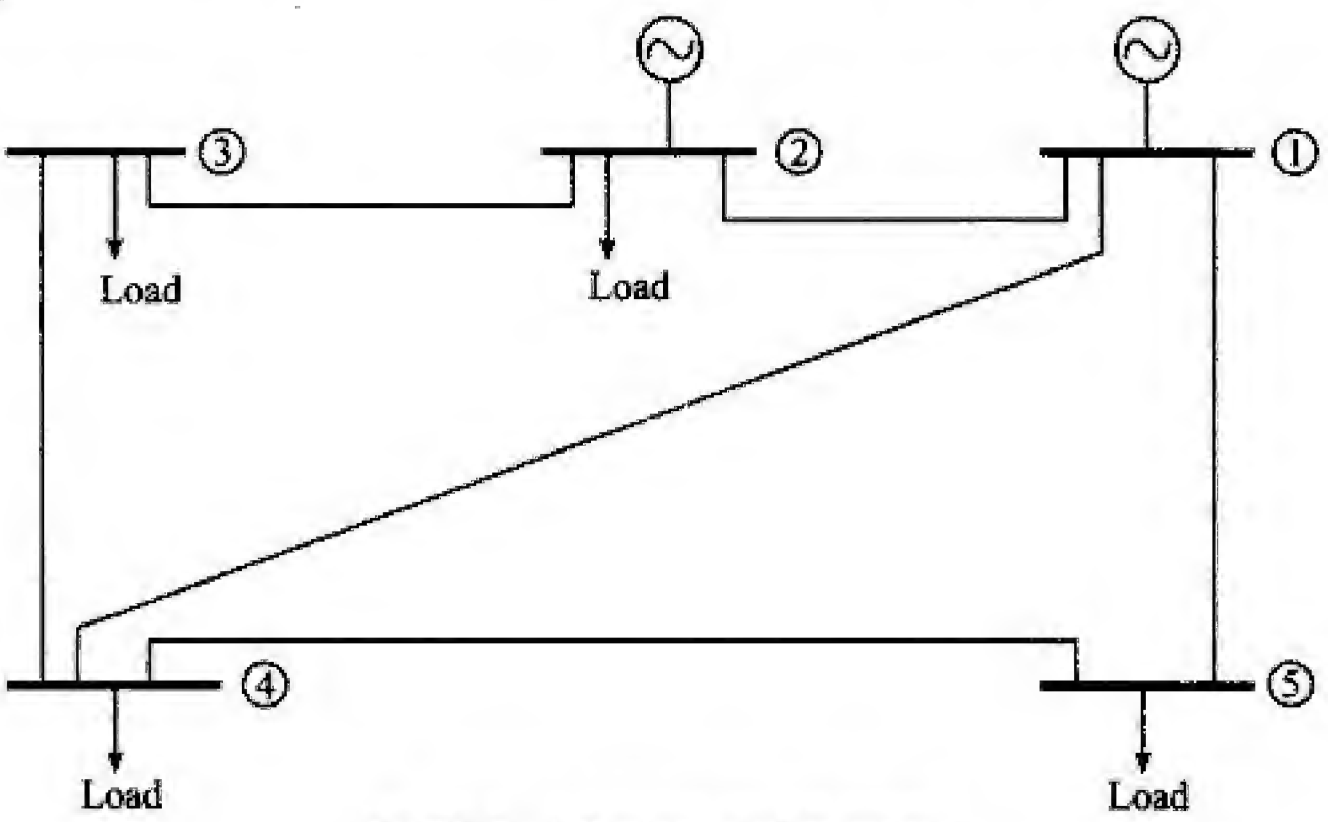


Fig. P4.2 A five-bus, six-line system.

TABLE P4.3

Line no.	*From bus*	*To bus*	*Line impedance (p.u.)*	*Shunt susceptance (B/2) (p.u.)*
1	1	2	$(0.01 + j0.05)$	$j0.02$
2	1	5	$(0.10 + j0.5)$	$j0.025$
3	1	4	$(0.15 + j0.55)$	$j0.025$
4	2	3	$(0.05 + j0.3)$	$j0.02$
5	3	4	$(0.08 + j0.5)$	$j0.02$
6	4	5	$(0.02 + j0.15)$	$j0.01$

TABLE P4.4

Bus	*Generation*		*Load*	
no.	MW	MVAR	MW	MVAR
1	–	–	–	–
2	50	20	15	5
3	0	0	20	10
4	0	0	25	10
5	0	0	30	15

Assume slack bus voltage $=1\angle 0°$ p.u. Perform G-S, N-R and FDLF methods of load flow study and compare the results.

Voltage in POLAR form

Bus no.	Voltage magnitude (p.u.)	Delta (Voltage angle in rad.)
1	1.000000	.000000
2	1.003492	.002711
3	.952690	-.079898
4	.925518	-.121505
5	.918446	-.128885

By N-R Method

FINAL BUS VOLTAGES ARE
Iteration reqd.,k = 3

Bus-code	VOLTAGE (p.u.)	VOLTAGE ANGLE (rad.)
1	1.000000	.000000
2	1.003490	.002689
3	.952696	-.079704
4	.925606	-.121155
5	.918640	-.128602

By FDLF Method

No. of Iterations reqd. k = 3
FINAL BUS VOLTAGES (in p.u.) ARE

Bus-code	VOLTAGE	VOLTAGE ANGLE (rad.)
1	1.000000	.000000
2	1.003936	.002625
3	.953983	-.079864
4	.927719	-.121589
5	.921310	-.129055

4. RESULT OF EXERCISE 4 ELNVOLT4.DAT

When V2:V4 = 1.02:1

Iteration reqd.,k = 3

Bus-code	VOLTAGE (p.u.)	VOLTAGE ANGLE (rad.)
1	1.000000	.000000
2	.997329	-.004191
3	.968431	-.066869
4	.973032	-.073258
5	.957731	-.090680

```
Base MVA = 100

            ************ LINEFLOWS ************

   Bus-code              MW                   MVAR
 ------------       ------------          ------------
    1 - 2             -.022417              -2.004093
    2 - 1              .022437              -1.996346
    1 - 5            -3.713629              -6.203759
    5 - 1             3.819432                .933006
    1 - 4            -3.156833              -5.952059
    4 - 1             3.257828                .784158
    2 - 3            -1.616288              -3.413584
    3 - 2             1.641201               -.598145
    3 - 4             -.213809              -2.203544
    4 - 3              .214354              -1.499838
    4 - 5             -.224699              -1.122858
    5 - 4              .225573               -.699430
    2 - 4            22.130110               8.169057
    4 - 2           -22.130110              -6.499916

   LINE LOSS
   =========

           .233150 MW ,     -22.307350 MVAR

SLACK BUS POWER (IN P.U.)

        (-6.892879E-02,-1.415991E-01)
```

6. RESULT OF EXERCISE 6 EDCVOLT.DAT

```
No. of Iterations reqd. k =  2

FINAL BUS VOLTAGES ARE

   Bus-code        VOLTAGE (p.u.)     VOLTAGE ANGLE (rad.)
 ------------     --------------     ---------------------
      1              1.040000               .000000
      2              1.000000              -.201515
      3              1.000000              -.065728
      4              1.000000              -.170765
      5              1.000000              -.178810
      6              1.000000              -.238048
```

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

5.5 INCREMENTAL FUEL COST (IFC) CURVE

This curve can be obtained from IFR curve by multiplying the 'IFR' by the cost of fuel per Kcal. As in a power station, fuel cost governs the actual total cost. Hence, IFC is very significant in economic loading of the generation unit. The IFC curves will be similar to the IFC characteristic in configuration.

It is obvious that the slopes of the input-output curve and incremental fuel rate curve do not change for different fuels or for changes in the cost of the same fuel. This time a multiplying factor may be used so that the actual cost is a realistic one. The unit of *IFC* (or simply the *IC*) is unit of cost/MWhr.

5.6 CONSTRAINTS IN ECONOMIC OPERATION OF POWER SYSTEM

5.6.1 Primary Constraints

These constraints arise out of the necessity for the system to balance the load demand and generation. They are also called *equality constraints*. If P_i and Q_i are the scheduled electrical generation, P_{load_i} and Q_{load_i} are the respective load demands, it is obvious that the following equations must be satisfied at the load bus (Fig. 5.4),

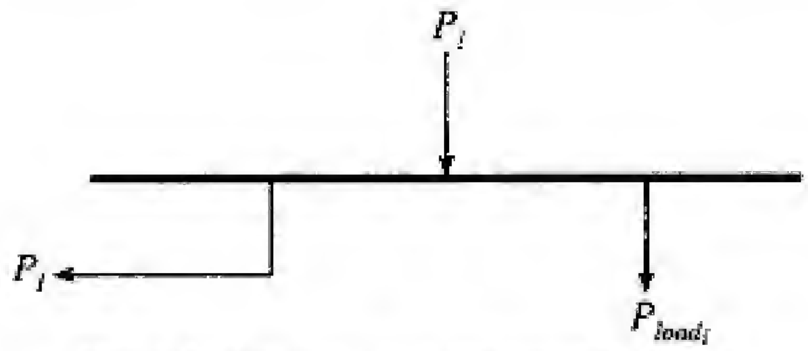


Fig. 5.4 Real power position at load bus.

$$P_i - P_{load_i} - P_l = M_i = 0 \tag{5.1}$$

$$Q_i - Q_{load_i} - Q_l = N_i = 0 \tag{5.2}$$

where M_i and N_i represent the power residuals at bus-i and P_l and Q_l the power flow to the neighbouring system given by

$$P_l = \sum_{j=1}^{N} V_i V_j Y_{ij} \cos\left(\delta_{ij} - \theta_{ij}\right) \tag{5.3}$$

$$Q_l = \sum_{j=1}^{N} V_i V_j Y_{ij} \sin\left(\delta_{ij} - \theta_{ij}\right) \tag{5.4}$$

5.6.2 Secondary Constraints

These constraints arise due to physical and operational limitations of respective units and components and are known as *inequality constraints. Power inequality constraints* are applicable for proper operation; for each generator we should have a minimum and maximum permissible output and the unit production should be constrained to ensure that

$$P_{i_{min}} \le P_i \le P_{i_{max}}, \quad i = 1, 2, ..., N_P$$

and

$$Q_{i_{min}} \le Q_i \le Q_{i_{max}}, \quad i = 1, 2, ..., N_Q$$

N_P and N_Q being the total number of real and reactive sources in the system.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

$$\frac{\partial L}{\partial P_i} = \frac{dF_i(P_i)}{dP_i} - \lambda = 0$$

or
$$\lambda = \frac{dF_i(P_i)}{dP_i} \tag{5.9}$$

The necessary condition for the existence of a *minimum* cost operating condition for the thermal-based energy generating system is thus obtained in equation (5.9) when the *incremental cost rate* $\frac{dF_i(P_i)}{dP_i}$ of all the units is equal to λ, the Lagrangian multiplier. For N number of such units, economic operation is thus attained when

$$\frac{dF_1(P_1)}{dP_1} = \frac{dF_2(P_2)}{dP_2} = \cdots = \frac{dF_i(P_i)}{dP_i} = \cdots = \frac{dF_N(P_N)}{dP_N} = \lambda \tag{5.10}$$

It may be noted that the equations (5.7) and (5.10) exactly suffice for finding $(N + 1)$ unknowns $(P_1, P_2, ..., P_i, ..., P_N$ and $\lambda)$.

5.9 COMPUTER SOLUTION OF THE ECONOMIC OPERATION PROBLEM

The simplest procedure for the economic scheduling of thermal power generating plants, *with losses neglected*, is conventionally the *λ-iteration method*, the algorithm being presented below:

Step 1: An initial estimate of λ^0 is to be assigned.

Step 2: To compute P_i^0 corresponding to the following numerical relation*.

$$P_i^0 = \alpha_i\left(\lambda_i^0\right)^2 + \beta_i\left(\lambda_i^0\right) + \gamma_i$$

(λ_i^0 being identical to the initial estimate of the λ of the i-th generator)

Step 3: To compute $\sum_{i=1}^{N} P_i^0$.

Step 4: To check whether $\sum_{i=1}^{N} P_i^0 = P_{load}$ is satisfied.

[Usually $\sum_{i=1}^{N} P_i - P_{load} = \varepsilon \le 0.001$ (a tolerance)]

Step 5: If $\sum_{i=1}^{N} P_i^0$ becomes less than P_{load}, it is required to assign a new value of λ^1 [$= \lambda^0 + \Delta\lambda$] and go to step 2. Computational loop is continued till $\sum_{i=1}^{N} P_i - P_{load} = \varepsilon$.

*A typical incremental cost rate curve for a thermal generator with varying power output can be expressed in the form of the following polynomial expression $P_i = \gamma_i + \beta_i(IC_i) + \alpha_i(IC_i)^2 + ...$, where α_i, β_i and γ_i are constants for the i-th plant and (IC_i) is the incremental cost of the i-th plant.

Solving equations (1), (2) and (3) with this condition and with the constraint equation

$P_A + P_B + P_C - 1000 = 0$, we obtain $\lambda = 9.345$ unit of cost /MWhr

and $P_A = 498.376$ MW, $P_B = 366.16$ MW, $P_C = 135.464$ MW.

Example 5.6: The input-output characteristic of two steam plants are analytically given as:

$$H_1 = (2.3P_1 + 0.0062P_1^2 + 25) \times 10^6 \text{ Kcal/hr}$$

$$H_2 = (1.5P_2 + 0.01P_2^2 + 35) \times 10^6 \text{ Kcal/hr}$$

Calorific value of coal at plant-1 is 4×10^3 Kcal/Kg, calorific value of coal at plant-2 is 5×10^3 Kcal/Kg while cost of coal at plant-1 is Rs 50 per ton and cost of coal at plant-2 is Rs 65 per ton.

Find the incremental fuel rate, incremental fuel cost and the incremental production cost, if the cost of other items such as water, labour and maintenance can be taken as 10% of the incremental fuel cost at each plant.

Solution: Incremental fuel rate at plant-1 is given by

$$\frac{dF_1}{dP_1} = (2.3 + 2 \times 0.0062P_1) \times 10^6 \text{ Kcal/MWhr}$$

Incremental fuel rate at plant-2 is given by

$$\frac{dF_2}{dP_2} = (1.5 + 2 \times 0.01P_2) \times 10^6 \text{ Kcal/MWhr}$$

Cost of fuel at plant-1 is given by

$$F_1 = \frac{50}{1000} \times \frac{1}{4000} = 13.75 \times 10^{-6} \text{ Rs/Kcal}$$

Cost of fuel at plant-2 is given by

$$F_2 = \frac{65}{1000} \times \frac{1}{5000} = 13 \times 10^{-6} \text{ Rs/Kcal}$$

Incremental fuel cost at plant-1 is then given by

$$(2.3 + 0.0124P_1) \times 10^6 \times 13.75 \times 10^{-6} = (31.625 + 0.1705P_1) \text{ Rs/MWhr}$$

Incremental fuel cost at plant-2 is then given by

$$(1.5 + 0.02P_2) \times 10^6 \times 13 \times 10^{-6} = (19.5 + 0.26P_2) \text{ Rs/MWhr}$$

Incremental production cost at plant-1 is

$$1.1(31.625 + 0.1705P_1) = (34.7875 + 0.18755P_1) \text{ Rs/MWhr}$$

Incremental production cost at plant-2 is

$$1.1(19.5 + 0.26P_2) = (21.45 + 0.286P_2) \text{ Rs/MWhr.}$$

(It may be noted that the cost given here is an indicative price only.)

Example 5.7: Determine the economic operation point for the three thermal units delivering a total load of 600 MW without considering generator limit as well as with considering generator limit. Data given are:

Output of computer program LLESS.FOR after execution (without considering prescribed generation limit): SOL11.DAT

```
FINAL ECONOMIC GENERATOR SCHEDULE
=================================

lambda =        8.732112

Net gen (MW) =  600.000100  Net load (MW) =  600.000000

Power tolerance =    .000122

               Bus no            Economic generation(MW)
               ======            =======================
                 1                     312.761100
                 2                     213.027800
                 3                      74.211170

Incremental Fuel Cost
=====================

            Generator no.          IFC

                 1               8.732112
                 2               8.732111
                 3               8.732111
```

Output of computer program LLESS.FOR after execution (with considering prescribed generation limit): SOL12.DAT

```
 **** PRESCRIBED GENERATION LIMIT IS CROSSED
BY GENERATOR NO. 3
```

[Generator-3 has crossed its minimum generation limit, hence its generation is kept fixed at its minimum value 75 MW.]

```
FINAL ECONOMIC GENERATOR SCHEDULE
=================================

lambda =        8.730685

Net gen (MW) =   600.000200  Net load (MW) =  600.000000
Power tolerance =      .000244
```

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Then $\lambda = AP + B = 0.003 \times 200 + 5.6 = 6.2$ unit of cost/MWhr,

$$P_1 = \frac{\lambda - \beta_1}{\alpha_1} = \frac{6.2 - 6}{0.005} = 40 \text{ MW}$$

and
$$P_2 = \frac{\lambda - \beta_2}{\alpha_2} = \frac{6.2 - 5}{0.0075} = 160 \text{ MW}$$

For a loading of 300 MW, $\lambda = AP + B = 6.5$ (when $P = 300$ MW);

$$\therefore \quad P_1 = \frac{\lambda - \beta_1}{\alpha_1} = 100 \text{ MW and } P_2 = \frac{\lambda - \beta_2}{\alpha_2} = 200 \text{ MW.}$$

Exactly in a similar way for a system load of 500 MW, $A = 0.003$, $B = 5.6$ and $\lambda = AP + B = 7.1$ [$\because P = 500$ MW]

$$\therefore \quad P_1 = \frac{\lambda - \beta_1}{\alpha_1} = 220 \text{ MW and } P_2 = \frac{\lambda - \beta_2}{\alpha_2} = 280 \text{ MW}$$

The reader is encouraged to observe Table 5.1.

Table 5.1: Plant scheduling

Operation no.	*Total load* (MW)	P_1 (MW)	P_2 (MW)	λ_1	λ_2
1	100	100	0	6.5	0
2	100	0	100	0	5.75
3	200	100	100	6.5	5.75
4	200	0	200	0	6.5
5	200	40	160	6.2	6.2
6	300	100	200	6.5	6.5
7	500	220	280	7.1	7.1

It may be observed that for single generator operation (at light loads) and up to 200 MW, it is economical to load unit-2 only [operation nos. 2 and 4 in the Table]. For load 200 MW and beyond, both the generators may be loaded to obtain economic operation [operation nos. 5 to 7 in the Table].

5.11 ECONOMIC ALLOCATION OF GENERATION BETWEEN DIFFERENT PLANTS IN A SYSTEM CONSIDERING SYSTEM TRANSMISSION LOSS (ECONOMIC DISPATCH)

Economic allocation of generation between different generating units in a plant has been considered previously. When considering the economic allocation of generation between different plants in an integrated system, the transmission losses are to be considered. This leads to the dispatch of power in an economical way so as to make the overall cost to be minimum. Let there be 'N' plants in a system interconnected by transmission lines and ties (Fig. 5.8).

Let $P_1, P_2, \ldots, P_N$ refer to the generation of the N plants, respectively, in MW. Let the total load be P_{load} (constant) and loss in the lines be P_l. The constraint equation is given by

$\therefore$ For generator-1, assuming it to be a slack bus,

$$(PF)_1 = \frac{1}{1-\frac{\partial P_l}{\partial P_{g_1}}} = \frac{1}{1-0} = 1$$

and
$$(PF)_2 = \frac{1}{1-\frac{\partial P_l}{\partial P_{g_B}}} = \frac{1}{1-0.0016P_{g_B}+0.16} = \frac{1}{1.16-0.0016P_{g_B}}$$

For optimal dispatch,

$$(PF)_1 \frac{dF_1}{dP_{g_1}} = (PF)_2 \frac{dF_2}{dP_{g_2}} = \lambda$$

i.e.
$$0.006P_{g_A} + 4 = \lambda \tag{1}$$

and
$$\frac{1}{1.16-0.0016P_{g_B}}\left(0.007P_{g_B}+4\right) = \lambda \tag{2}$$

also
$$P_{g_A} + P_{g_B} = P_l + P_{\text{load}}$$

or
$$P_{g_A} + P_{g_B} = 0.0008\left(P_{g_B}-100\right)^2 + 500 \tag{3}$$

Solving for (1), (2) and (3), we get

$$P_{g_A} = 227.7 \text{ MW},\ P_{g_B} = 117.65,\ \lambda = 5.694 \text{ and } P_l = 4.58 \text{ MW}$$

Example 5.11: Find the incremental transmission losses for a two-area power system, where the bus voltages are kept fixed and the line power flow is a function of line angle. Power loss is a function of generation of area B only.

Solution: It is evident that

$$P_l = f\,(P_{g_B})$$

This also suggests that the incremental transmission loss for grid A will be zero and the incremental transmission loss of the line will be governed by the grid B only. Thus,

$$(ITL)_A = 0$$

$$(ITL)_B = \frac{\partial P_l}{\partial P_{g_B}} = \frac{\partial P_l}{\partial \delta} \times \frac{\partial \delta}{\partial P_{g_B}}$$

Economic operation being dictated by the criterion

$$\lambda = \frac{(IFC)_A}{1-(ITL)_A} = \frac{(IFC)_B}{1-(ITL)_B}$$

In this case, for economic operation,

$$\lambda = \frac{(IFC)_A}{1-0} = \frac{(IFC)_B}{1-(ITL)_B}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

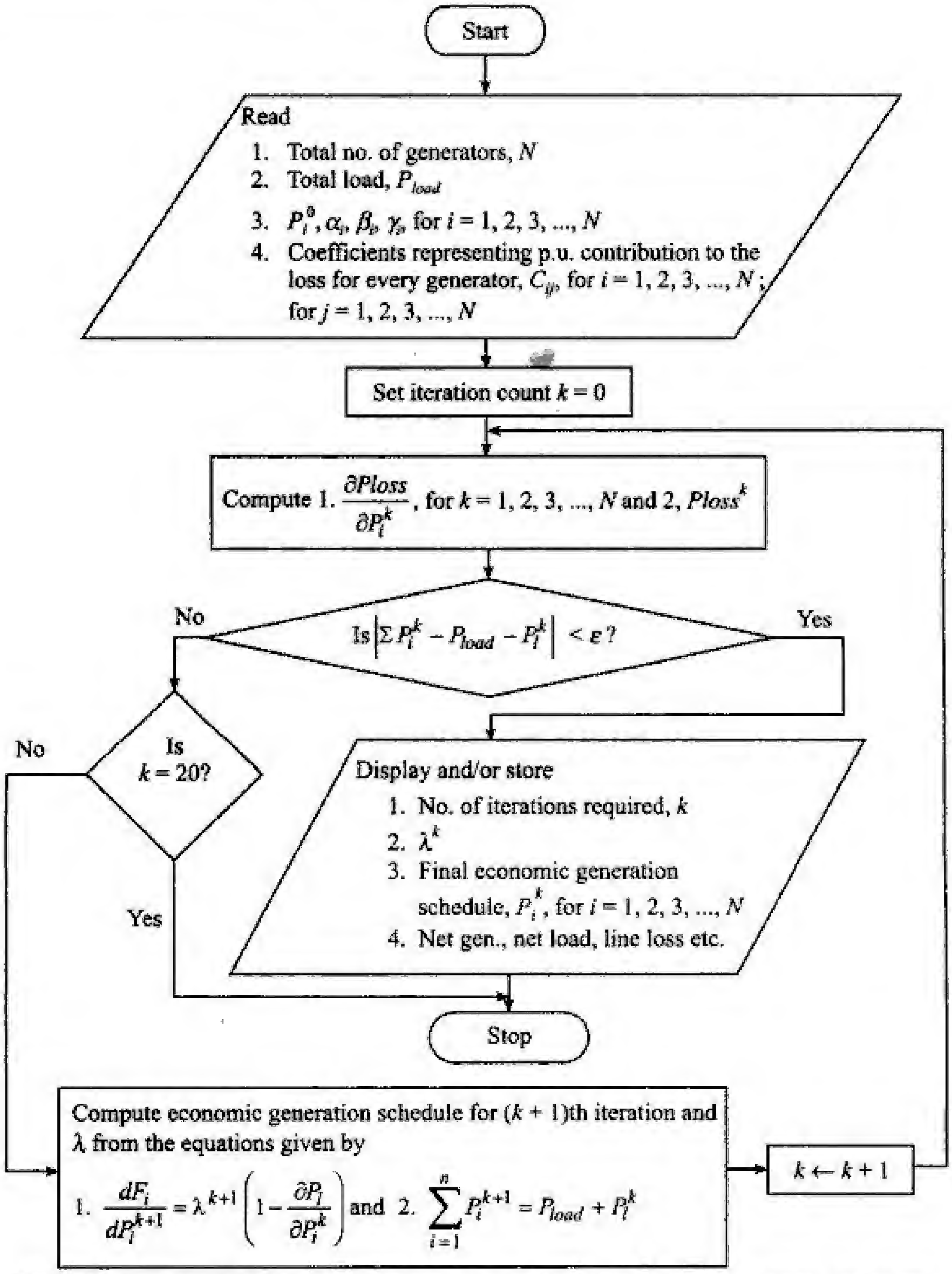


Fig. 5.9 Flowchart to find economic generation schedule considering transmission line losses.

Example 5.14: In a three-plant system the cost functions are given by

$$F_1(P_1) = 400 + 6.8P_1 + 0.002P_1^2$$

$$F_2(P_2) = 300 + 6.7P_2 + 0.003P_2^2$$

$$\begin{bmatrix} V_{1n} \\ V_{2n} \\ V_{3n} \\ V_{4n} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

Expanding for the first row,

$$V_{1n} = Z_{11}I_1 + Z_{12}I_2 + Z_{13}I_3 + Z_{14}I_4 \tag{5.26}$$

Substituting for I_3 and I_4 from equation (5.24) we can write,

$$I_D = \frac{-Z_{11}}{K_3Z_{13} + K_4Z_{14}} I_1 + \frac{-Z_{12}}{K_3Z_{13} + K_4Z_{14}} I_2 + \frac{-Z_{11}}{K_3Z_{13} + K_4Z_{14}} I_n^0 \tag{5.27}$$

where $\quad I_n^0 = -\dfrac{V_{1n}}{Z_{11}}$

We next write equation (5.27) as

$$I_D = -m_1I_1 - m_2I_2 - m_1I_n^0 \tag{5.27a}$$

where
$$\left.\begin{aligned} m_1 &= \frac{-Z_{11}}{K_3Z_{13} + K_4Z_{14}} \\ m_2 &= \frac{-Z_{12}}{K_3Z_{13} + K_4Z_{14}} \end{aligned}\right\} \tag{5.28}$$

Substitution of (5.27(a)) in (5.24) yields

$$I_3 = -K_3m_1I_1 - K_3m_2I_2 - K_3m_1I_n^0 \tag{5.29a}$$

and
$$I_4 = -K_4m_1I_1 - K_4m_2I_2 - K_4m_1I_n^0 \tag{5.29b}$$

Let us now relate current I_1, I_2 and I_n^0 with the bus current through the *connection matrix* (C) utilising equations (5.29a) and (5.29b) when I_1 and I_2 remain *invariant*.

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 1 & & \\ & 1 & \\ -K_3m_1 & -K_3m_2 & -K_3m_1 \\ -K_4m_1 & -K_4m_2 & -K_4m_1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_n^0 \end{bmatrix} = C \begin{bmatrix} I_1 \\ I_2 \\ I_n^0 \end{bmatrix} \tag{5.30}$$

However, we know from the concepts of *power invariant transformation* that total power loss S_l, in terms of transformed (new) current, can be expressed as

$$S_l = I_{new}^T Z_{Bus(new)} I_{new}^* \tag{5.31}$$

But,
$$Z_{Bus(new)} = C^T Z_{Bus} C^* \tag{5.32}$$

where, $Z_{Bus} = (R_{Bus} + jX_{Bus})$. Thus, equation (5.31) can be written for real load power as

$$P_l = I_{new}^T C^T R_{Bus} C^* I_{new}^* \tag{5.33}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

which, with loss formula yields

$$\frac{\partial L}{\partial P_i} = \frac{dF_i(P_i)}{dP_i} - \lambda\left(1 - 2\sum_{j=1}^{N} B_{ij}P_j - B_{i0}\right) \tag{5.40}$$

Equation (5.40) can be used to find the economic operation schedule of the thermal generators in a system as shown in the following algorithm.

5.14.1 Algorithm for Determination of Optimal Generation Using Loss Formula

Method 1

Step 1: Assume starting values of P_i ($i = 1, 2, ..., N$) [Given: Total Load (P_{load}); [B] matrix]

Step 2: Assume λ^0

Step 3: Compute P_l using B-matrix formula [optional: Calculate P_{demand} (= $P_{load} + P_l$)]

Step 4: Find bus penalty factor PF_i, for $i = 1, 2, \ldots, N$ $\left[PF_i = \dfrac{1}{1 - 2\sum_{j=1}^{N} B_{ij}P_j - B_{i0}}\right]$.

Step 5: Solve for P_i such that $PF_i \times \dfrac{dF_i(P_i)}{dP_i} = \lambda$ for $i = 1, 2, ..., N$.

$$\left[\because\ PF_i\left(\alpha_i P_i + \beta_i\right) = \lambda, \quad \therefore\ \alpha_i P_i + \beta = \frac{\lambda}{PF_i}, \text{ i.e. } P_i = \left(\frac{\lambda}{PF_i} - \beta_i\right)\Big/\alpha_i\right]$$

Step 6: Check whether $\left|\sum P_i - P_{demand}\right| < \varepsilon_1$ (ε_1 being the total demand tolerance). If yes, go to step 8. Otherwise go to step 7.

Step 7: Adjust λ such that $\lambda^1 = \lambda^0 \pm \Delta\lambda$ and go to step 5.

Step 8: Compare P_i^{k-1} to P_i^k (P_i of the last iteration) and store max $|P_i^{k-1} - P_i^k|$.

Here (k) is the iteration count.

Step 9: Check whether Max. $|P_i^{k-1} - P_i^k| < \varepsilon_2$ (ε_2 being the solution convergence tolerance). If yes, go to step 10. Otherwise go to step 3.

Step 10: Stop.

Method 2

During economic operation of a power system when transmission losses are considered, the incremental cost of received power λ, in unit of cost/MWhr is given by

$$\lambda = \left(\frac{1}{1 - \dfrac{\partial P_l}{\partial P_i}}\right)\frac{dF_i(P_i)}{dP_i} \quad \text{[equation (5.19), rewritten]}$$

This equation can also be expressed as,

$$\frac{dF_i(P_i)}{dP_i} - \lambda + \lambda\frac{\partial P_l}{\partial P_i} = 0 \tag{5.41}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

[for $\tau = 1, 2, ..., \tau_{max}$, ϕ and ψ are constraint functions]. It may be noted in this context that the term $\left[\sum_{\tau=1}^{\tau_{max}} n_\tau F_{m\tau}\right]$ is a constant term as the total quantity of fuel used by the mth plant is fixed as per the contract. Hence this $\left[\sum_{\tau=1}^{\tau_{max}} n_\tau F_{m\tau}\right]$ term can be ignored from the scheduling problem objective function.

Following Lagrangian mathematics, the objective function is to be added to the constraint functions after the constraint functions are multiplied by undetermined multipliers λ_τ and γ_f in order to get the necessary condition for optimum value of the objective function

$$L = \sum_{\tau=1}^{\tau_{max}} n_\tau \sum_{\substack{i=1\\ i\neq m}}^{N} F_{i\tau} + \sum_{\tau=1}^{\tau_{max}} \lambda_\tau \left(P_{d\tau} - \sum_{\substack{i=1\\ i\neq m}}^{N} P_{i\tau} - P_{m\tau} \right) + \gamma_f \left(\sum_{\tau=1}^{\tau_{max}} n_\tau p_{m\tau} - p_{total} \right)$$

It may be observed that assuming the load demand to be constant at the specified interval of time, $P_{d\tau}$ is constant. The independent variables being only $P_{i\tau}$ and $P_{m\tau}$, for any given period of time $\tau = K$, the objective function is optimised when the first derivative of the Lagrangian function with respect to each of the independent variables is set to zero.

i.e. when
$$\frac{\partial L}{\partial P_{ik}} = 0$$

or
$$n_k \frac{dF_{ik}(P_{ik})}{dP_{ik}} - \lambda_k = 0 \text{ for } i = 1, 2, ..., N;\ i \neq m$$

and
$$\frac{\partial L}{\partial P_{mk}} = 0$$

or
$$-\lambda_k + \gamma_f n_k \frac{dp_{mk}}{dP_{mk}} = 0$$

Thus, for a limited resource fuel supply system having contract of 'take or pay' for the stipulated amount of fuel, the economic operation of the units of the utility is governed by the following two criteria:

$$n_k \frac{dF_{ik}(P_{ik})}{dP_{ik}} - \lambda_k = 0, \quad i = 1, 2, ..., N\ ;\ i \neq m \tag{5.52}$$

$$-\lambda_k + \gamma_f n_k \frac{dp_{mk}}{dP_{mk}} = 0 \tag{5.53}$$

It may be observed that γ_f is constant and has dimensions of Rs/ton. Besides this, we have also ignored the unit's minimum or maximum limits in the scheduling process.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

5.25 SCHEDULING OF HYDRAULICALLY COUPLED UNITS (HYDRO-UNITS IN SERIES)

Let us consider m number of hydro plants in series (Fig. 5.17) and they are being operated with a thermal system. Hydro-thermal coordination is now required to be done for economic operation of such a system.

Let, p_i = water inflow,

v_i = volume of reservoir,

s_i = water spillage rate,

q_i = hydro plant water discharge,

n_i = number of hours in scheduling interval τ,

P_{h_i} = hydro power output,

P_{th} = thermal power output,

P_{load} = load demand,

and $F(P_{th_r})$ = thermal plant fuel cost.

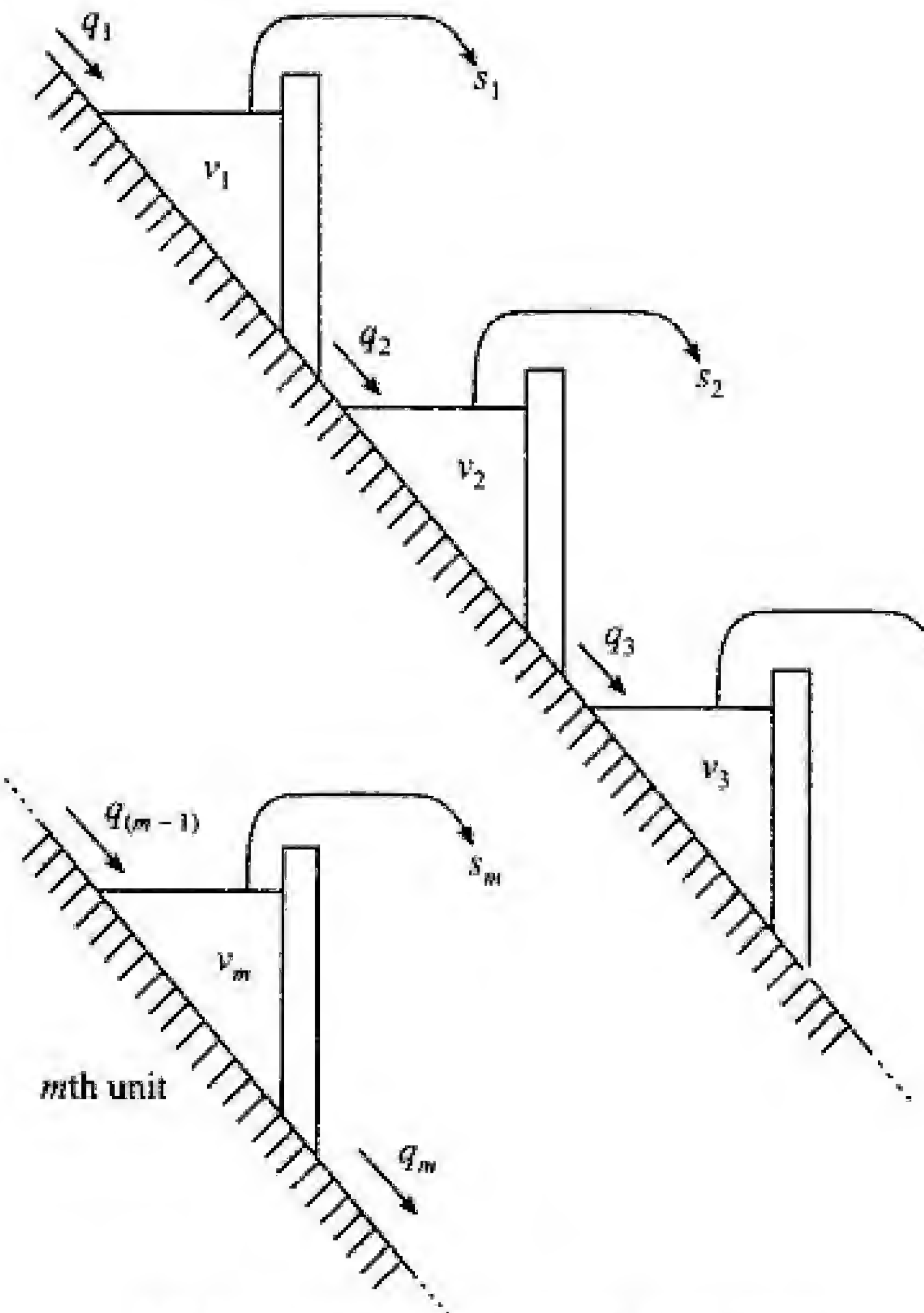


Fig. 5.17 Hydraulically coupled hydel plants.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

The total Lagrangian function L is given by for the entire period of scheduling,

$$L = L_{t_1} + L_{t_2}$$

If reservoir constraints are added, the initial and final volumes being given by

$$v_{in} = v_0 \quad \text{and} \quad v_{fin} = v_{24}$$

the constraint equation can be written as

$$(v_{in} - v_0) \text{ and } (v_{fin} - v_{24}) = 0$$

Finally, the system Lagrangian (L) is then given by

$$L = L_{t_1} + L_{t_2} + [\varepsilon_0 (v_{in} - v_0) + \varepsilon_{24}(v_{fin} - v_{24})] \tag{5.100}$$

Here $[\varepsilon_0 (v_{in} - v_0) + \varepsilon_{24}(v_{fin} - v_{24})]$ are called *end point storage constraints* and may be neglected for eliminating complexity in scheduling problem.

For economisation with volume parameter for the i-th interval in the day ($i \neq$ 0-th or 24-th hour), we can write

$$\frac{\partial L}{\partial v_i} = 0 = \gamma_i - \gamma_{i+1}$$

For $i = 0$,

$$\frac{\partial L}{\partial v_0} = 0 - \gamma_1 + \varepsilon_0$$

while for $i = 24$,

$$\frac{\partial L}{\partial v_{24}} = 0 = \gamma_{24} + \varepsilon_{24}$$

Computer programs using λ–γ iteration can thus be developed in the conventional way and is left for the reader as an exercise. The following assumptions can be stated which require attention while developing the computer program:

(*a*) hydro reservoir has constant head during operation,
(*b*) the thermal plant represents equivalent unit for all steam plants in the system,
(*c*) operating schedule is for 24 hrs while each time interval is for one hour,
(*d*) during any interval of time either the plant is generating or pumping [the plant may also be taken as idle keeping $P_{h(t_1)} = P_{h(t_2)} = 0$],
(*e*) beginning and ending storage volumes are specified,
(*f*) pump and generator ratings are identical,
(*g*) pumping can be done continuously, and
(*h*) cycle efficiency remains constant.

5.27 SHORT-TERM FIXED HEAD HYDRO-THERMAL SCHEDULING CONSIDERING TRANSMISSION LINE LOSS AND INVOLVING MULTIPLE THERMAL AND HYDRO GENERATORS (CLASSICAL METHOD)

Let us consider a power system containing N number of thermal units and M number of hydro units supplying total load P_{load_τ} for τ-th subinterval. The fuel cost curve of each thermal generator is given by

$$F_{c\tau_i} = \alpha_{th_i} (P_{th\tau_i})^2 + \beta_{th_i} P_{th\tau_i} + \gamma_{th_i} \text{ unit of cost/hr} \tag{5.101}$$

for $i = 1, 2, 3, \ldots, N$

in next iteration the iterative process converges for the load of 350 MW and we get the final results as given below.

$$P_{11}^{(1)} = 177.513557 \text{ MW}, P_{21}^{(1)} = 175.240266 \text{ MW and } \lambda_1^{(1)} = 10.881984$$

$$\begin{aligned} P_{\ell_1}^{(1)} &= 4.5 \times 10^{-5} \times 177.513557^2 + 2(-0.05) \times 10^{-5} \times 177.513557 \times 175.240266 \\ &\quad + 4.45 \times 10^{-5} \times 175.240266^2 + 0.23 \times 10^{-5} \times 177.513557 \\ &\quad + 0.15 \times 10^{-5} \times 175.240266 + 0.094 \times 10^{-5} \\ &= 2.754120 \text{ MW} \end{aligned}$$

The power tolerance,

$$Ptol = \sum_{i=1}^{2} P_{i1} - (P_{load\,1} + P_{\ell 1})$$

$$= 177.513557 + 175.240266 - 350 - 2.754120 = -0.000297 \cong -0.0003$$

Cost of thermal generation for $\tau = 1$, is obtained from

$$F_c = (0.002P_{th}^2 + 10P_{th} + 1000)$$

$$\therefore \quad F_{c1_1} = 0.002(177.513557)^2 + 10 \times 177.513557 + 1000 = 2838.158 \text{ unit of cost.}$$

Water release for hydel power generation can be obtained from

$$W = (0.2P_h^2 + 2.5P_h + 1.5 \times 10^4)$$

$$\therefore \quad W_{1_1} = 0.2(175.240266)^2 + 2.5 \times 175.240266 + 1.5 \times 10^4$$

$$= 0.215799 \times 10^5 \text{ m}^3/\text{hr.}$$

Consider scheduling interval $\tau = 2$.

From equation (5.117a), initial generations are,

$$P_{12}^{(0)} (\text{i.e. } P_{th\,2_1}^{(0)}) = P_{12}^{(0)} (\text{i.e. } P_{h\,21}^{0}) = \frac{P_{load\,2}}{N+M} = \frac{700}{2} = 350 \text{ MW}$$

From equation (5.117b),

$$\lambda_2^0 = 2\alpha_{th_1} P_{th\,2_1}^{(0)} + \beta_{th_1} = 2 \times 0.002 \times 350 + 10 = 11.4$$

γ-iteration count being same,

$$\therefore \quad \gamma_1^0 = 0.147586$$

Following the theory developed in the text, the matrix equation to be solved is as follows.

$$\begin{bmatrix} \dfrac{\partial^2 L}{\partial P_{th\,2_1}^2} & \dfrac{\partial^2 L}{\partial P_{th\,2_1} \partial P_{h\,2_1}} & \dfrac{\partial^2 L}{\partial P_{th\,2_1} \partial \lambda_2} \\ \dfrac{\partial^2 L}{\partial P_{h\,2_1} \partial P_{th\,2_1}} & \dfrac{\partial^2 L}{\partial P_{h\,2_1}^2} & \dfrac{\partial^2 L}{\partial P_{h\,2_1} \partial \lambda_2} \\ \dfrac{\partial^2 L}{\partial \lambda_2 \partial P_{th\,2_1}} & \dfrac{\partial^2 L}{\partial \lambda_2 \partial P_{h\,2_1}} & \dfrac{\partial^2 L}{\partial \lambda_2^2} \end{bmatrix} \begin{bmatrix} \Delta P_{th\,2_1} \\ \Delta P_{h\,2_1} \\ \Delta \lambda_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{\partial L}{\partial P_{th\,2_1}} \\ -\dfrac{\partial L}{\partial P_{h\,2_1}} \\ -\dfrac{\partial L}{\partial \lambda_2} \end{bmatrix}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

```
   250.000000       1.780437       11.506920        -.000011

Iteration completed at k=  3   , for Load=  550.000000
=======================================================

Tolerance =    .000043

Final Economic Generation Schedule for tow=   2  &  kg= 16

   i        P        Cost(unit of cost/hr)    Gama     Water release

   1    387.545400       4669.801000
   2    172.491300                           .748692    .199235E+05

       Pload           Ploss          Lambda           Ptol

    550.000000      10.036660      13.390410        -.000043

Iteration completed at k=  3   , for Load=  625.000000
=======================================================

Tolerance =    .000031

Final Economic Generation Schedule for tow=   3  &  kg= 16

   i         P        Cost(unit of cost/hr)    Gama     Water release

   1     450.818800       5493.310000
   2     187.438400                           .748692    .201895E+05

      Pload           Ploss           Lambda           Ptol

   625.000000      13.257220      13.882820        -.000018

Iteration completed at k=  3   , for Load=  340.000000
=======================================================

Tolerance =    .000029

Final Economic Generation Schedule for tow=   4  &  kg= 16

   i        P        Cost(unit of cost/hr)   Gama     Water release

   1    211.806500      2508.554000
   2    131.688100                         .748692    .192427E+05
```

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Chapter 6

Computer-Aided Economic Load Dispatch and Optimal Power Flow

6.1 INTRODUCTION

The concept of *optimal power flow* (OPF) was first introduced in the early 1960s. However, it took nearly twenty years for the concept to become an effective algorithm to be applied in power system operation and planning studies.

The previous chapter dealt with the concept of economic load dispatch. In this chapter, attempts have been made to develop the digital method for optimising the economic load dispatch problem, using more rigorous mathematical models. Higher order differentials have been used in optimisation solutions, both for economic dispatch and optimal power flow.

In economic load dispatch problems, the principal constraint is that the total generation is equal to total load plus total loss in lines. In optimal power flow, the economic load dispatch schedule is calculated in terms of generation costs while the entire set of equations needed for the power flow itself acts as constraints.

In Chapter 4, we have introduced the power flow equations and in the following sections of OPF, these power flow equations would act as constraints while we will formulate the OPF problem. We can solve the optimal power flow problem for minimum generation cost and require that the optimisation calculation also balances the entire power flow simultaneously. Though the objective function can take different forms other than minimising the generation cost, in this text attempts have been made to minimise the generation cost (the objective function in this chapter) subjected to the presence of entire set of power constraints.

Optimal scheduling of electric power system is a major activity, and it is a large-scale problem when the constraints of the electrical network are taken into account. An optimal power flow (OPF) problem schedules the power system controls to optimize an objective function while satisfying a set of non-linear equality and inequality constraints. The equality constraints are the conventional power flow equations, while the inequality constraints are limit or controls and operating variables of the system. Mathematically, OPF is formulated as a constrained nonlinear optimization procedure. Basically, it is a static constrained nonlinear optimization problem whose development is made with the advances in numerical optimization techniques and computer software.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

The second order derivatives of equation (6.9b) with respect to l are obtained as:

$$\frac{\partial^2 L}{\partial \lambda^2} = 0 \tag{6.10d}$$

Economic load dispatch problem may be solved by solving equation (6.8a or 6.8b) using equations (6.9a–6.10d). The flowchart, in order to find economic generation schedule using Newton–Raphson method, is shown in Fig. 6.1.

Example 6.1: The fuel cost characteristics of the two generators of a system are as under:

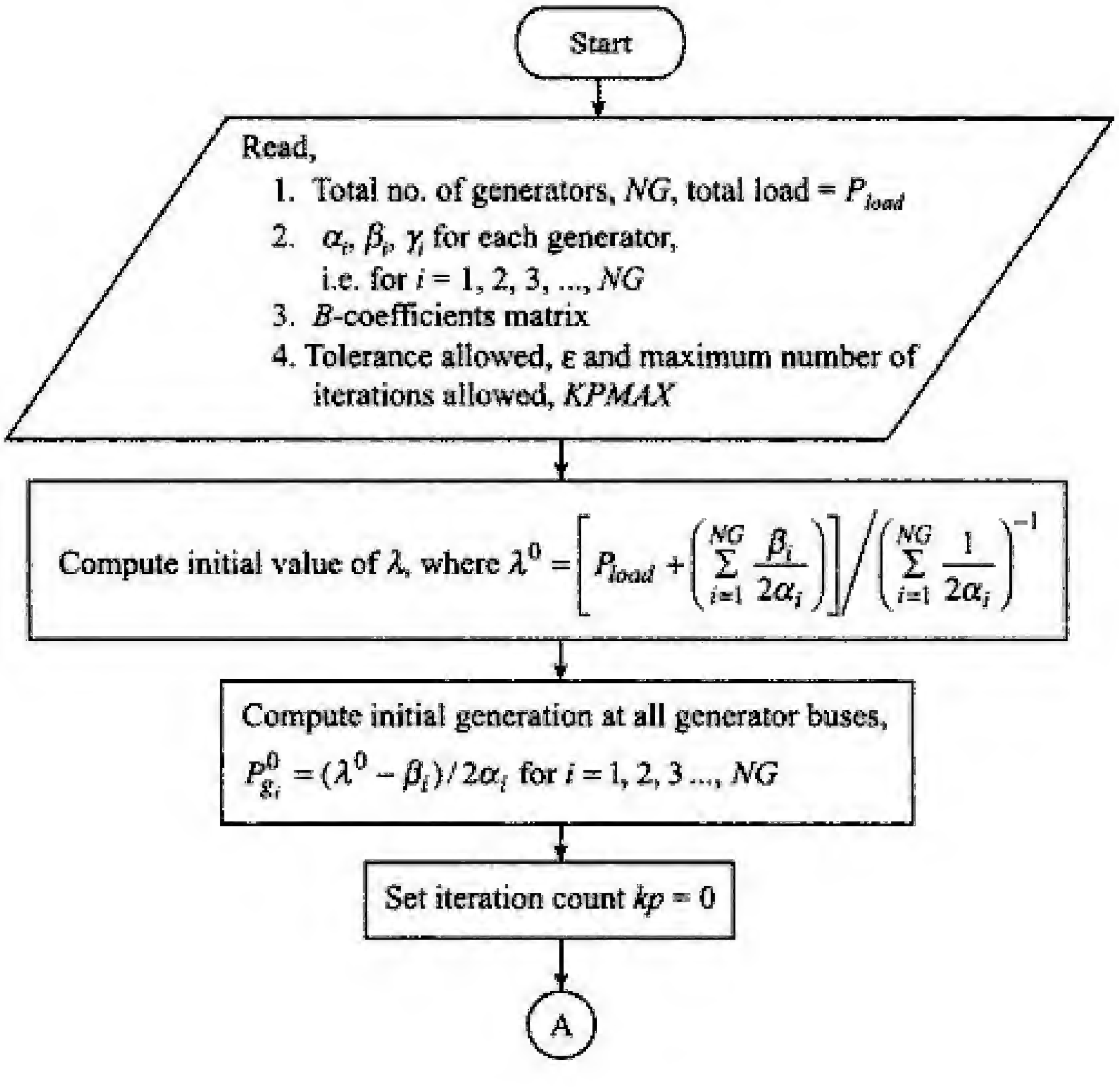
$$F_{c_1}(P_{g_1}) = 20P_{g_1}^2 + 175P_{g_1} + 50 \text{ unit of cost/hr}$$

$$F_{c_2}(P_{g_2}) = 30P_{g_2}^2 + 180P_{g_2} + 40 \text{ unit of cost/hr}$$

where P_{g_1} and P_{g_2} are in p.u. on 100 MVA base.

Determine economic generation schedule to supply a load of 0.55 p.u. using Newton-Raphson method. The $[B]$ matrix of the system is given as:

$$[\mathrm{B}] = \begin{bmatrix} 0.007 & 0.0002 & 0.0006 \\ 0.0002 & 0.006 & 0.0005 \\ 0.0006 & 0.0005 & 0.0000042 \end{bmatrix} \text{ p.u.}$$



$$\frac{\partial^2 L}{\partial P_{g_2}^0 \partial \lambda^0} = \frac{\partial^2 L}{\partial \lambda^0 \partial P_{g_2}^0} = B_{20} + \sum_{k=1}^{2} 2B_{2k} P_{g_k}^0 - 1$$

$$= 0.001 + 2\times 0.0002\times 0.38 + 2\times 0.006\times 0.17 - 1 = -0.996808$$

Second order partial derivatives of L with respect to λ^0:

From equation (6.10d), we find $\dfrac{\partial^2 L}{\partial (\lambda^0)^2} = 0$

The matrix equation to be solved for this problem is given below:

$$\begin{bmatrix} \dfrac{\partial^2 L}{\partial P_{g_1}^2} & \dfrac{\partial^2 L}{\partial P_{g_1} \partial P_{g_2}} & \dfrac{\partial^2 L}{\partial P_{g_1} \partial \lambda} \\ \dfrac{\partial^2 L}{\partial P_{g_2} \partial P_{g_1}} & \dfrac{\partial^2 L}{\partial P_{g_2}^2} & \dfrac{\partial^2 L}{\partial P_{g_2} \partial \lambda} \\ \dfrac{\partial^2 L}{\partial \lambda \partial P_{g_1}} & \dfrac{\partial^2 L}{\partial \lambda \partial P_{g_2}} & \dfrac{\partial^2 L}{\partial \lambda^2} \end{bmatrix} \begin{bmatrix} \Delta P_{g_1} \\ \Delta P_{g_2} \\ \Delta \lambda \end{bmatrix} = \begin{bmatrix} -\dfrac{\partial L}{\partial P_{g_1}} \\ -\dfrac{\partial L}{\partial P_{g_2}} \\ -\dfrac{\partial L}{\partial \lambda} \end{bmatrix}$$

or,
$$\begin{bmatrix} 42.6628 & 0.07608 & -0.993412 \\ 0.07608 & 60.2824 & -0.996808 \\ -0.993412 & -0.996808 & 0 \end{bmatrix} \begin{bmatrix} \Delta P_{g_1} \\ \Delta P_{g_2} \\ \Delta \lambda \end{bmatrix} = \begin{bmatrix} -1.253038 \\ -0.607118 \\ -0.001840 \end{bmatrix}$$

By solving the above equation by Gauss elimination method, we obtain

$$\Delta P_{g_1}^0 = -0.005112,\ \Delta P_{g_2}^0 = 0.006941 \text{ and } \Delta\lambda^0 = 1.0423$$

Therefore, updated values of control variables for next iteration (kp = 1) are:

$$P_{g_1}^1 = P_{g_1}^0 + \Delta P_{g_1}^0 = 0.38 - 0.005112 = 0.374888 \text{ p.u.}$$

$$P_{g_2}^1 = P_{g_2}^0 + \Delta P_{g_2}^0 = 0.17 + 0.006941 = 0.176941 \text{ p.u.}$$

$$\lambda^1 = \lambda^0 + \Delta\lambda^0 = 190.2 + 1.042344 = 191.2423 \text{ unit of cost/p.u. power} \cdot \text{hr.}$$

Tolerance in this iteration (i.e. $kp = 0$) is obtained as follows:

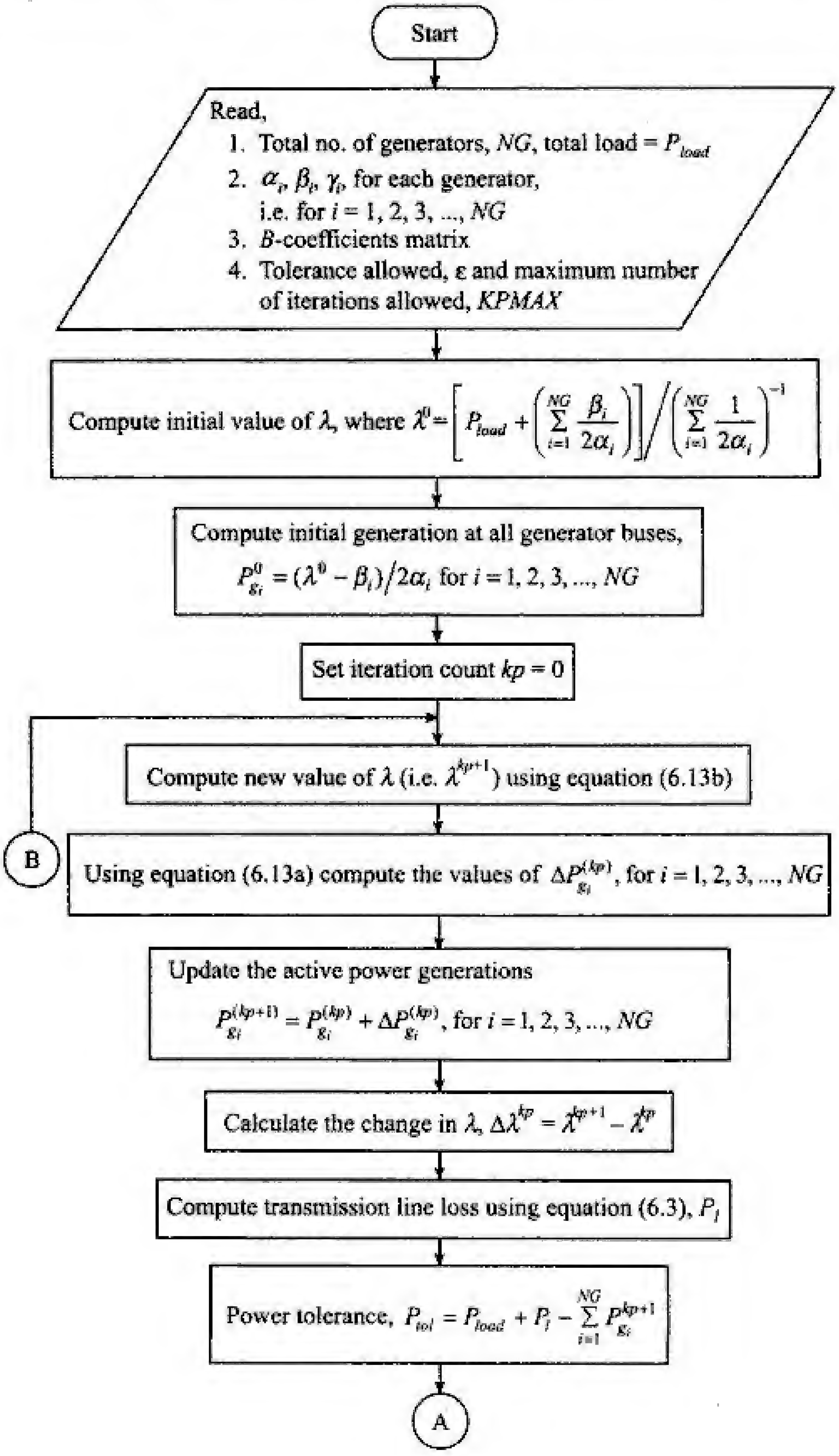
$$tol = \left[\sum_{i=1}^{2}\left(\Delta P_{g_i}^0\right)^2 + \left(\Delta\lambda^0\right)^2\right]^{\frac{1}{2}} = \left[\sum_{i=1}^{2}\left(\Delta P_{g_i}^0\right)^2 + \left(\lambda^0\right)^2\right]^{\frac{1}{2}}$$

$$= 1.042313$$

We find tolerance > ε, (ε = 0.0001) is the tolerance allowed for convergence. Thus, with these updated values of control variables (λ, P_{g_1} and P_{g_2}) obtained above we have to go for next iteration starting form calculation of elements of *Jacobian* matrix to obtain next set of control variables and this iterative process will continue till the tolerance becomes less than ε (= 0.0001).

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Equations (6.13a) and (6.13e) can be solved by iterative method to obtain the economic load dispatch schedule. The stepwise method of solution in the form of a flowchart is given in Fig. 6.2.



```
Final Economic Generation
==========================

            Bus no.                 Pg (p.u.)
           ---------              ------------
               1                     .374890
               2                     .176939

Transmission line loss =  1.829182E-03

Schedule of generation during iterations

 k     Pg(1)     Pg(2)      Lambda       Ploss      Ptol     dLambda
 -     -----     -----      ------      ------      ----     -------
 1   .380000   .170000  190.200000   .001840   .001840    .000000
 2   .374897   .176932  191.242200   .001829   .000000   1.042191
 3   .374890   .176940  191.242400   .001829   .000000    .000168
 4   .374890   .176939  191.242300   .001829   .000000   -.000015

All results are in p.u.

Lambda is in unit of cost/p.u. power.hr.

Total cost of generation
=========================
           Bus no.           Cost(Unit of cost/hr)
          ---------        -----------------------
              1                     118.417
              2                      72.788

Total cost is 191.204800 unit of cost/hr.
```

6.4 ECONOMIC LOAD DISPATCH USING EXACT LOSS FORMULA

6.4.1 Formation of Exact Loss Formula

The total system loss (here assumed to be the total transmission line loss) is the sum of the bus powers of the power system, i.e.

$$P_l + jQ_l = \sum_{i=1}^{N} S_i = \sum_{i=1}^{N} V_i I_i^* \tag{6.14}$$

where, P_l = real power loss in transmission line,

Q_l = reactive power loss in transmission line,

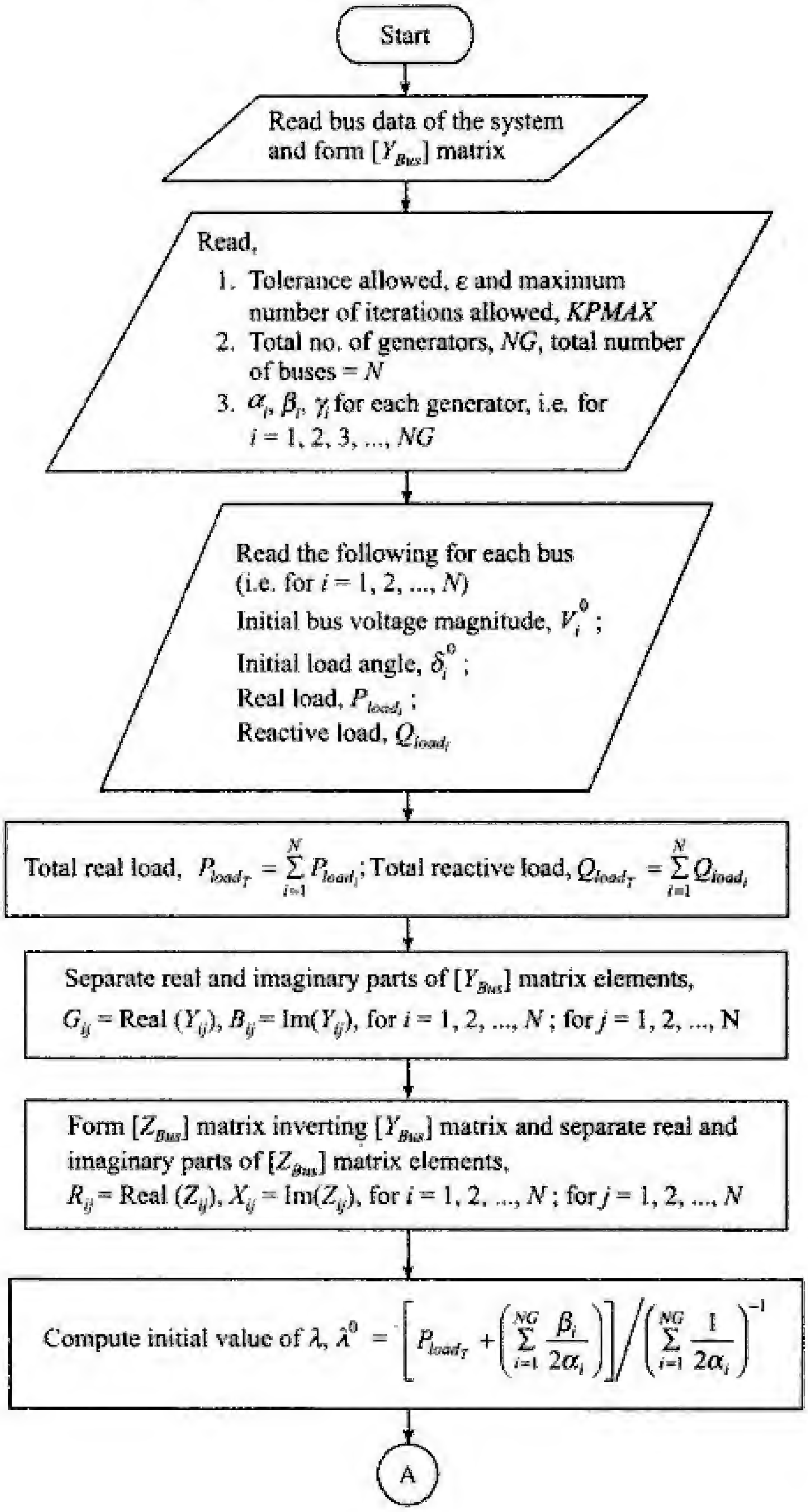
V_i = the complex voltage of i-th bus,

I_i = injected current at i-th bus,

S_i = injected power at i-th bus,

Equation (6.9b) is reproduced for reference as equation (6.27c)

$$\frac{\partial L(P_{g_i},\lambda)}{\partial P_{g_i}} = \sum_{i=1}^{N} P_{load_i} + P_l - \sum_{i=1}^{NG} P_{g_i} \tag{6.27c}$$



Example 6.3: Consider a three-bus, three-line power system as shown in Fig. E6.3. The system has two generators. Find out the economic load dispatch schedule using exact loss formula. The fuel cost characteristics of two generators are as under:

$$F_{c_1}(P_{g_1}) = 20P_{g_1}^2 + 175P_{g_1} + 50 \text{ unit of cost/hr}$$

$$F_{c_2}(P_{g_2}) = 30P_{g_2}^2 + 180P_{g_2} + 40 \text{ unit of cost/hr}$$

where P_{g_1} and P_{g_2} are in p.u. on 100 MVA base. The line data and bus data of the system are as follows.

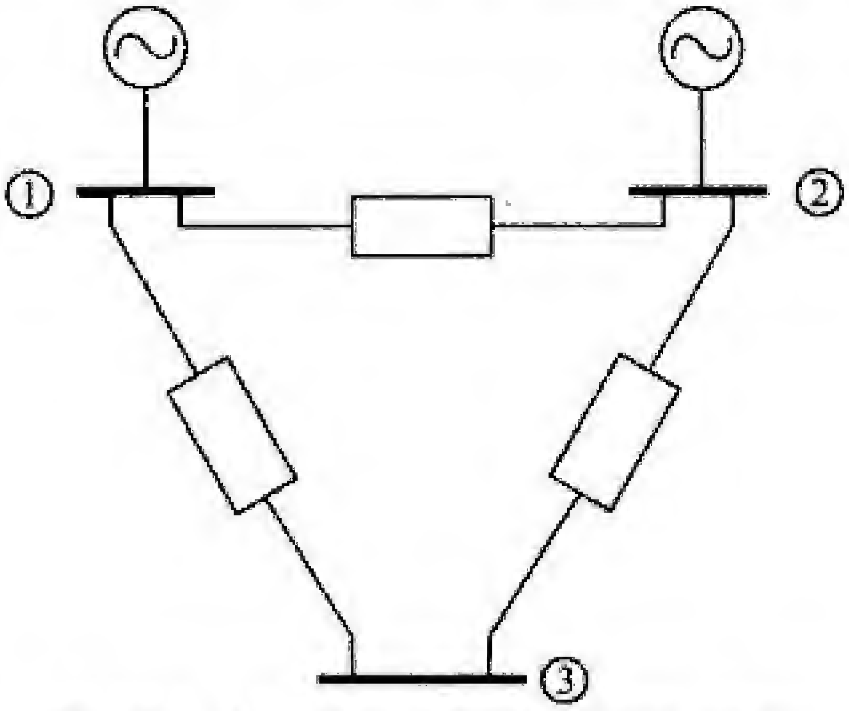


Fig. E6.3 A three-bus, three-line power system.

Line data of the system:

Line no.	*From bus*	*To bus*	*Line impedance (p.u.)*	*B/2 (p.u.)*
1	1	2	$(0.05 + j0.3)$	$j0.01$
2	1	3	$(0.05 + j0.3)$	$j0.01$
3	2	3	$(0.05 + j0.3)$	$j0.01$

The power and bus voltages of the system:

Bus no.	*Bus type*	*V (p.u.)*	P_g *(p.u.)*	Q_g *(p.u.)*	P_d *(p.u.)*	Q_d *(p.u.)*
1	Slack	$1.02\angle 0°$	?	?	0.2	0
2	PV	$\|1.01\|$	?	?	0.1	0.15
3	PQ	?	0	0	0.25	0.1

Solution: Number of generator buses (or PV buses) in the system (NG) = 2 (including slack bus)

Total number of buses in the system (N) = 3

$\therefore$ Number of PQ buses in the system (NP) = 3 – 2 = 1

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Output of computer program ELD.FOR after execution: ELDS1.DAT

```
Iteration completed at kp =  5
Iteration completed at k =  3

Tolerance in P = .000000  Tolerance = .000013  Epsil = .000100

Lambda (unit of cost/p.u.power.hr) =191.208700

Power loss = .001868

Final Economic Generation
=========================
    i      Pg       Qg      Pload    Qload      P        Q
   ---   ------   ------   ------   ------     ---      ---
    1   .373217  .068524  .200000  .000000  .173217  .068524
    2   .178651  .131627  .100000  .150000  .078651 -.018373
    3   .000000  .000000  .250000  .100000 -.250000 -.099997
    i        V           Delta        Lambda
   ---      ---         ------       -------
    1     1.020000      .000000     191.208700
    2     1.010000     -.007703     191.208700
    3      .996020     -.038951     191.208700

All powers and voltages are in p.u.

Angles are in radian.

Lambda is in unit of cost/p.u.power·hr.

Total cost of generation
=========================
          Bus no.         Cost(unit of cost/hr)
          --------       ----------------------
             1                  118.099
             2                   73.115

Total cost is 191.213400 unit of cost/hr.
```

6.5 ECONOMIC LOAD DISPATCH USING LOSS FORMULA WHICH IS A FUNCTION OF REAL AND REACTIVE POWER

6.5.1 Derivation of Real and Reactive Power Governed Loss Formula

The real power loss in the integrated power system has been obtained in equation (6.21a) and is reproduced here,

$$\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{N} P_{load_i} + P_l - \sum_{i=1}^{NG} P_{g_i} \tag{6.36c}$$

The second order partial derivatives of equation (6.36*a*) *with respect to* P_{g_i} *are given below:*

$$\frac{\partial^2 L}{\partial P_{g_i}^2} = 2\alpha_i + \lambda \frac{\partial^2 P_l}{\partial P_{g_i}^2} = 2\alpha_i + 2\lambda a_{ii} \tag{6.37a}$$

$$\frac{\partial^2 L}{\partial P_{g_i} \partial P_{g_k}} = \lambda \frac{\partial^2 P_l}{\partial P_{g_i} \partial P_{g_k}} = \lambda(a_{ik} + a_{ki}) = \frac{\partial^2 L}{\partial P_{g_k} \partial P_{g_i}} \tag{6.37b}$$

$$\frac{\partial^2 L}{\partial P_{g_i} \partial Q_{g_k}} = \lambda \frac{\partial^2 P_l}{\partial P_{g_i} \partial Q_{g_k}} = \lambda(b_{ik} - b_{ki}) \tag{6.37c}$$

$$\frac{\partial^2 L}{\partial P_{g_i} \partial \lambda} = \frac{\partial P_l}{\partial P_{g_i}} - 1 = 2a_{ii}P_i + \sum_{\substack{k=1 \\ k \neq i}}^{N} \left[(a_{ik} + a_{ki}) P_k + (b_{ki} - b_{ik}) Q_k \right] - 1 \tag{6.37d}$$

The second order partial derivatives of equation (6.36*b*) *with respect to* Q_{g_i} *are given below:*

$$\frac{\partial^2 L}{\partial Q_{g_i} \partial P_{g_k}} = \lambda \frac{\partial^2 P_l}{\partial Q_{g_i} \partial P_{g_k}} = \lambda(b_{ik} - b_{ki}) = -\frac{\partial^2 L}{\partial P_{g_i} \partial Q_{g_k}} \tag{6.38a}$$

$$\frac{\partial^2 L}{\partial Q_{g_i}^2} = \lambda \frac{\partial^2 P_l}{\partial Q_{g_i}^2} = 2\lambda a_i \tag{6.38b}$$

$$\frac{\partial^2 L}{\partial Q_{g_i} \partial Q_{g_k}} = \lambda \frac{\partial^2 P_l}{\partial Q_{g_i} \partial Q_{g_k}} = \lambda(a_{ik} + a_{ki}) = \frac{\partial^2 L}{\partial Q_{g_k} \partial Q_{g_i}} = \frac{\partial^2 L}{\partial P_{g_i} \partial P_{g_k}} \tag{6.38c}$$

$$\frac{\partial^2 L}{\partial Q_{g_i} \partial \lambda} = \frac{\partial P_l}{\partial Q_{g_i}} = 2a_{ii}Q_i + \sum_{\substack{k=1 \\ k \neq i}}^{N} \left[(a_{ik} + a_{ki}) Q_k + (b_{ik} - b_{ki}) P_k \right] \tag{6.38d}$$

while *the second order partial derivatives of equation* (6.36*c*) *with respect to* λ *is obtained as:*

$$\frac{\partial^2 L}{\partial \lambda^2} = 0 \tag{6.39}$$

Thus, economic load dispatch problem may also be solved by solving equation (6.34a or 6.34b) using equations (6.36a–6.39). The flowchart of this method is displayed in Fig. 6.4.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

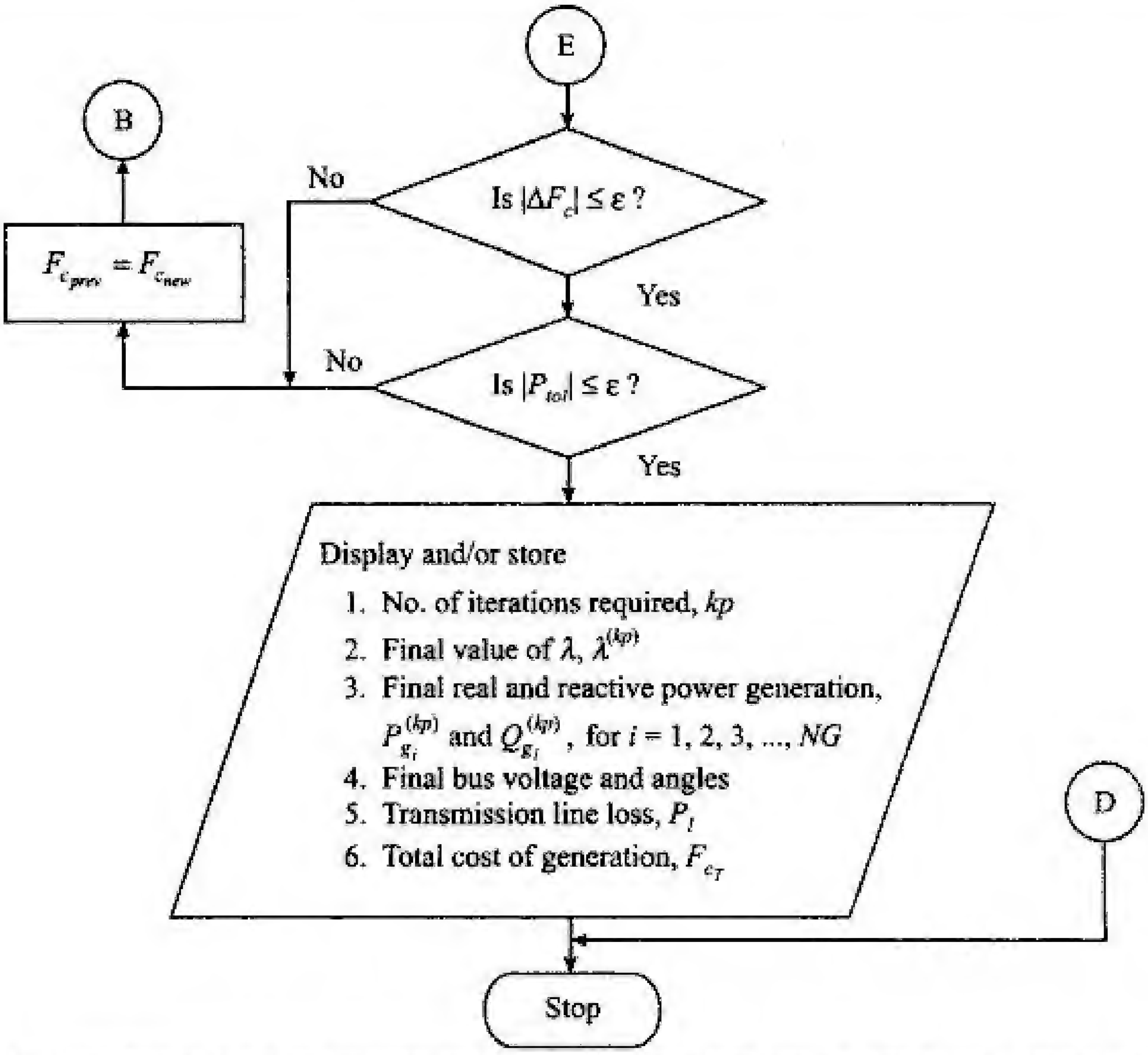


Fig. 6.4 Flowchart to find economic generation schedule using loss formula, which is a function of real and reactive power.

In this method, the active power loss in transmission lines is assumed to be a function of both active and reactive power injection of all the buses in a power system. Therefore, this method is more accurate than earlier methods with only one disadvantage that reactive power loss has not been considered here.

Example 6.4: Consider a three-bus, three-line power system as shown in Fig. E6.3. Find out the economic load dispatch schedule using loss formula which is a function of real and reactive power.

Solution: Number of generator buses or PV buses (i.e. generator buses) in the system $(NG) = 2$ (including slack bus).

Total number of buses in the system $(N) = 3$

$\therefore$ Number of PQ buses in the system $(NP) = 3 - 2 = 1$.

Let kp be the iteration count for P_g iteration and kv be the iteration count for V, δ iteration in load flow study.

$[Y_{Bus}]$ and $[Z_{Bus}]$ matrices (in p.u.) of the system are given in Example 6.3.

$$\therefore \qquad \frac{\partial L}{\partial \lambda^0} = 0.55 + 0.001893 - 0.381892 - 0.17 = 0.000001$$

Let us now calculate the elements of Hessian matrix [using equations (6.37a) to (6.39)].

Second order partial derivatives of L with respect to P_g^0:

From equations (6.37a) and (6.37b), for the given problem, we can write

$$\frac{\partial^2 L}{\partial (P_{g_1}^0)^2} = 2\alpha_1 + \lambda^0 \frac{\partial^2 P_l^0}{\partial (P_{g_1}^0)^2} = 2\alpha_1 + 2\lambda^0 a_{11} = 44.089734;$$

$$\frac{\partial^2 L}{\partial (P_{g_2}^0)^2} = 2\alpha_2 + 2\lambda^0 a_{22} = 64.170900;$$

$$\frac{\partial^2 L}{\partial P_{g_1}^0 \partial P_{g_2}^0} = \frac{\partial^2 L}{\partial P_{g_2}^0 \partial P_{g_1}^0} = \lambda^0 \times (a_{12} + a_{21}) = -2.048628$$

Second order partial derivatives of L with respect to P_g^0 and Q_g^0:

Using equation (6.37c) for the given problem,

$$\frac{\partial^2 L}{\partial P_{g_1}^0 \partial Q_{g_1}^0} = \lambda^0 \frac{\partial^2 P_l^0}{\partial P_{g_1}^0 \partial Q_{g_1}^0} = \lambda^0 (b_{11} - b_{11}) = 0;$$

$$\frac{\partial^2 L}{\partial P_{g_1}^0 \partial Q_{g_2}^0} = \lambda^0 \frac{\partial^2 P_l^0}{\partial P_{g_1}^0 \partial Q_{g_2}^0} = \lambda^0 (b_{21} - b_{12}) = 0.019339;$$

$$\frac{\partial^2 L}{\partial P_{g_2}^0 \partial Q_{g_1}^0} = \lambda^0 \frac{\partial^2 P_l^0}{\partial P_{g_2}^0 \partial Q_{g_1}^0} = \lambda^0 (b_{12} - b_{21}) = -0.019339;$$

$$\frac{\partial^2 L}{\partial P_{g_2}^0 \partial Q_{g_2}^0} = 0$$

Second order partial derivatives of L with respect to P_g^0 and λ^0:

Next, using equation (6.37d), we can write

$$\frac{\partial^2 L}{\partial P_{g_1}^0 \partial \lambda^0} = \frac{\partial^2 L}{\partial \lambda^0 \partial P_{g_1}^0} = \frac{\partial P_l^0}{\partial P_{g_1}^0} - 1 = 0.005840 - 1 = -0.994160;$$

$$\frac{\partial^2 L}{\partial P_{g_2}^0 \partial \lambda^0} = \frac{\partial^2 L}{\partial \lambda^0 \partial P_{g_2}^0} = \frac{\partial P_l^0}{\partial P_{g_2}^0} - 1 = 0.002292 - 1 = -0.997708$$

Bus data and generator fuel cost data (Input data of computer program, named ELDLOSS.FOR based on the flowchart given in Fig. 6.4): NPROB1.DAT

Given in Example 6.3

Output of computer program ELDLOSS.FOR after execution: ELOSS1.DAT

```
Iteration completed at kp =  3
Iteration completed at k =  3

Tolerance in P = -.000515  Tolerance in cost = .000397

Epsilon = .001000

Lambda (unit of cost/p.u.power.hr) = 191.069000

Active power loss = .001871

Final Economic Generation
====================
 i       Pg        Qg       Pload     Qload       P          Q
---    ------    ------    ------    ------      ---        ---
 1    .374910  -.109968  .200000   .000000   .174910   .068200
 2    .176445   .041102  .100000   .150000   .076445  -.018012
 3    .000000   .000000  .250000   .100000  -.250000  -.099997

 i             V            Delta          Lambda
---           ---          ------         -------
 1         1.020000        .000000      191.069000
 2         1.010000      -.008147       191.069000
 3          .996019      -.039172       191.069000

All powers and voltages are in p.u.

Angles are in radian.

Lambda is in unit of cost/p.u.power.hr

Total cost of generation
=========================

          Bus no.          Cost(unit of cost/hr)
         ---------         ----------------------
             1                    118.420
             2                     72.694

Total cost is 191.114600 unit of cost/hr.
```

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

The second order derivatives required for equations (6.41a–6.41d) are given below:
From equation (6.44a), we can write

$$\frac{\partial^2 L}{\partial P_{g_i}^2} = 2\alpha_i + \lambda_p \frac{\partial^2 P_l}{\partial P_{g_i}^2} + \lambda_q \frac{\partial^2 Q_l}{\partial P_{g_i}^2} = 2\alpha_i + 2a_{ii}\lambda_p + 2c_{ii}\lambda_q \tag{6.45a}$$

$$\frac{\partial^2 L}{\partial P_{g_i}\partial P_{g_k}} = \lambda_p \frac{\partial^2 P_l}{\partial P_{g_i}\partial P_{g_k}} + \lambda_q \frac{\partial^2 Q_l}{\partial P_{g_i}\partial P_{g_k}} = \frac{\partial^2 L}{\partial P_{g_k}\partial P_{g_i}}$$

$$= \lambda_p (a_{ik} + a_{ki}) + \lambda_q (c_{ik} + c_{ki}) \tag{6.45b}$$

$$\frac{\partial^2 L}{\partial P_{g_i}\partial Q_{g_k}} = \lambda_p \frac{\partial^2 P_l}{\partial P_{g_i}\partial Q_{g_k}} + \lambda_q \frac{\partial^2 Q_l}{\partial P_{g_i}\partial Q_{g_k}}$$

$$= \lambda_p (b_{ki} - b_{ik}) + \lambda_q (d_{ki} - d_{ik}) \tag{6.45c}$$

$$\frac{\partial^2 L}{\partial P_{g_i}\partial \lambda_p} = \frac{\partial P_l}{\partial P_{g_i}} - 1 = \sum_{k=1}^{N} \{(a_{ik} + a_{ki})P_k + (b_{ki} - b_{ik})Q_k\} - 1 = \frac{\partial^2 L}{\partial \lambda_p \partial P_{g_i}} \tag{6.45d}$$

$$\frac{\partial^2 L}{\partial P_{g_i}\partial \lambda_q} = \lambda_q \frac{\partial Q_l}{\partial P_{g_i}} = \sum_{k=1}^{N} \{(c_{ik} + c_{ki})P_k + (d_{ki} - d_{ik})Q_k\} = \frac{\partial^2 L}{\partial \lambda_q \partial P_{g_i}} \tag{6.45e}$$

From equation (6.44b), we can write

$$\frac{\partial^2 L}{\partial Q_{g_i}\partial P_{g_k}} = \lambda_p \frac{\partial^2 P_l}{\partial Q_{g_i}\partial P_{g_k}} + \lambda_q \frac{\partial^2 Q_l}{\partial Q_{g_i}\partial P_{g_k}}$$

$$= \lambda_p (b_{ik} - b_{ki}) + \lambda_q (d_{ik} - d_{ki}) = -\frac{\partial^2 L}{\partial P_{g_i}\partial Q_{g_k}} \tag{6.46a}$$

$$\frac{\partial^2 L}{\partial Q_{g_i}^2} = \lambda_p \frac{\partial^2 P_l}{\partial Q_{g_i}^2} + \lambda_q \frac{\partial^2 Q_l}{\partial Q_{g_i}^2} = 2a_{ii}\lambda_p + 2c_{ii}\lambda_q \tag{6.46b}$$

$$\frac{\partial^2 L}{\partial Q_{g_i}\partial Q_{g_k}} = \lambda_p \frac{\partial^2 P_l}{\partial Q_{g_i}\partial Q_{g_k}} + \lambda_q \frac{\partial^2 Q_l}{\partial Q_{g_i}\partial Q_{g_k}}$$

$$= \lambda_p (a_{ik} + a_{ki}) + \lambda_q (c_{ik} + c_{ki}) \tag{6.46c}$$

$$\frac{\partial^2 L}{\partial Q_{g_i}\partial \lambda_p} = \frac{\partial P_l}{\partial Q_{g_i}} = \sum_{k=1}^{N} \{(a_{ik} + a_{ki})Q_k + (b_{ik} - b_{ki})P_k\} = \frac{\partial^2 L}{\partial \lambda_p \partial Q_{g_i}} \tag{6.46d}$$

$$\frac{\partial^2 L}{\partial Q_{g_i}\partial \lambda_q} = \frac{\partial Q_l}{\partial Q_{g_i}} - 1 = \sum_{k=1}^{N} \{(c_{ik} + c_{ki})Q_k + (d_{ik} - d_{ki})P_k\} - 1 = \frac{\partial^2 L}{\partial \lambda_q \partial Q_{g_i}} \tag{6.46e}$$

From equation (6.44c), we obtain

$$\frac{\partial^2 L}{\partial \lambda_p^2} = \frac{\partial^2 L}{\partial \lambda_p \partial \lambda_q} = \frac{\partial^2 L}{\partial \lambda_q \partial \lambda_p} = \frac{\partial^2 L}{\partial \lambda_q^2} = 0 \tag{6.46f}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

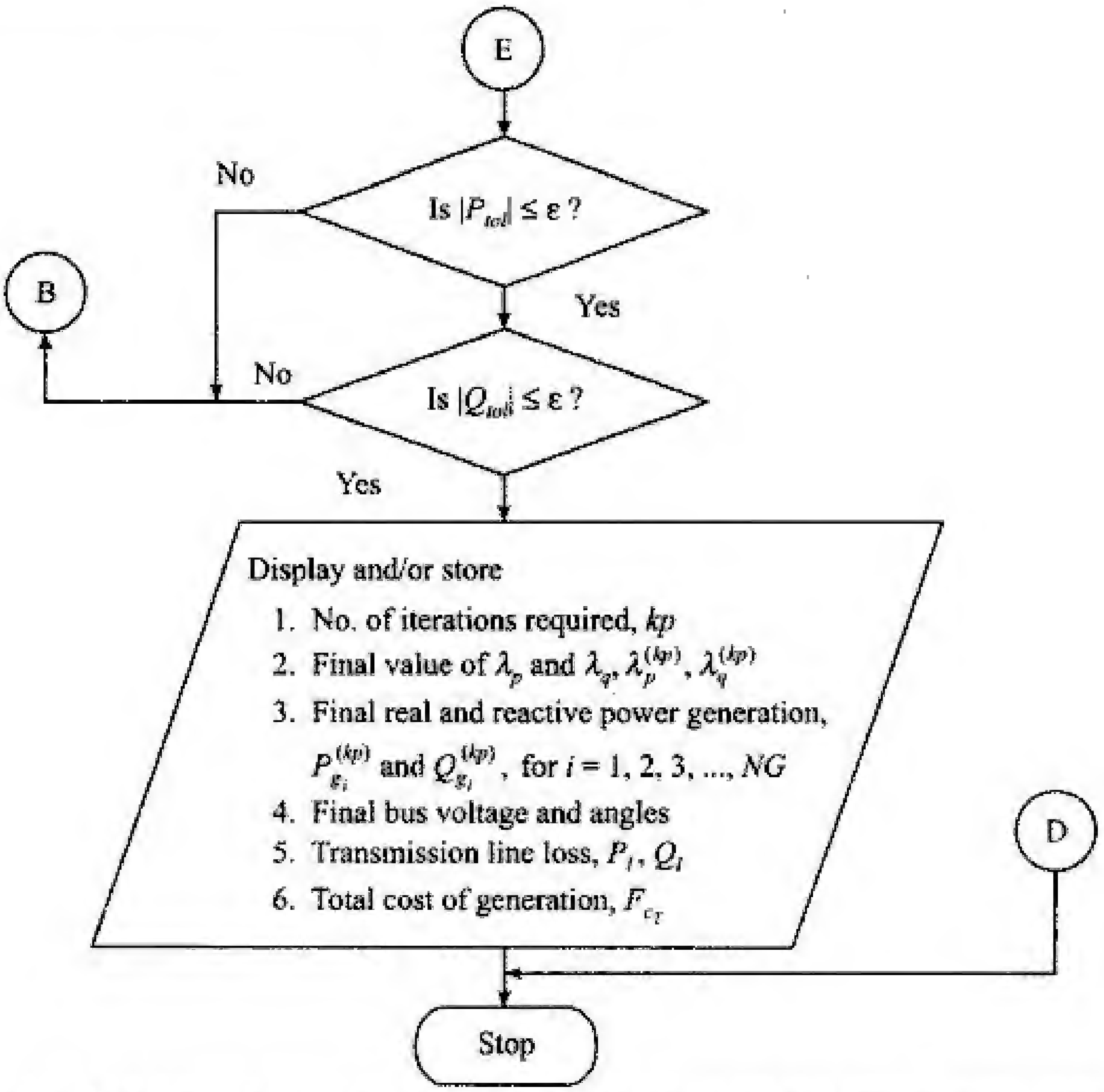


Fig. 6.5 Flowchart to find economic generation schedule for real and reactive power balance.

Example 6.5: Consider a three-bus, three-line power system as shown in Fig. E6.3. Find out the economic load dispatch for Real and Reactive Power Balance.

Solution: The first iteration count, $kp = 0$.

Here, $\lambda_q^0 = 0$. All other initial values are calculated in earlier examples.

It has also been assumed here that total reactive power demand of the system is distributed among the generators in such a manner that power factor of all the above generator buses remains the same.

$$\therefore \qquad Q_{g_1}^0 = 1.72727 \text{ p.u.} \quad \text{and} \quad Q_{g_2}^0 = 0.077273 \text{ p.u.}$$

Since for load buses both active and reactive power generations are zero, hence

$$P_{g_3} = 0 \quad \text{and} \quad Q_{g_3} = 0$$

Total cost of generation, $F_T^0 (= F_{c_1} + F_{c_2}) = 190.855$ unit of cost/hr

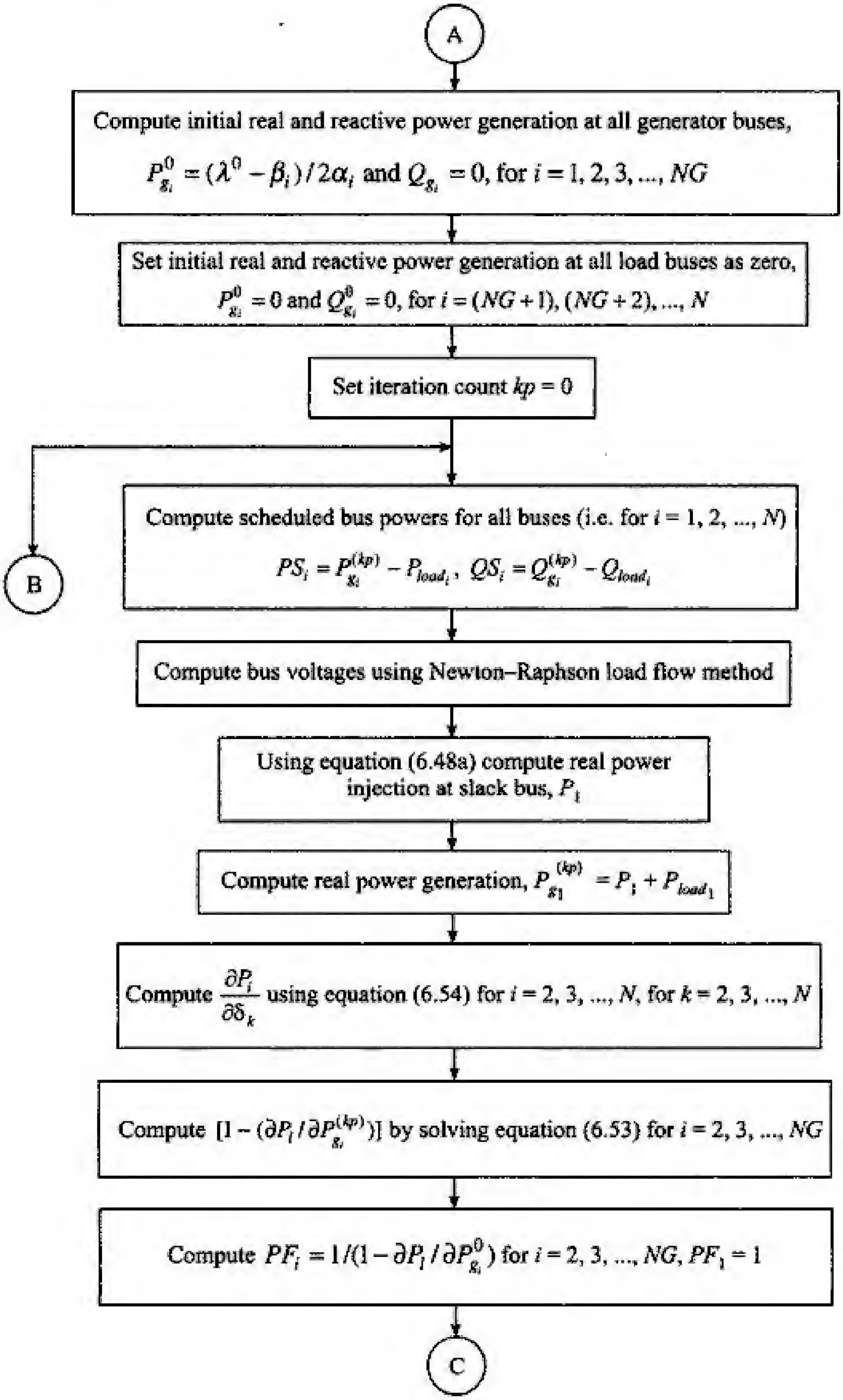
After performing load flow study, the following results are obtained:

$$V_1^{(final)} = 1.02 \text{ , } \delta_1^{(final)} = 0$$

$$V_2^{(final)} = 1.01 \text{ , } \delta_2^{(final)} = -0.009440$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.


A
Compute initial real and reactive power generation at all generator buses,
$P_{g_i}^0 = (\lambda^0 - \beta_i)/2\alpha_i$ and $Q_{g_i} = 0$, for $i = 1, 2, 3, ..., NG$
Set initial real and reactive power generation at all load buses as zero,
$P_{g_i}^0 = 0$ and $Q_{g_i}^0 = 0$, for $i = (NG+1), (NG+2), ..., N$
Set iteration count $kp = 0$
B
Compute scheduled bus powers for all buses (i.e. for $i = 1, 2, ..., N$)
$PS_i = P_{g_i}^{(kp)} - P_{load_i}$, $QS_i = Q_{g_i}^{(kp)} - Q_{load_i}$
Compute bus voltages using Newton–Raphson load flow method
Using equation (6.48a) compute real power injection at slack bus, P_1
Compute real power generation, $P_{g_1}^{(kp)} = P_1 + P_{load_1}$
Compute $\frac{\partial P_i}{\partial \delta_k}$ using equation (6.54) for $i = 2, 3, ..., N$, for $k = 2, 3, ..., N$
Compute $[1 - (\partial P_l / \partial P_{g_i}^{(kp)})]$ by solving equation (6.53) for $i = 2, 3, ..., NG$
Compute $PF_i = 1/(1 - \partial P_l / \partial P_{g_i}^0)$ for $i = 2, 3, ..., NG$, $PF_1 = 1$
C

$V_3^{(final)} = 0.996680,\ \delta_3^{(final)} = -0.039598.$

(All voltages are in p.u. and angles are in radian.)

Again, from equation (6.48a) slack bus power for $kp = 1$ is given by

$$\begin{aligned}
&= \sum_{k=1}^{N} |V_1^{(final)} V_k^{(final)}| [G_{1k} \cos (\delta_1^{(final)} - \delta_k^{(final)}) \\
&\quad + B_{1k} \sin (\delta_1^{(final)} - \delta_k^{(final)})] \\
&= |V_1^{(final)}| \Big[|V_1^{(final)}| G_{11} + |V_2^{(final)}| \{G_{12} \cos (\delta_1^{(final)} - \delta_2^{(final)}) \\
&\quad + B_{12} \sin (\delta_1^{(final)} - \delta_2^{(final)})\} + |V_3^{(final)}| \{G_{13} \cos (\delta_1^{(final)} - \delta_3^{(final)}) \\
&\quad + B_{13} \sin (\delta_1^{(final)} - \delta_3^{(final)})\} \Big] \\
&= 1.02\, [1.02 \times 1.081081 + 1.01\, \{-0.540541 \cos (0.009298) \\
&\quad + 3.243243 \sin (0.009298)\} + 0.99668\, \{-0.540541 \cos (0.039598) \\
&\quad + 3.243243 \sin (0.039598)\}] \\
&= 1.124757 - 0.525775 - 0.418565 \\
&= 0.180417 \text{ p.u.}
\end{aligned}$$

Therefore, total power generation at slack bus (for $kp = 1$) is given by

$$P_{g_1}^1 = P_{1_{scheduled}}^1 + P_{load_1} = 0.180417 + 0.2 = 0.380417 \text{ p.u.}$$

It is now required to find out $\partial P_i / \partial \delta_i$ for all buses. We proceed as follows:

$$\begin{aligned}
\frac{\partial P_{g_1}^0}{\partial \delta_2^{(final)}} &= |V_1^{(final)}||V_2^{(final)}| [G_{12} \sin (\delta_1^{(final)} - \delta_2^{(final)}) - B_{12} \cos (\delta_1^{(final)} - \delta_2^{(final)})] \\
&= |1.02||1.01| [-0.540541 \sin (0.009298) - 3.243243 \cos (0.009298)] \\
&= -3.346222
\end{aligned}$$

$$\begin{aligned}
\frac{\partial P_{g_2}^0}{\partial \delta_2^{(final)}} &= \sum_{\substack{k=1 \\ k \neq 2}}^{N} |V_2^{(final)}||V_k^{(final)}| [-G_{2k} \sin (\delta_2^{(final)} - \delta_k^{(final)}) \\
&\quad + B_{2k} \cos (\delta_2^{(final)} - \delta_k^{(final)})] \\
&= |1.01||1.02| [0.540541 \sin (-0.009298) + 3.243243 \cos (-0.009298)] \\
&\quad + |1.01||0.99668| [0.540541 \sin (0.0303) + 3.243243 \cos (0.0303)] \\
&= 3.335867 + 3.279787 \\
&= 6.615654
\end{aligned}$$

$$\begin{aligned}
\frac{\partial P_3^0}{\partial \delta_2^{(final)}} &= |V_3^{(final)}||V_2^{(final)}| [G_{32} \sin (\delta_3^{(final)} - \delta_2^{(final)}) - B_{32} \cos (\delta_1^{(final)} - \delta_2^{(final)})] \\
&= |0.99668||1.01| [-0.540541 \sin (-0.0303) - 3.243243 \cos (-0.0303)] \\
&= -3.246818
\end{aligned}$$

```
Total cost of generation
==========================
          Bus no.          Cost(unit of cost/hr)
          --------         -----------------------
             1                  120.578300
             2                   70.356220

Total cost is 190.934600 unit of cost/hr.
```

6.8 MODERN APPROACH TO OPTIMAL POWER FLOW SOLUTION

6.8.1 Newton–Raphson (N-R) Method

Let us consider a N-bus power system having NG number of thermal power generators. Then the aim of optimal power flow problem is *to minimise the cost of thermal power generation,*

$$F_{c_{total}} = \sum_{i=1}^{NG} F_{c_i} = \sum_{i=1}^{NG} \alpha_i (P_{g_i})^2 + \beta_i P_{g_i} + \gamma_i \text{ unit of cost/hr}$$

subjected to

(*i*) active power balance in the network

$$P_i(|V|,\delta) - P_{g_i} - P_{load_i} = 0 \quad \text{for } i = 1, 2, 3, \ldots, N$$

where P_i = active power injection at i-th bus and is a function of $|V|$ and δ. For load buses [i.e. for $i = (NG + 1), (NG + 2), \ldots, N$], $P_{g_i} = 0$;

(*ii*) reactive power balance in the network

$$Q_i(V,\delta) - Q_{g_i} - Q_{load_i} = 0 \quad \text{for } i = (NG + 1), (NG + 2), \ldots, N$$

where Q_i = reactive power injection at i-th load bus and is also a function of $|V|$ and δ; Q_{g_i} = reactive power generation at i-th bus;

(*iii*) security related constraints (also called *soft constraints*). These constraints are as follows:

(*a*) limits on real power generations, i.e. $P_{g_{i_{min}}} \le P_{g_i} \le P_{g_{i_{max}}}$ for $i = 1, 2, 3, \ldots, NG$;

(*b*) limits on reactive power generations, i.e. $Q_{g_{i_{min}}} \le Q_{g_i} \le Q_{g_{i_{max}}}$ for $i = 1, 2, 3, \ldots, NG$;

(*c*) limits on voltage magnitudes of load buses $|V_i|_{min} \le |V_i| \le |V_i|_{max}$ for $i = (NG + 1), (NG + 2), \ldots, N$;

(*d*) limits on voltage angles of all the above buses excluding slack bus $\delta_{i_{min}} \le \delta_i \le \delta_{i_{max}}$ for $i = 2, 3, \ldots, N$.

The reactive power injection at bus-i is given by [as given in equation (6.48b)]

$$Q_i = \sum_{k=1}^{N} |V_i V_k| \left[G_{ik} \sin(\delta_i - \delta_k) - B_{ik} \cos(\delta_i - \delta_k) \right]$$

The constraint minimisation problem can be transformed into an unconstrained one by augmenting the load flow constraints into objective function. The additional variables are known as *Lagrange multiplier*

Different values of $\partial^2 P_r/\partial\delta_i\partial|V_k|$ and $\partial^2 Q_r/\partial\delta_i\partial|V_k|$ can be calculated similarly using equations (6.71) and (6.72).

Next, second order partial derivatives required for equation (6.57c) are obtained by differentiating equation (6.56c) with respect to control variables:

$$\frac{\partial^2 L}{\partial\lambda_{p_i}\partial\delta_k} = \frac{\partial P_i}{\partial\delta_k} \text{ for } i = 1, 2, 3, \ldots, N; k = 2, 3, \ldots, N \tag{6.60a}$$

$$\frac{\partial^2 L}{\partial\lambda_{p_i}\partial\lambda_{p_k}} = 0 \text{ for } i = 1, 2, 3, \ldots, N; k = 1, 2, 3, \ldots, N \tag{6.60b}$$

$$\frac{\partial^2 L}{\partial\lambda_{p_i}\partial\lambda_{q_k}} = 0 \text{ for } i = 1, 2, 3, \ldots, N; k = (NG + 1), \ldots, N \tag{6.60c}$$

$$\frac{\partial^2 L}{\partial\lambda_{p_i}\partial|V_k|} = \frac{\partial P_i}{\partial|V_k|} \text{ for } i = 1, 2, 3, \ldots, N; k = (NG + 1), \ldots, N \tag{6.60d}$$

Also, second order partial derivatives required for equation (6.57d) are obtained by differentiating equation (6.56d) with respect to control variables, and are as follows:

$$\frac{\partial^2 L}{\partial|V_i|\partial\delta_k} = \sum_{r=1}^{N}\lambda_{p_r}\frac{\partial^2 P_r}{\partial|V_i|\partial\delta_k} + \sum_{r=NG+1}^{N}\lambda_{q_r}\frac{\partial^2 Q_r}{\partial|V_i|\partial\delta_k} \tag{6.61a}$$

$$\text{for } i = (NG + 1), \ldots, N; k = 2, 3, \ldots, N$$

Different values of $\partial^2 P_r/\partial|V_i|\partial\delta_k$ and $\partial^2 Q_r/\partial|V_i|\partial\delta_k$ can be calculated using equations (6.73) and (6.74) derived later in this chapter.

$$\frac{\partial^2 L}{\partial|V_i|\partial\lambda_{p_k}} = \frac{\partial P_k}{\partial|V_i|} \text{ for } i = (NG + 1), \ldots, N; k = 1, 2, 3, \ldots, N \tag{6.61b}$$

$$\frac{\partial^2 L}{\partial|V_i|\partial\lambda_{q_k}} = \frac{\partial Q_k}{\partial|V_i|} \text{ for } i = (NG + 1), \ldots, N; k = (NG + 1), \ldots, N \tag{6.61c}$$

$$\frac{\partial^2 L}{\partial|V_i|\partial|V_k|} = \sum_{r=1}^{N}\lambda_{p_r}\frac{\partial^2 P_r}{\partial|V_i|\partial|V_k|} + \sum_{r=NG+1}^{N}\lambda_{q_r}\frac{\partial^2 Q_r}{\partial|V_i|\partial|V_k|} \tag{6.61d}$$

$$\text{for } i = (NG + 1), \ldots, N; k = (NG + 1), \ldots, N$$

Different values of $\partial^2 P_r/\partial|V_i|\partial|V_k|$ and $\partial^2 Q_r/\partial|V_i|\partial|V_k|$ can also be calculated using equations (6.75) and (6.76) of this chapter.

Second order partial derivatives required for equation (6.57e) are obtained by differentiating equation (6.56e) with respect to control variables and are as follows:

$$\frac{\partial^2 L}{\partial\lambda_{q_i}\partial\delta_k} = \frac{\partial Q_i}{\partial\delta_k} \text{ for } i = (NG + 1), \ldots, N; k = 2, 3, \ldots, N \tag{6.62a}$$

$$\frac{\partial^2 L}{\partial\lambda_{q_i}\partial\lambda_{p_k}} = 0 \text{ for } i = (NG + 1), \ldots, N; k = 1, 2, 3, \ldots, N \tag{6.62b}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

C

Update the control variables

1. $P_{g_i}^{(kp+1)} = P_{g_i}^{(kp)} + \Delta P_{g_i}^{(kp)}$, for all generator buses (i.e. $i = 1, 2, 3, \ldots, NG$),
2. $\delta_i^{(kp+1)} = \delta_i^{(kp)} + \Delta\delta_i^{(kp)}$, for all buses excluding slack bus (i.e. $i = 2, 3, \ldots, N$),
3. $\lambda_{p_i}^{(kp+1)} = \lambda_{p_i}^{(kp)} + \Delta\lambda_{p_i}^{(kp)}$, for all buses (i.e. $i = 1, 2, 3, \ldots, N$),
4. $\lambda_{q_i}^{(kp+1)} = \lambda_{q_i}^{(kp)} + \Delta\lambda_{q_i}^{(kp)}$, for all load buses (i.e. $i = (NG+1), (NG+2), \ldots, N$),
5. $\left|V_i^{(kp+1)}\right| = \left|V_i^{(kp)}\right| + \Delta\left|V_i^{(kp)}\right|$, for all load buses (i.e. $i = (NG+1), (NG+2), \ldots, N$)

$$tol = \left[\sum_{i=1}^{NG}(\Delta P_{gi}^0)^2 + \sum_{i=2}^{N}(\Delta\delta_i^0)^2 + \sum_{i=1}^{N}(\Delta P_{pi}^0)^2 + \sum_{i=NG+1}^{N}(\Delta\lambda_{qi}^0)^2 + \sum_{i=NG+1}^{N}(\Delta|V_i|^0)^2\right]^{1/2}$$

Reactive power generations $Q_{g_i}^{(kp)} = Q_i + Q_{load_i}$, i.e. for $i = 1, 2, 3, \ldots, NG$

$kp \leftarrow kp + 1$

Is $kp > KPMAX$? — Yes → D; No ↓

Is $tol \leq \varepsilon$? — Yes → E; No → B

where, from equation (6.54)

$$\frac{\partial P_1^0}{\partial \delta_3^0} = |V_1^0||V_3^0|[G_{13}\sin(\delta_1^0-\delta_3^0) - B_{13}\cos(\delta_1^0-\delta_3^0)]$$

$$= -1.02 \times 1.0 \times 3.243243 \cos 0 = -3.308108$$

$$\frac{\partial P_2^0}{\partial \delta_3^0} = |V_2^0||V_3^0|[G_{23}\sin(\delta_2^0-\delta_3^0) - B_{23}\cos(\delta_2^0-\delta_3^0)]$$

$$= -1.01 \times 1.0 \times 3.243243 \cos 0 = -3.275675$$

$$\frac{\partial P_3^0}{\partial \delta_3^0} = \sum_{\substack{k=1 \\ k\neq 3}}^{N} |V_3^0||V_k^0|[-G_{3k}\sin(\delta_3^0-\delta_k^0) + B_{3k}\cos(\delta_3^0-\delta_k^0)]$$

$$= |V_3^0||V_1^0|B_{31}\cos(\delta_3^0-\delta_1^0) + |V_3^0||V_2^0|B_{32}\cos(\delta_3^0-\delta_2^0)$$

$$= 1.0 \times 1.02 \times 3.243243 \cos 0 + 1.0 \times 1.01 \times 3.243243 \cos 0$$

$$= 6.583783$$

From equation (6.66), we find

$$\frac{\partial Q_3^0}{\partial \delta_3^0} = \sum_{\substack{k=1 \\ k\neq 3}}^{N} |V_3^0||V_k^0|[G_{3k}\cos(\delta_3^0-\delta_k^0) + B_{3k}\sin(\delta_3^0-\delta_k^0)]$$

$$= |V_3^0||V_1^0|G_{31}\cos(\delta_3^0-\delta_1^0) + |V_3^0||V_2^0|G_{32}\cos(\delta_3^0-\delta_2^0)$$

$$= 1.0 \times 1.02 \times (-0.540541)\cos 0 + 1.0 \times 1.01 \times (-0.540541)\cos 0$$

$$= -1.097298$$

$$\therefore \quad \frac{\partial L}{\partial \delta_3^0} = \lambda_{p_1}^0 \frac{\partial P_1^0}{\partial \delta_3^0} + \lambda_{p_2}^0 \frac{\partial P_2^0}{\partial \delta_3^0} + \lambda_{p_3}^0 \frac{\partial P_3^0}{\partial \delta_3^0} + \lambda_{q_3}^0 \frac{\partial Q_3^0}{\partial \delta_3^0}$$

$$= 190.2 \times (-3.308108) + 190.2 \times (-3.275675) + 0 \times (6.583783) + 0 \times (-1.097298)$$

$$= -1252.235527$$

Partial derivatives of L with respect to $\lambda_{p_1}^0$ *:*

From equation (6.56c)

$$\frac{\partial L}{\partial \lambda_{p_1}^0} = P_1^0(|V^0|, \delta^0) - P_{g_1}^0 + P_{load_1}$$

Now, from equation (6.48a)

$$P_1^0 = \sum_{k=1}^{3} |V_1^0 V_k^0|[G_{1k}\cos(\delta_1^0-\delta_k^0) + B_{1k}\sin(\delta_1^0-\delta_k^0)]$$

$$= |V_1^0|^2 G_{11} + |V_1^0 V_2^0| G_{12}\cos 0 + |V_1^0 V_3^0| G_{13}\cos 0$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

and from equation (6.69)

$$\frac{\partial^2 P_1^0}{\partial(\delta_2^0)^2} = |V_1^0 V_2^0|[-G_{12}\cos(\delta_1^0-\delta_2^0)-B_{12}\sin(\delta_1^0-\delta_2^0)]$$

$$= -|V_1^0 V_2^0|\,G_{12}\cos(\delta_1^0-\delta_2^0)$$

$$= -1.02\times 1.01\times(-0.540541)\cos 0 = 0.556865$$

$$\frac{\partial^2 P_2^0}{\partial(\delta_2^0)^2} = \sum_{\substack{i=1\\ i\neq 2}}^{N}|V_2 V_i|[-G_{2i}\cos(\delta_2^0-\delta_i^0)-B_{2i}\sin(\delta_2^0-\delta_i^0)]$$

$$= -|V_2^0 V_1^0|\,G_{21}\cos(\delta_2^0-\delta_1^0)-|V_2^0 V_3^0|\,G_{23}\cos(\delta_2^0-\delta_3^0)$$

$$= -1.01\times 1.02\times(-0.540541)\cos 0 - 1.01\times 1.0\times(-0.540541)\cos 0$$

$$= 1.102812$$

$$\frac{\partial^2 P_3^0}{\partial(\delta_2^0)^2} = |V_3^0 V_2^0|[-G_{32}\cos(\delta_3^0-\delta_2^0)-B_{32}\sin(\delta_3^0-\delta_2^0)]$$

$$= -|V_3^0 V_2^0|\,G_{32}\cos(\delta_3^0-\delta_2^0)$$

$$= -1.0\times 1.01\times(-0.540541)\cos 0 = 0.545946$$

Also from equation (6.70),

$$\frac{\partial^2 Q_3^0}{\partial(\delta_2^0)^2} = |V_3 V_2|[-G_{32}\sin(\delta_3^0-\delta_2^0)+B_{32}\cos(\delta_3^0-\delta_2^0)]$$

$$= |V_3 V_2|\,B_{32}\cos(\delta_3^0-\delta_2^0)$$

$$= 1.0\times 1.01\times 3.243243\cos 0 = 3.275675$$

$$\therefore\quad \frac{\partial^2 L}{\partial(\delta_2^0)^2} = \lambda_{p_1}^0\frac{\partial^2 P_1^0}{\partial(\delta_2^0)^2}+\lambda_{p_2}^0\frac{\partial^2 P_2^0}{\partial(\delta_2^0)^2}+\lambda_{p_3}^0\frac{\partial^2 P_3^0}{\partial(\delta_2^0)^2}+\lambda_{q_3}^0\frac{\partial^2 Q_3^0}{\partial(\delta_2^0)^2}$$

$$= 190.2\times 0.556865 + 190.2\times 1.102812 + 0 + 0 = 315.670565$$

Second order partial derivatives of L with respect to δ_2^0 *and* δ_3^0 *:*

From equation (6.59a), we get

$$\frac{\partial^2 L}{\partial\delta_2^0\partial\delta_3^0} = \sum_{r=1}^{3}\lambda_{p_r}^0\frac{\partial^2 P_r^0}{\partial\delta_2^0\partial\delta_3^0}+\sum_{r=3}^{3}\lambda_{q_r}^0\frac{\partial^2 Q_r^0}{\partial\delta_2^0\partial\delta_3^0}$$

Now from equation (6.69),

$$\frac{\partial^2 P_1^0}{\partial\delta_2^0\partial\delta_3^0} = 0$$

and from equation (6.72),

$$\frac{\partial^2 Q_3^0}{\partial \delta_2^0 \partial |V_3|^0} = |V_2|^0 [-G_{32}\cos(\delta_3^0 - \delta_2^0) - B_{32}\sin(\delta_3^0 - \delta_2^0)]$$

$$= -|V_2|^0 G_{32}\cos(\delta_3^0 - \delta_2^0) = -1.01 \times (-0.540541)\cos 0$$

$$= 0.545946$$

$$\therefore \quad \frac{\partial^2 L}{\partial \delta_2^0 \partial |V_3|^0} = \lambda_{p_1}^0 \frac{\partial^2 P_1^0}{\partial \delta_2^0 \partial |V_3|^0} + \lambda_{p_2}^0 \frac{\partial^2 P_2^0}{\partial \delta_2^0 \partial |V_3|^0} + \lambda_{p_3}^0 \frac{\partial^2 P_3^0}{\partial \delta_2^0 \partial |V_3|^0} + \lambda_{q_3}^0 \frac{\partial^2 Q_3^0}{\partial \delta_2^0 \partial |V_3|^0}$$

$$= 0 + 190.2 \times 3.275675 + 0 + 0 = 623.033385$$

Again from equation (6.59d)

$$\frac{\partial^2 L}{\partial \delta_3^0 \partial |V_3|^0} = \sum_{r=1}^{3} \lambda_{p_r}^0 \frac{\partial^2 P_r^0}{\partial \delta_3^0 \partial |V_3|^0} + \sum_{r=3}^{3} \lambda_{q_r}^0 \frac{\partial^2 Q_r^0}{\partial \delta_3^0 \partial |V_3|^0}$$

Now from equation (6.71),

$$\frac{\partial^2 P_1^0}{\partial \delta_3^0 \partial |V_3|^0} = |V_1|^0 [G_{13}\sin(\delta_1^0 - \delta_3^0) - B_{13}\cos(\delta_1^0 - \delta_3^0)]$$

$$= -|V_1|^0 B_{13}\cos(\delta_1^0 - \delta_3^0) = -1.02 \times 3.243243\cos 0$$

$$= -3.308108$$

$$\frac{\partial^2 P_2^0}{\partial \delta_3^0 \partial |V_3|^0} = |V_2|^0 [G_{23}\sin(\delta_2^0 - \delta_3^0) - B_{23}\cos(\delta_2^0 - \delta_3^0)]$$

$$= -|V_2|^0 B_{23}\cos(\delta_2^0 - \delta_3^0) = -1.01 \times 3.243243\cos 0$$

$$= -3.275675$$

$$\frac{\partial^2 P_3^0}{\partial \delta_3^0 \partial |V_3|^0} = \sum_{\substack{k=1 \\ k\neq 3}}^{3} |V_k|^0 [-G_{3k}\sin(\delta_3^0 - \delta_k^0) + B_{3k}\cos(\delta_3^0 - \delta_k^0)]$$

$$= |V_1|^0 B_{31}\cos(\delta_3^0 - \delta_1^0) + |V_2|^0 B_{32}\cos(\delta_3^0 - \delta_2^0)$$

$$= 1.02 \times 3.243243\cos 0 + 1.01 \times 3.243243\cos 0$$

$$= 6.583783$$

Also from equation (6.72),

$$\frac{\partial^2 Q_3^0}{\partial \delta_3^0 \partial |V_3|^0} = \sum_{\substack{k=1 \\ k\neq 3}}^{3} |V_k|^0 [G_{3k}\cos(\delta_3^0 - \delta_k^0) + B_{3k}\sin(\delta_3^0 - \delta_k^0)]$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Bus powers:

Bus no.	P_g (*p.u.*)	Q_g (*p.u.*)	P_{load} (*p.u.*)	Q_{load} (*p.u.*)	$P_{calculated}$ (*p.u.*)	$Q_{calculated}$ (*p.u.*)
1	0.374900	0.068270	0.2	0	0.174960	0.068270
2	0.176972	0.131912	0.1	0.15	0.076912	– 0.018088
3	0	0	0.25	0.1	– 0.250000	– 0.099999

Bus voltages and values of λ_p and λ_q:

Bus no.	V (*p.u.*)	δ (*p.u.*)	λ_p	λ_q
1	1.020000	0	189.996000	0
2	1.010000	– 0.008040	190.618300	0
3	0.996019	– 0.039119	192.725300	0.876832

The costs of generation are as follows:

For generator-1,

$$\text{Cost } (F_{c_1}) = 20\,(0.3749)^2 + 175 \times 0.3749 + 50 \text{ unit of cost/hr}$$
$$= 118.4185 \text{ unit of cost/hr.}$$

For generator-2,

$$\text{Cost } (F_{c_2}) = 30\,(0.176972)^2 + 180 \times 0.176972 + 40 \text{ unit of cost/hr}$$
$$= 72.794533 \text{ unit of cost/hr.}$$

$$\therefore \quad \text{Total cost} = F_{c_1} + F_{c_2} = 118.4185 + 72.794533$$
$$= 191.213033 \text{ unit of cost/hr.}$$

The application of computer method (flowchart of Fig. 6.7) in this problem (Example 6.7) yields the following results:

Execution of computer algorithm given in Fig. 6.7 for Example 6.7

LINE DATA of the system: NZPROB1.DAT

Given in Example 6.3

[Y_{Bus}] of the system: NYPROB1.DAT

Given in Example 6.3

Bus data and generator fuel cost data (Input data of computer program, named NEWOPT.FOR based on the flowchart given in Fig. 6.7): NPROB1.DAT

Given in Example 6.3

Bus data and generator fuel cost data: NPROB2.DAT

3, 6 [No. of generator buses, total no. of buses]
120, 175, 75 [α, β, γ for generator-1]
1.05, 0, 0.1, 0 [$V_1^0, \delta_1^0, P_{load_1}, Q_{load_1}$]
110, 180, 50 [α, β, γ for generator-2]
1, 0, 0.3, 0.25 [$V_2^0, \delta_2^0, P_{load_2}, Q_{load_2}$]
160, 110, 80 [α, β, γ for generator-3]
1.02, 0, 0.1, 0.3 [$V_3^0, \delta_3^0, P_{load_3}, Q_{load_3}$]
1.0, 0, 0.15, 0.2 [$V_4^0, \delta_4^0, P_{load_4}, Q_{load_4}$]
1.0, 0, 0.35, 0.45 [$V_5^0, \delta_5^0, P_{load_5}, Q_{load_5}$]
1.0; 0, 0.3, 0.5 [$V_6^0, \delta_6^0, P_{load_6}, Q_{load_6}$]

Output of computer program NEWOPT.FOR after execution: NSOL2.DAT

Iteration completed at k = 11
Tolerance = .004747
Final Economic Generation

i	Pg	Qg	Pload	Qload	P	Q
1	.397804	.991192	.100000	.000000	.297805	.991192
2	.407596	.029375	.300000	.250000	.107596	-.220625
3	.497392	.685601	.100000	.300000	.397391	.385601
4	.000000	.000000	.150000	.200000	-.150000	-.200011
5	.000000	.000000	.350000	.450000	-.350000	-.450014
6	.000000	.000000	.300000	.500000	-.300000	-.500014

i	V	Delta	Plam	Qlam
1	1.050000	.000000	270.473100	
2	1.000000	.000816	269.671100	
3	1.020000	.011358	269.165300	
4	.979173	-.016777	270.038800	1.457464
5	.982133	-.033335	271.511700	.872780
6	.994906	-.022762	270.315900	.585502

All the powers and voltages are in p.u.

Angles are in radian.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Output of computer program FDOPT.FOR after execution: FSOL2.DAT

Iteration completed at k = 7

Tolerance-1 = .005323   Tolerance-2 = .004755

Final Economic Generation
==========================

i	Pg	Qg	Pload	Qload	P	Q
1	.397801	.991506	.100000	.000000	.297777	.991506
2	.407599	.029514	.300000	.250000	.107651	-.220486
3	.497393	.685984	.100000	.300000	.397418	.385984
4	.000000	.000000	.150000	.200000	-.150017	-.200223
5	.000000	.000000	.350000	.450000	-.350041	-.450280
6	.000000	.000000	.300000	.500000	-.299994	-.500293

i	V	Delta	Plam	Qlam
1	1.050000	.000000	270.472400	
2	1.000000	.000814	269.671900	
3	1.020000	.011357	269.165800	
4	.979190	-.016777	270.038000	1.456649
5	.982144	-.033335	271.510000	.871919
6	.994916	-.022763	270.315200	.585506

All the powers and voltages are in p.u.
Angles are in radian.

All lambdas are in unit of cost/p.u.power.hr.

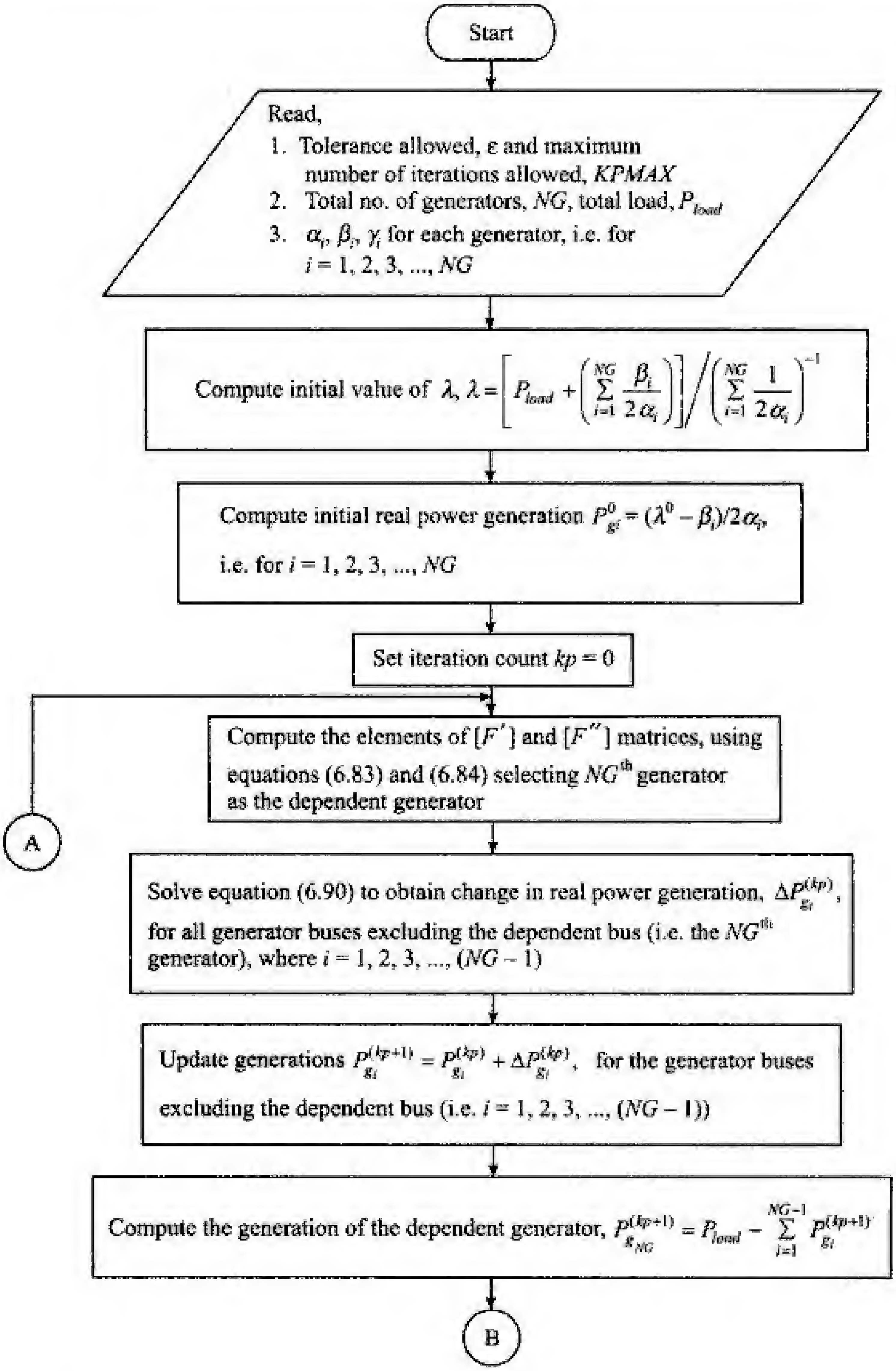
Total cost of generation
=========================

Bus no.	Cost(unit of cost/hr)
1	163.604800
2	141.643000
3	174.297200

Total cost is 479.545000 unit of cost/hr.

The Fast Decoupled method is found to be faster than N–R method for large interconnected power system and it requires lesser memory size.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.


Start
Read,
1. Tolerance allowed, ε and maximum number of iterations allowed, $KPMAX$
2. Total no. of generators, NG, total load, P_{load}
3. α_i, β_i, γ_i for each generator, i.e. for $i = 1, 2, 3, \ldots, NG$
Compute initial value of λ, $\lambda = \left[P_{load} + \left(\sum_{i=1}^{NG} \frac{\beta_i}{2\alpha_i}\right)\right] \Big/ \left(\sum_{i=1}^{NG} \frac{1}{2\alpha_i}\right)^{-1}$
Compute initial real power generation $P_{gi}^0 = (\lambda^0 - \beta_i)/2\alpha_i$, i.e. for $i = 1, 2, 3, \ldots, NG$
Set iteration count $kp = 0$
Compute the elements of $[F']$ and $[F'']$ matrices, using equations (6.83) and (6.84) selecting NG^{th} generator as the dependent generator
A
Solve equation (6.90) to obtain change in real power generation, $\Delta P_{gi}^{(kp)}$, for all generator buses excluding the dependent bus (i.e. the NG^{th} generator), where $i = 1, 2, 3, \ldots, (NG - 1)$
Update generations $P_{gi}^{(kp+1)} = P_{gi}^{(kp)} + \Delta P_{gi}^{(kp)}$, for the generator buses excluding the dependent bus (i.e. $i = 1, 2, 3, \ldots, (NG - 1)$)
Compute the generation of the dependent generator, $P_{g_{NG}}^{(kp+1)} = P_{load} - \sum_{i=1}^{NG-1} P_{gi}^{(kp+1)}$
B

```
       Bus no.   Economic generation    lambda
       --------  --------------------   ------
          1             .394100        269.584000
          2             .407200        269.584000
          3             .498700        269.583900

All the powers generations are in p.u.
Lambda is in unit of cost/p.u.power.hr.
Total cost of generation
=========================
       Bus no.          Cost(unit of cost/hr)
      ---------        ----------------------
          1                  162.605300
          2                  141.535300
          3                  174.649200

Total cost is 190.855000 unit of cost/hr.
```

EXERCISES

1. The fuel cost characteristics of the two generators of a system are as under:

$$F_{c_1}(P_{g_1}) = 50P_{g_1}^2 + 200P_{g_1} + 100 \text{ unit of cost/hr}$$

$$F_{c_2}(P_{g_2}) = 90P_{g_2}^2 + 120P_{g_2} + 150 \text{ unit of cost/hr}$$

$$F_{c_2}(P_{g_3}) = 70P_{g_3}^2 + 170P_{g_3} + 50 \text{ unit of cost/hr}$$

where P_{g_1}, P_{g_2} and P_{g_3} are in p.u. with 100 MVA base.

Determine economic generation schedule to supply a load of 1.65 p.u. using Newton–Raphson method as well as Approximate Newton-Raphson method. The [B] matrix of the system is given below:

$$[B] = \begin{bmatrix} 0.006 & 0.0002 & 0.0002 & 0.0008 \\ 0.0002 & 0.003 & -0.0001 & 0.0006 \\ 0.0002 & -0.0001 & 0.004 & 0.0005 \\ 0.0008 & 0.0006 & 0.0005 & 0.000002 \end{bmatrix} \text{p.u.}$$

2. Consider the six-bus, seven-line power system shown in Fig. E6.8. Perform the following computations:
   (*i*) find out economic generation schedule using exact loss formula,
   (*ii*) find out economic generation schedule using loss formula which is a function of real and reactive power.

```
3,4,(0.02,0.2),(0.0,0.010)
1,3,(0.04,0.2),(0.0,0.010)
```

[Y_{Bus}] of the system:

NYPROB3.DAT

4	
(1.398220,-8.062800) Y_{11}	(-4.366812E-01,3.275109) Y_{12}
(-9.615384E-01,4.807692)	(0.000000E+00,0.000000E+00)
(-4.366812E-01,3.275109)	(1.170177,-8.145084)
(-7.334963E-01,4.889976)	(0.000000E+00,0.000000E+00)
(-9.615384E-01,4.807692)	(-7.334963E-01,4.889976)
(2.190084,-14.618160)	(-4.950495E-01,4.950495)
(0.000000E+00,0.000000E+00)	(0.000000E+00,0.000000E+00)
(-4.950495E-01,4.950495)	(4.950495E-01,-4.940495) Y_{44}

Bus data and generator fuel cost data: NPROB3.DAT

3,4	[No. of generator buses, total no. of buses]
120,175,75	[α, β, γ for generator-1]
1.04,0,0.2,0	[$V_1^0, \delta_1^0, P_{load_1}, Q_{load_1}$]
110,180,50	[α, β, γ for generator-2]
1.01,0,0.15,0.05	[$V_2^0, \delta_2^0, P_{load_2}, Q_{load_2}$]
140,110,80	[α, β, γ for generator-3]
1.02,0,0.15,0.05	[$V_3^0, \delta_3^0, P_{load_3}, Q_{load_3}$]
1.0,0,0.25,0.1	[$V_4^0, \delta_4^0, P_{load_4}, Q_{load_4}$]

Output of computer program ELDPQ.FOR after execution: EPQS2.DAT

```
Iteration completed at kp = 2
Iteration completed at k = 2
Tolerance in P = -.000845  Tolerance in Q = .006577
Epsilon = .010000
Lambda for active power (unit of cost/p.u.power.hr) = 227.441800
Lambda for reactive power(unit of cost/p.u.power.hr) = -.087891
```

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Role of central control centre (CCC): Network management and decision-making.

Role of regional control centre (RCC) (or Regional Load Dispatch Centre): Implementation of CCC directive and control of HV transmission and sub-transmission network directly. It also controls captive power generation. It is responsible for directing LCC as per the guidelines of CCC and to govern distribution network.

Role of local control centre (LCC): To control generation as per directives. It also has control on sub-transmission network.

7.3 PLANNING OBJECTIVE

For controlling a power system properly, it is very much essential to plan the system operation with proper scheduling objectives. The scheduling may include short-term as well as long-term scheduling. In short-term scheduling the program may be for hourly interval for 24 hr period, while for long-term scheduling the schedule of interval is for months or on a weekly basis. The chief aim of the scheduling is to find the optimal mix in hydro-thermal generation and to decide upon the exchanges with the neighbouring systems.

The planning objective includes the following:

(i) Selection of generating units to serve the normal load demand and to serve during emergency
(ii) Anticipated load demand
(iii) Power system network structure to cope with the demand and available generation
(iv) Evaluation of possible contingencies and schemes to tackle them.

7.4 FUNCTIONS OF CONTROL CENTRES

Control centres (or load dispatch centres) perform a wide variety of functions under three broad categories:

(i) Planning
(ii) Monitoring of the interconnected system
(iii) Data acquisition and system control.

7.4.1 Planning

The most important planning aspect of the control centre is load forecasting and generation scheduling. Usually, short-term load forecasting is preferred where the forecasting is done on a daily basis. The forecasting mechanism ensures the prediction of the load curve as accurately as possible with reference to the load curves of the previous day, the corresponding day in the previous week and year and also taking into account weather conditions. A forecasting program includes some allowances in order to cope with special events, if any. An attempt is made to make forecasting method effective by incorporating the objective of economic operation in the forecasting program.

In some forecasting programs, sometimes the skill of a human operator and his/her experience is included, but usually load forecasting is attempted following automatic computer algorithms based on extrapolation methods. Implementation of forecasting programs based on artificial neural networks (ANN) is more effective and takes into account the role of weather, labour problem possibilities, disputes and contingencies.

The next planning aspect is to determine the power reserve in the system. A certain amount of power reserve is usually maintained in the interconnected system in excess of the load demand in order to provide the scope for maintenance of units and to have some spare capacity to meet minor contingencies. It also covers the errors in the estimation of the load demands. However, the capacity of reserve depends on the number and size of the generating units, maintenance requirements, characteristics of plants,

7.8.1 Power Line Carrier Communication (PLCC)

PLCC is frequently used for communication over medium and long distance power transmission networks. It is very much economical and a reliable method of communication. Figure 7.2 represents a simple scheme PLCC system. Since telephonic and data communication system hardwares operate at low dc voltage, it cannot be directly connected to the HV or EHV power line. Suitable coupling devices (coupling capacitors, matching units) are used for the purpose. Radio frequency data/ voice are prevented to enter the sub-station bus and are directed towards the carrier equipment. On the other hand, the power frequency voltage or current is allowed to enter the station bus while they are prevented to enter the carrier equipment. Wave trap and coupling capacitor are employed to achieve these objectives. Since any standard power system textbook at undergraduate level describes PLCC, we are not going into the technical details of PLCC here.

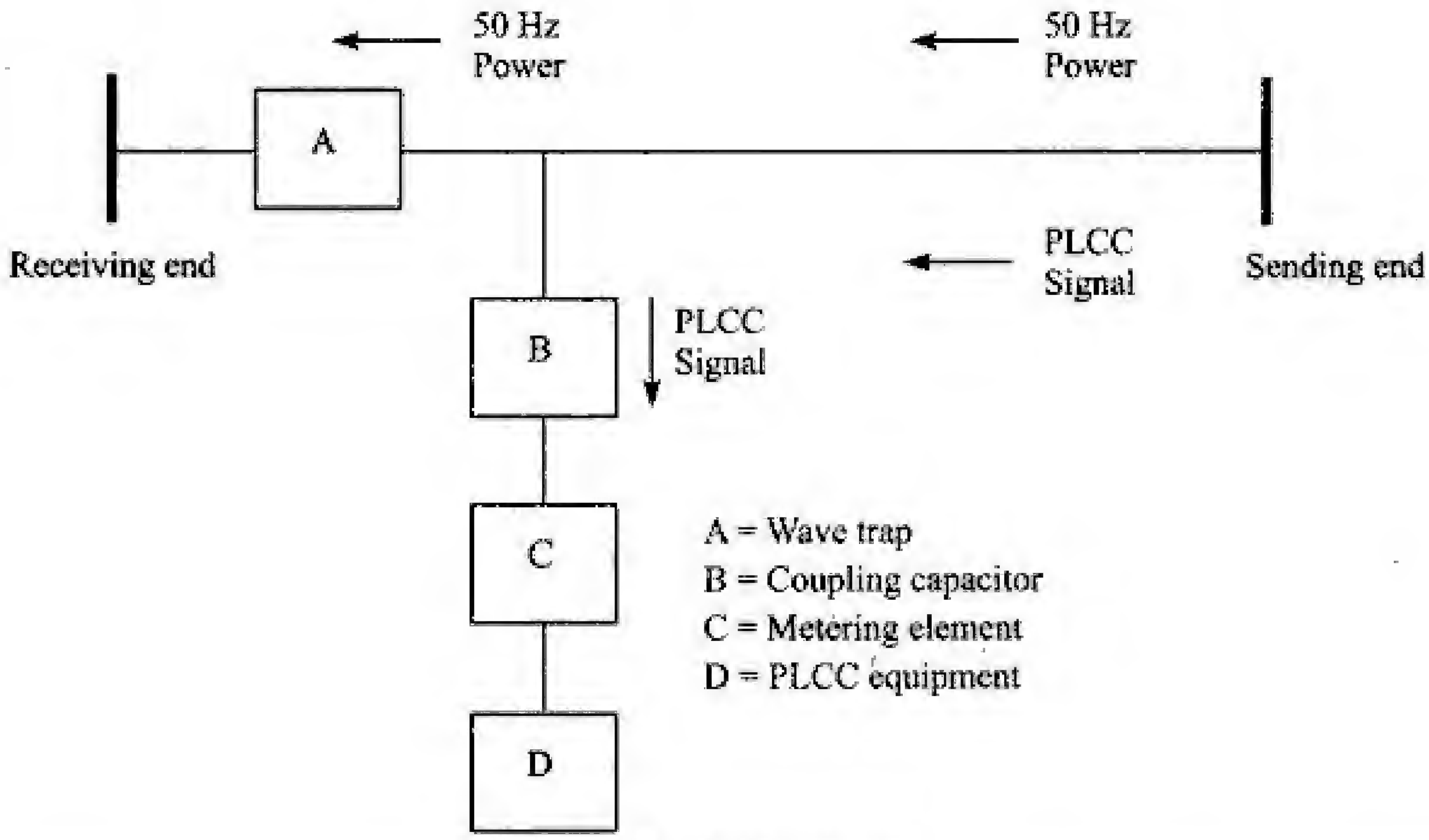


Fig. 7.2 PLCC scheme.

In PLCC communication, the carrier wave is generally transmitted with two side bands. This enables the use of very simple equipment but offers broad band transmission facility. Single side band transmission is also seen in installations above 132 V. Though the frequency band reduces in it, the signal/noise ratio increases in it.

Usually, the carrier frequencies are in the range of 50–300 Hz while the audio bandwidth of the PLCC is 200–3400 Hz. This includes the bandwidth for data transmission in the range of 2000–3400 Hz. The characteristic impedance of a long transmission power line being around 400 ohms, the signal attenuation is in the order of 0.1 dB per km for a 220 kV line. The signal frequency selection is largely governed by the condition that PLCC signal should not interfere with radio broadcasting or telephone systems. Also, it is to be noted that both very low and very high frequencies are not recommended as very low frequency will increase the size of coupling capacitor, while very high frequency will increase radiation loss and may create interference with other communication channels.

Noise in high voltage lines is created due to corona discharge and may affect signal/noise ratio. Necessary electronic hardwares are regained to be installed to maintain proper signal/noise ratio for PLCC applications in HV systems. PLCC is very much suitable for application in case of lines connecting

with the input signal. An analog transmitter converts this dc signal into a variable frequency output signal. This variable frequency signal is transmitted to the analog receiver at the other end of the system. The receiver at the other end converts the varying frequency signal back to a dc signal giving the replica of the original signal and is displayed/recorded.

The analog telemetry is a continuous signalling process while the digital process of telemetry is mainly based on sampling techniques of the desired quantity to be telemetered at fixed intervals. The digital telemetry is reliable and very precise. In this system the input is first converted into an equivalent dc current signal and then using analog-digital converters, this dc signal is converted to digital outputs. This digital form of data is then transmitted and at the receiving end using digital to analog converter, it is reconverted to analog value for indication or recording. In a number of schemes the central station scans the local control centres serially and sequentially and receives the telemetered data. The telemetry channel may be telephone wire, microwave links or carrier links.

7.10 EMERGENCY CONTROL

Emergency control is a closed loop control that automatically actuates load shedding (or generation reduction) in order to cope with overloads (or underloads) to restore generation load balance. The main purpose of emergency control is to relieve overloads in transmission lines and transformers and to ensure that throughout the system the generation is equal to the sum of load and loss. Emergency control prevents disruption of normal grid operation and deterioration of quality of power supply.

The emergency control may be automatic or manual on the assessment of the situation by the operator. Emergency control may be needed when the system frequency declines in a system because of tripping of transmission lines or generators. In case of sudden reactive power deficits like tripping of compensators or generators, emergency control is also needed.

Normally, the frequency variation up to ± 0.5% may be tolerated before any emergency control is actuated. On the basis of the same concept, a voltage change of ± 3% in the transmission sector and ± 5% in the sub-transmission sector may be tolerated. Once the system frequency declines beyond ± 0.5%, hydro plants are pressed into service to give their peak output and pumped storage plants may be started. In case of frequency overshoot beyond 0.5%, hydro plants may be asked to reduce their generation. In case the frequency fall is beyond 0.1%, automatic start-up of some quick start generator (gas turbine) is initiated. Beyond a frequency decline of 0.3%, load shedding is restored. Usually, the amount of load to be shed is 10% of the peak demand. With further decline, the magnitude of load to be shed may be increased to 15% or higher.

For accelerated load shedding in a graded load shedding, scheme rate of frequency change relays (df/dt) may be utilised. Still the (df/dt) value is not very high, load shedding can be continued. These relays are used to actuate stage by stage load shedding and to isolate a healthy system from an affected system in case of sharp frequency decline in a short span of time. However, inadvertent operation of such a relay may create problem of unwanted load shedding. Hence the maximum of load which can be connected to any (df/dt) relay is generally limited to a specific value. For any (df/dt) relay operation, operator inspection is a must.

For effective emergency control, the following philosophy is usually adopted:

(i) Load shedding is usually spread throughout the system.
(ii) Load shedding is initiated in steps.
(iii) Attempts are to be made at regular intervals to restore the system once the corrective measures are taken
(iv) Load shedding is automatically carried out by present relays and is a function of frequency and time.

(df/dt) relay scheme is superior in the concept that it has the advantage of anticipating the shedding action and has a clear effect for faster restoration of power balance. It is possible to design a load shedding scheme that would give adequate load relief during emergencies.

The setting of (df/dt) relay is very important and it should be done after a prolonged negotiation with the different users. A number of steps are to be taken taking into account the convenience of the consumers. Attempts are made so that the inconvenience caused by the consumers is minimum. (df/dt) relay is installed at points of interconnection such that if the frequency declines below 5%, the generating units are separated from the network automatically.

The main role of (df/dt) being the detection of rate of decline of frequency, its another aim should be helping the operator indirectly to restore loads and bring the system to normal state.

The best way to maintain system voltage is to keep primary regulation low. In addition to this, it is very much recommended that reactive compensators are installed at strategic locations in the network. In case of voltage decline beyond 3% in the HV transmission network (primary network), under voltage relays become active and load shedding may be resorted to. In case the receiving end voltage declines further, it is then recommended to shed some major loads at the receiving end.

Secondary voltage regulation is adopted in some systems. This scheme involves location of some strategic nodes in the network. This strategic node acts as the representative of the surrounding network. The actual voltage at the strategic node is measured and compared to the scheduled value. The difference is used to initiate a voltage regulator that will correct the voltage to nominal value at the strategic location.

Once the system blackout (condition of partial or total de-energisation) occurs, the restoration procedure should start. In a number of systems in developing countries, the restoration is mostly based on the experience of the operators and engineers and fully guided by circumstances and human skill. However, attempts are nowadays being made to implement the restoration procedure through computerised schemes due to vastness of the network. The restoration procedure is normally based on the strategy of re-energising the network gradually starting the quick start units and hydro-stations first and then thermal units.

When the power system gets separated from the generating units, the various breakers normally remain in the closed or in open position as they were before blackout. Thus, it is dangerous to attempt re-energisation of the network as a whole simultaneously. On the other hand, the restoration should start through pre-drawn restoration routes. Only the breakers in the restoration route should be in closed position and other breakers should be in open position. On successful restoration of one route, the next route should be attempted. Supervision of the synchronisation of the routes must be ensured and the generation is to be adjusted so that the frequency and voltage remains within prescribed limits following restoration of routes. Proper feedback is also necessary to guide the dispatcher for restarting of the generator and to apply load of the restored route gradually. The first attempt is restoration of the network and then reconstituting the larger part of the network.

EXERCISES

1. What are the main functions of a control centre? What are the roles of RCC, CCC and LCC?
2. How does a control centre function?
3. Why is the location of a control centre important?
4. What are the control facilities available in the control centre?
5. What are the different power system communication modes? What is PLCC? Describe it briefly.

Chapter 8

AUTOMATIC GENERATION CONTROL

8.1 INTRODUCTION

Electric energy is required to be supplied to the consumers within permissible limits of voltage and frequency around the rated values. As the demand changes, the system voltage and frequency deviate from the initial values causing an unpredictable, small amount of change in the state of the system. An automatic control system is assigned to detect the change and it initiates a set of counter-control actions in order to nullify effectively and at the earliest any deviation in the state of the system. Large deviations also affect the transmission and distribution of electrical energy due to the negative effect on regulation, power balance and stability. Also, in any interconnected system, deviation of the state of the system may well disturb the state of economic operation and may even cause overloads on the interconnecting ties, with the risk of having lost the continuity of operation. The obvious way to maintain a perfect power balance at each bus would be to continuously keep the generated powers (P_{gi} and Q_{gi}) in *balance* with the changing load powers P_D and Q_D. This, then, would maintain all ΔP_i and ΔQ_i at zero levels and thus all busbar voltages and line powers at constant values. However, this is neither exactly desirable nor possible. It is not desirable because constant line powers would defeat the real purpose of transmission lines, which is to make possible (at each moment) the most economical transfer of power. Neither would it be possible since most buses lack both real and reactive power sources.

The most important and bulk power source is the synchronous generator (alternator). But usually generators are available only at less than five per cent of the network buses. At these generator buses, both P_i and Q_i can be controlled. The real power is controlled through the turbine torque, while the reactive power is controlled via the exciter field winding.

Generators obviously represent the most important means for system control. Automatic control of generators involves two major control loops with which the large generators in the power system are equipped. These two major loops have been termed as *Automatic Voltage Regulator* (AVR) loop and *Automatic Load Frequency Control* (ALFC) loop [Fig. 8.1]. *Automatic Generation Control* (AGC) is a centralised operation that operates in real time and in closed-loop with a strong interface to other functions oriented towards the economy and security of the power system. AGC was operated with analog controls for a number of years and digital versions came later. Both the systems perform well.

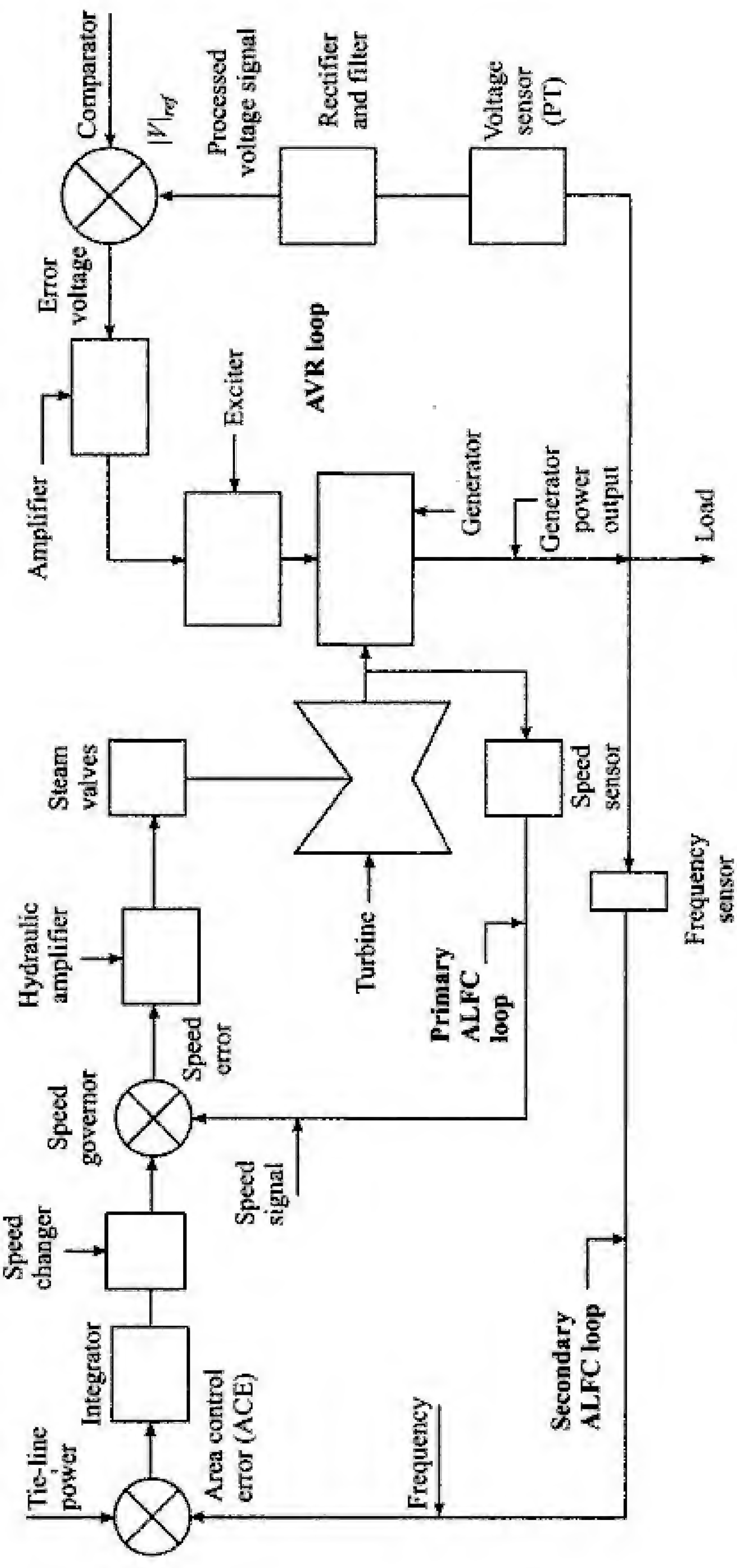


Fig. 8.1 AVR and ALFC loops of large generators.
[ALFC-Automatic load frequency control; AVR-Automatic voltage regulator; PT-Potential transformer]

AVR loop is assigned to control the magnitude of the terminal voltage (V) of the generator, which, in turn, maintains bus voltage manipulating the reactive power output. The process involves continuous sensing of the terminal voltage, its rectification, smoothening and comparison with a preset dc reference ($|V|_{ref}$). Then this compared result "error voltage", after amplification and shaping, is used to control the alternator field excitation.

The ALFC loop, in turn, regulates the real power output and the corresponding frequency of the generator power output. The primary ALFC loop senses the turbine speed and controls the operation of the control valves of turbine power input via the speed governor. This loop is relatively faster than the secondary ALFC loop which senses the electrical frequency of the generator output and maintains proper power interchange with the interconnections. This loop is slower in response and is insensitive to rapid load and frequency changes. Usually, the primary ALFC loop operates in order of seconds while the secondary ALFC loop operates in order of minutes. All these loops are designed to operate around normal state with small variable excursions. Thus, the loops may be modelled with linear, constant coefficient differential equations and represented with linear transfer functions.

8.2 TYPES OF ALTERNATOR EXCITERS

Depending upon the size and type of generators, excitation systems come in several different models. Here, we briefly point out the important features of the excitation system where the most important is the AVR loop. Since the exciter of the alternator is the main component in the AVR loop, it is important to have a brief discussion about the types of exciter before proceeding to model the exciter.

8.2.1 Primitive Type Exciters

In old type of slow response exciters, the exciter was basically a dc generator being driven by the alternator shaft (Fig. 8.2). This scheme required the dc power to be injected to the field of the alternator through slip rings and brushes.

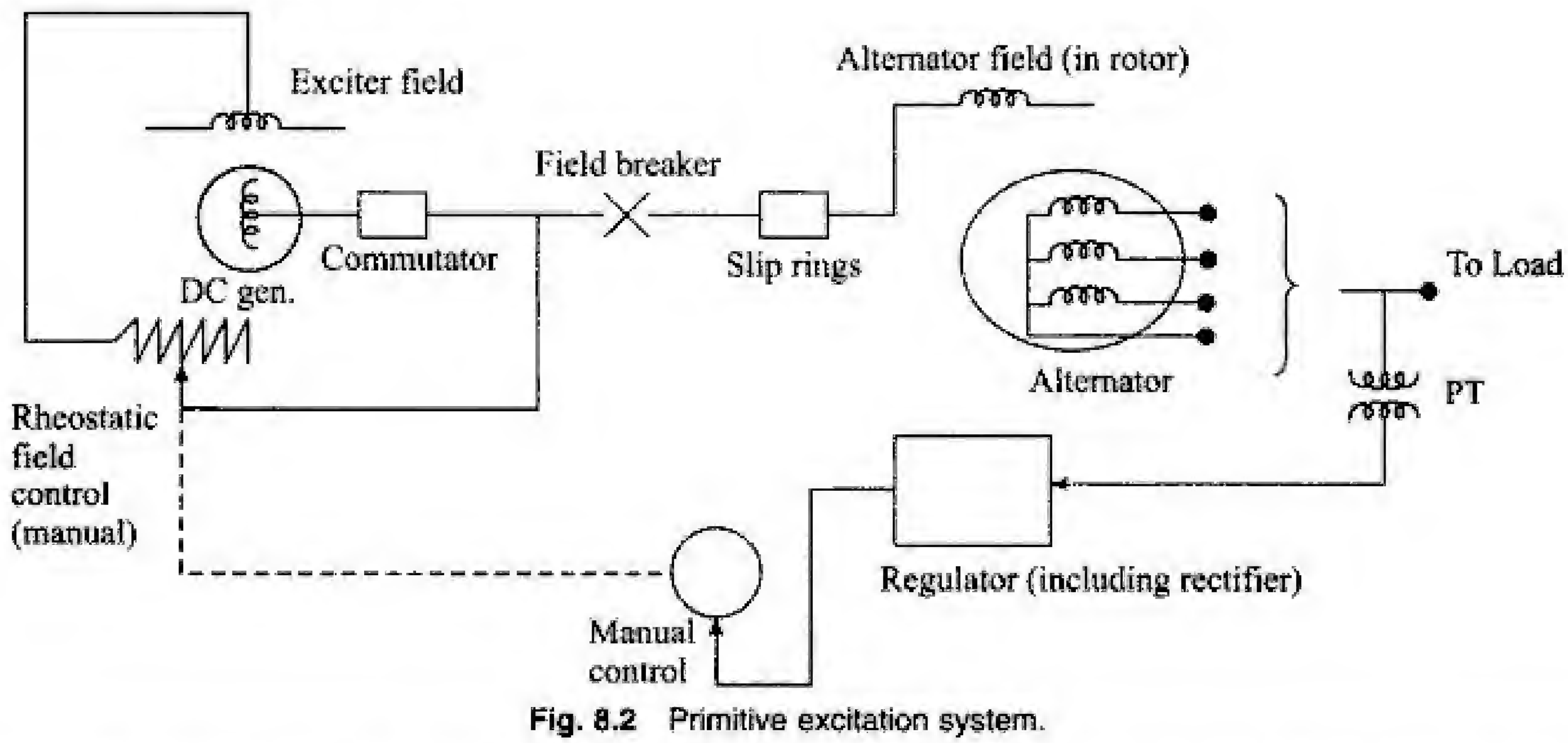


Fig. 8.2 Primitive excitation system.

The regulator, in this case, detects the voltage level and includes a mechanical element to alter the control of the rheostatic resistance of the exciter field. An increase in generator output voltage would cause an increase in the dc voltage from the rectifier (included in the regulator). It causes an increase in current through the regulator coil that mechanically (or manually) inserts resistance in the exciter field

circuit. This, then, reduces the excitation flux thereby lowering the dc generator output voltage. Thus the alternator excitation being decreased, alternator output voltage reduces. This system was slow in response because of manual intervention in the exciter control.

8.2.2 Modern Exciters

(a) Pilot-main exciter system

The next level development was the inclusion of pilot exciter along with the main exciter (Fig. 8.3). This system responses much faster than the self-excited main exciter described above because of lesser intervention by human operators. The pilot exciter supplies the field voltage of the main exciter which operates with the same philosophy as described above. The excitation of the pilot exciter could be obtained from a separate source.

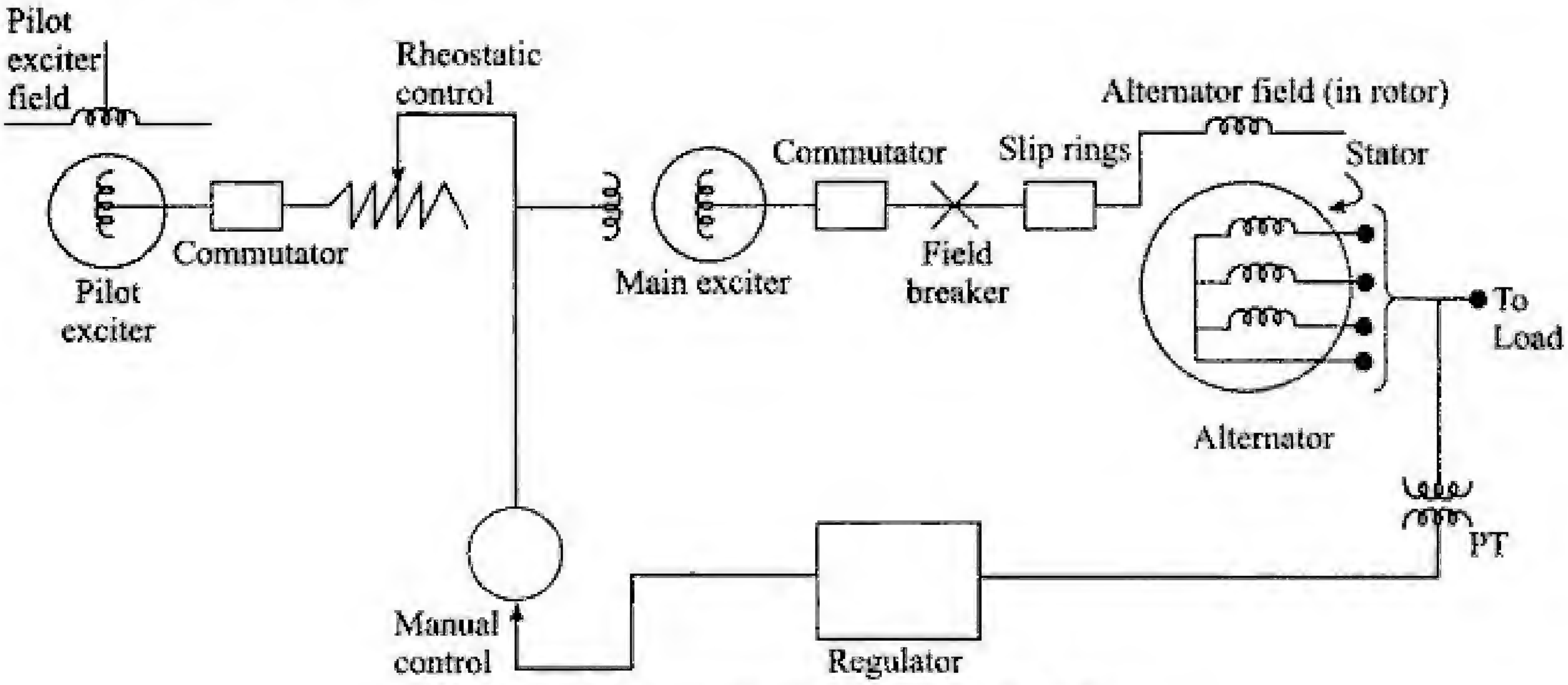


Fig. 8.3 Main exciter and pilot exciter scheme.

(b) Brushless exciter

With the passage of time, generators have become larger, interconnected system operation has become common, and the excitation control systems have become more complex. Static or brushless excitation systems were developed with the advent of solidstate devices. A typical brushless excitation system with a ac exciter has been shown in Fig. 8.4. Here the ac armature voltage is rectified in diodes mounted on the rotating shaft and is then fed directly in the alternator field.

(c) SCR controlled static exciter

Further development in the exciter design led to SCR (*Silicon Controlled Rectifier or Thyristor*) controlled static excitation system (Fig. 8.5). In this system the alternator output is rectified and injected to the alternator field by means of slip rings. This shunt excitation system completely disposes the presence of rotating exciter and the rectifier is usually a full wave SCR bridge circuit. The alternator excitation can thus be easily controlled by varying the SCR triggering angles. The response is obviously the fastest of all the exciters.

In these types of exciters, an additional hardware is included (*series compounding feature*). This helps to retain the voltage in the field of the alternator in case the alternator faces a short circuit in the bus side and alternator voltage starts collapsing. The series compounding feature helps in keeping the excitation, thus maintaining output voltage at the alternator and helping the important voltage operated relays to act.

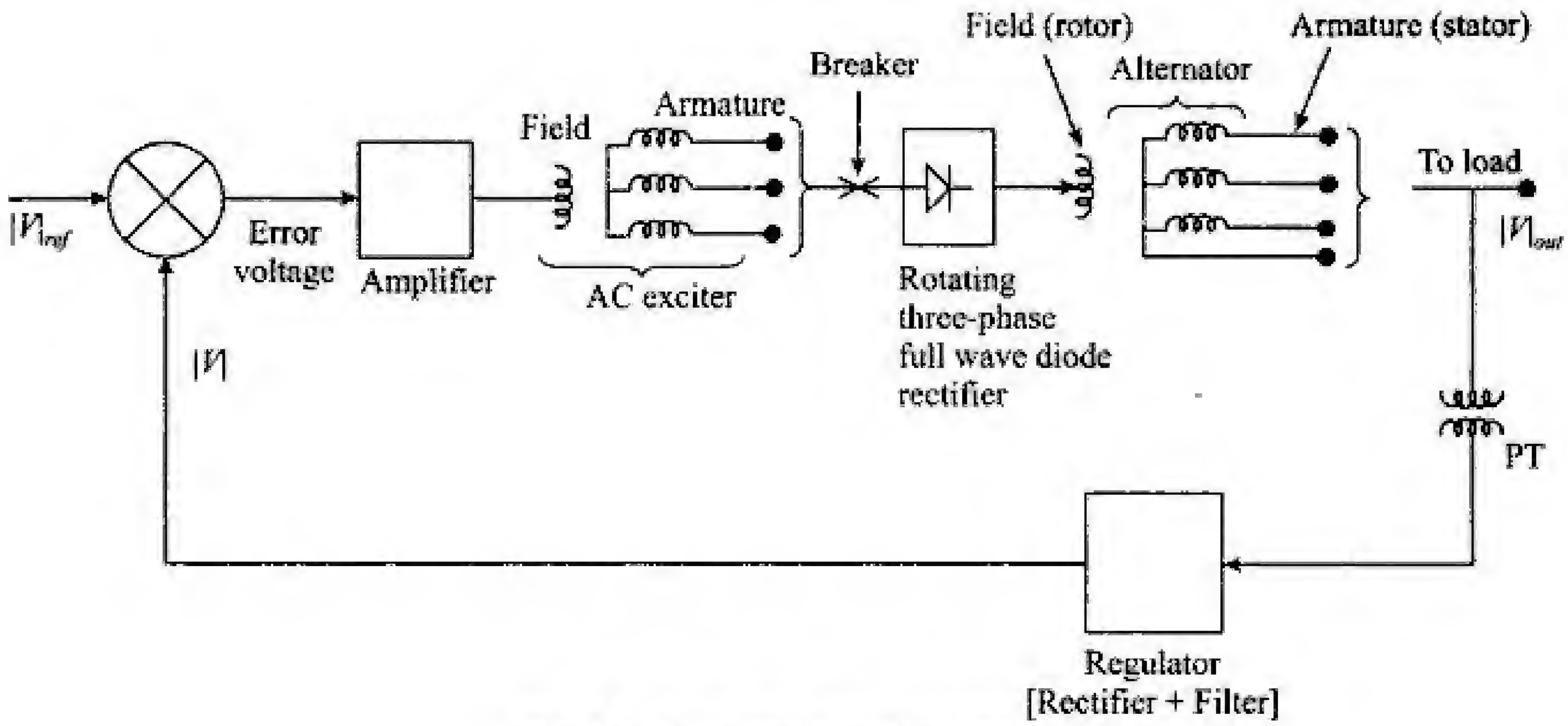


Fig. 8.4 Brushless excitation system.

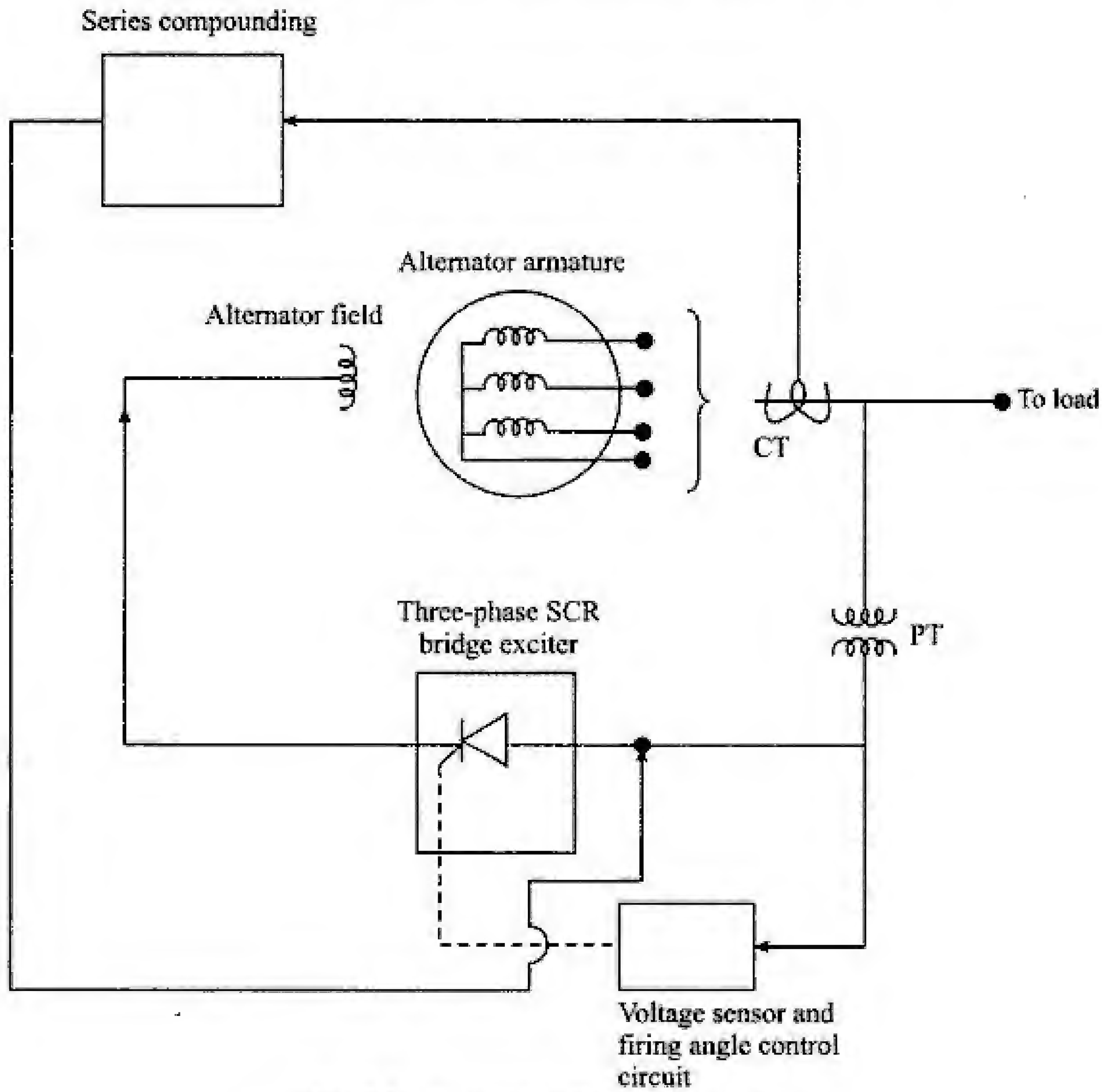


Fig. 8.5 Complete static excitation system.

8.3 EXCITER MODELLING

Let us assume a modern static excitation system where the bus voltage is measured by a potential transformer (PT). After rectification and filtering, the output is compared with a reference. The resulting error voltage, after amplification, serves as input to an exciter, which feeds directly into the generator field. A drop in the terminal voltage causes a boost in the field current. This increases the reactive power output of the machine thus tending to offset the initiating voltage drop. Denoting

$|V|_{ref_c}$ = dc reference voltage in the exciter control loop

$|V|$ = magnitude of the proportionally scaled down rectified terminal voltage of the alternator

e_c = error voltage (output of comparator)

v_{ex} = exciter internal voltage

i_{ex} = exciter current

v_f = excitation voltage of the alternator

R_{ex} = resistance of the exciter circuit

L_{ex} = inductance of the exciter circuit

for any small perturbation in the output voltage of the alternator, there is a corresponding change of the exciter internal voltage. At any steady state condition,

$$\Delta v_{ex} = R_{ex}\Delta i_{ex} + L_{ex}\frac{d}{dt}(\Delta i_{ex}) \tag{8.1}$$

and $\Delta v_f \propto \Delta i_{ex}$ (assuming no saturation)

or,
$$\Delta v_f = K_f \Delta i_{ex} \tag{8.2}$$

K_f is a constant denoting the dc armature voltage per unit field current.

Laplace transformation of equations (8.1) and (8.2) gives

$$\Delta V_{ex}(s) = R_{ex}\,\Delta I_{ex}(s) + sL_{ex}[\Delta I_{ex}(s)]$$

and
$$\Delta V_f(s) = K_f \Delta I_{ex}(s)$$

Elimination of $\Delta I_{ex}(s)$ yields

$$\frac{\Delta V_f(s)}{\Delta V_{ex}(s)} = \frac{K_{ex}}{1+sT_{ex}} (= G_{ex}(s))$$

where $K_{ex} = (K_f / R_{ex})$, K_{ex} being the exciter gain and $T_{ex} = (L_{ex} / R_{ex})$; T_{ex} is exciter time constant and ranges from 0.5 to 1.0 sec. G_{ex} denotes the exciter transfer function.

In order to model the exciter control circuit, we first write down the loop (consisting of the comparator and the amplifier) equations (8.3 and 8.4)

$$\Delta|V|_{ref_c} - \Delta|V| = \Delta e_c \tag{8.3}$$

and $\Delta v_{ex} \propto \Delta e_c$ (assuming no saturation)

giving,
$$\Delta v_{ex} = K_A \Delta e_c \tag{8.4}$$

(K_A being the gain of the amplifier).

Laplace transformation of these two equations yields

$$\Delta|V|_{ref_c}(s) - \Delta|V|(s) = \Delta e_c(s)$$

and
$$\Delta V_{ex}(s) = K_A \Delta e_c(s)$$

Amplifier transfer function is then given by

$$G_A(s) = \frac{\Delta V_{ex}(s)}{\Delta e_c(s)} = K_A \tag{8.5}$$

Though the amplifier response shown here to be instantaneous, in practice, the time response of amplifier being T_A (ranging from 0.02–0.08 sec); the amplifier transfer function is given by

$$G_A(s) = \frac{K_A}{1 + sT_A} = \frac{\Delta V_{ex}(s)}{\Delta e_c(s)} \tag{8.6}$$

8.4 MODELLING OF ALTERNATOR (SYNCHRONOUS GENERATOR)

The alternator field equation at steady state is given by

$$\Delta v_f = R_f \Delta i_f + L_f \frac{d}{dt}(\Delta i_f)$$

Without considering saturation for the alternator field, $i_f \propto E$, (the induced (*internal*) voltage of the alternator being E).

Thus,

$$\Delta v_f = (\text{constant}) \times \left[R_f \Delta E + L_f \frac{d}{dt}(\Delta E) \right]$$

Using Laplace transformation,

$$\frac{\Delta E(s)}{\Delta v_f(s)} = \frac{\dfrac{1}{R_f \times \text{constant}}}{1 + \dfrac{L_f}{R_f} s} = \frac{K_F}{1 + sT_F} = G_F(s) \tag{8.7}$$

where, $K_F = \dfrac{1}{R_f \times \text{constant}}$ and $T_F = \dfrac{L_f}{R_f}$.

The complete block diagram of the generator-exciter system gives birth to *AVR loop*, which is shown in Fig. 8.6(a).

The block diagram of the AVR loop, in final condensed form, is shown in Fig. 8.6(b) where

$$\frac{\Delta|V|}{\Delta|V|_{ref_c}} = \frac{G(s)}{1 + G(s)}$$

and
$$G(s) = \frac{K_A K_{ex} K_F}{(1 + sT_A)(1 + sT_{ex})(1 + sT_F)} \tag{8.8}$$

The AVR loop maintains reactive power balance at a generator bus by indirectly maintaining a constant voltage level. The generator bus can be considered *voltage controlled node* in the network.

With $s \to 0$ (for constant input condition),

$$\Delta e_c = \Delta|V|_{ref_c}\left[1 - \frac{G(0)}{1+G(0)}\right] = \frac{1}{1+G(0)}\Delta|V|_{ref_c}$$

$$= \frac{1}{1+g}\Delta|V|_{ref_c} = \frac{1}{1+K}\Delta|V|_{ref_c} \tag{8.11}$$

$[\because\ g \equiv G(0) = K/(1+0\times T) = K]$

It is now evident that greater the *loop gain* (K), lesser the static frequency error.

Comparing equations (8.9) and (8.11) we can write,

$$\frac{1}{1+K}\Delta|V|_{ref_c} < \frac{a_c}{100}\Delta|V|_{ref_c}\text{, i.e. } K > \frac{100}{a_c} - 1 \tag{8.12}$$

Example 8.1: Find the open loop gain of an AVR loop if the static error does not exceed 2%.

Solution: From the question, the maximum static accuracy error limit is 2%, i.e. $a_c = 2$.

$\therefore$ The open loop gain of the AVR loop is given by

$$K > \frac{100}{2} - 1 \quad \text{or} \quad K = 49$$

Thus, if the open loop gain of the AVR loop exceeds 49 then only it is possible to keep the static accuracy error limit within 2%.

8.6 DYNAMIC PERFORMANCE OF THE AVR LOOP

Since $$\frac{\Delta|V|(s)}{\Delta|V|_{ref}(s)} = \frac{G(s)}{1+G(s)}$$

$\therefore$ $$\Delta|V|(t) = L^{-1}\left[\Delta|V|_{ref}(s)\frac{G(s)}{1+G(s)}\right] \tag{8.13}$$

Usually, the AVR open loop transfer function *characteristic equation* $[1 + G(s) = 0]$ is of third order having three roots s_1, s_2 and s_3. If the roots are distinct and real, the *transient response components* are of the form $M_1\varepsilon^{s_1t}$, $M_2\varepsilon^{s_2t}$ and $M_3\varepsilon^{s_3t}$; where M_1, M_2 and M_3 are the *amplitude factors* expressing the relative size of the transient terms.

For the AVR loop to be stable, the transient terms should be dying out with time. This is possible if all the three roots are located on the *left-hand side of s-plane*. A high speed loop should possess the location of roots at the left half of the s-plane with dominating real values.

8.7 COMPENSATION IN AVR LOOP

Higher values of AVR loop gain K being preferred for static accuracy, this causes undesirable dynamic response and may lead to *instability* of the AVR loop.

In order to tackle the situation, let a *series compensator* (phase lead compensator) be added in cascade to the final AVR loop. Let the transfer function of the phase lead compensator be given by,

$$G_p(s) = (1 + sT_c)$$

∴ The open loop transfer function of the AVR loop then becomes

$$G(s) = \frac{K(1+sT_c)}{(1+sT_A)(1+sT_{ex})(1+sT_F)} \tag{8.14}$$

It becomes evident that the transfer function now consists of a zero. This gives better dynamic response. Further, with $T_c = T_{ex}$, the open loop transfer function becomes

$$G(s) = \frac{K}{(1+sT_A)(1+sT_F)} \tag{8.14a}$$

This makes the two roots only for $G(s)$. Situation of the two poles at the left half of the s-plane with real values improves the stability to a considerable extent.

8.8 AUTOMATIC LOAD FREQUENCY CONTROL (ALFC)

Actually, the ALFC loop has been bifurcated into two loops—*primary ALFC loop* and *secondary ALFC loop*. The purpose of both these loops is to achieve real power balance in the system. Just as AVR loop achieves Q-balance by maintaining a constant voltage, the ALFC loop achieves power balance by maintaining constant frequency. However, there is an important difference between the two—the AVR loop is able to maintain perfect Q-balance only at those buses which are voltage controlled. The ALFC loops maintain primarily P-balance at the generator buses, but because the frequency is the same throughout the system, they thus in totality achieve overall P-balance in the system.

The chief objective of the ALFC (known as automatic load frequency control) is to maintain the desired real power output of a synchronous generator unit and to assist the process of frequency control of the interconnected network. ALFC also helps in meeting the specified power changes among the members of interconnection. ALFC loop is functional only during small and slow changes in load and frequency. It is inadequate to supply or control real power during transient instability conditions.

In a typical thermal power plant, the primary ALFC loop uses the *speed governing system* to execute the desired control on the MW output of the generators. The schematic of such a system being shown in Fig. 8.7, the positive displacements of the respective 'linkings' have been marked by *arrows*.

Logic of operation: (Fig. 8.7)

To raise the steam entry, speed changer setting is raised, 'A' lowers, 'C' and 'D' raises, and subsequently 'E' goes down. High pressure oil enters through upper opening of *main* piston, which, then, moves downwards. This opens main control valve. To reduce the steam entry, speed changer is lowered and reverse operation takes place.

Once the turbine speed increases, balls of the speed governor B_1 and B_2 move outer, 'B' lowers, lowering point 'D'. High pressure oil enters through lower opening raising the main piston, closing the steam valve. This reduces the steam entry. The downward movement of linkage point 'E' results in an increment in the control valve power given by ΔP_{cv} which is then transferred to increment of turbine power ΔP_T. This increase being caused by change in ΔP_{ref} (say) initiated by change in reference power setting, it is evident that an increase in generated output power ΔP_e results from an increase in ΔP_{ref} and a decrease in Δf; ΔP_e is the increment in generator power output while Δf is the incremental change in the system frequency being characterised here by change of speed. Thus for small increments,

$$\Delta P_e = \Delta P_{ref} - \frac{1}{r}\Delta f \text{ (in MW)} \tag{8.15}$$

[negative sign for Δf appears to represent frequency drop];

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

or, $$\Delta f_c = -r\Delta P_{T_c} = -\left(r_{p.u.}\frac{f_{rated}}{P_{g_{rated}}}\right)\Delta P_{T_c} \tag{8.22a}$$

i.e. for a fixed setting of the speed changer, the static increase in the turbine power output is directly proportional to the steady state drop in frequency; also from equation (8.22), $r = -(\Delta f_c/\Delta P_{T_c})$ (i.e. the *regulation* in this case is the ratio of drop of frequency to the change of turbine power setting).

Let us now suppose that a unit is supplying output power P_{g_1} at frequency f_1 when the load increases to P_{g_2} $(P_{g_1} + \Delta P_g)$. As the speed of the unit decreases, the speed governor allows more steam from the boiler (or water for hydro units) to the turbine to arrest the decline in speed. Equilibrium between input and output power occurs at the new frequency $f\ (= f_1 + \Delta f)$. For the fixed setting of reference power, we then write

$$\Delta f = -r\Delta P_g = -r\Delta P_T$$

or, $$\Delta f = -r_{p.u.} \times \frac{f_{rated}}{P_{rated}}\Delta P_T \text{ Hz} \tag{8.23}$$

The isolated unit would continue to operate at the reduced frequency except for the supplementary controls of the speed governor.

Next, we illustrate the concept—it is evident that 5% regulation means that the turbine power P_T will increase 1 p.u. (or 100 MW) for a drop in frequency of 0.05 p.u.

($\because$ $\Delta P_{T_c} = \frac{-\Delta f_c}{r} = \frac{0.05}{5}\times 100 = 100$ MW, or, 2.5 Hz for a 50 Hz power system and for a 100 MW alternator).

$\therefore$ $$r = \frac{2.5}{100} = 0.025 \text{ Hz/MW } [\Delta f_c = -2.5 \text{ Hz}]$$

Thus, if there is a change in frequency of $\Delta f = -0.2$ Hz, the change in the turbine power would be

$$\Delta P_{T_c} = -\frac{1}{r}\Delta f_c = -\frac{1}{0.025}(-0.2) = 8.0 \text{ MW}$$

Next, we take the change in reference power setting as well as change in frequency. The change in turbine power output is then given by (equation 8.21).

$$\Delta P_{T_c} = \Delta P_{ref_c} - \frac{1}{r}\Delta f_c$$

Figure 8.12 represents the family of sloping straight lines. Each line corresponds to a fixed reference power setting.

In this context, it may also be noted that following the drop in frequency due to increase in system load, the reference setting must be lowered provided one wishes to keep the turbine output unchanged $(\Delta P_{T_c} = 0)$.

Thus, from $$\Delta P_{T_c} = \Delta P_{ref_c} - \frac{1}{r}\Delta f_c,$$

for $$\Delta P_{T_c} = 0,\ \Delta P_{ref_c} = \frac{1}{r}\Delta f_c \tag{8.24}$$

or, $$\left(\Delta P_T - \Delta P_D\right)\text{p.u.} = \left[\frac{2H}{f_o}\frac{d}{dt}(\Delta f) + d\Delta f\right]\text{p.u.} \tag{8.30}$$

$$\left[\text{obviously, } H = \frac{E_{kin_{(o)}}}{P_g}\right]$$

Here, ΔP_T, ΔP_D are measured in p.u. on the generator rating while (d) is measured in p.u. MW/Hz. H, the inertia constant, is measured in seconds.

Laplace transformation of equation (8.30) yields

$$\Delta P_T(s) - \Delta P_D(s) = \frac{2H}{f_o}s\,\Delta f(s) + d\Delta f(s)$$

or, $$\Delta f(s) = G_p(s)\left[\Delta P_T(s) - \Delta P_D(s)\right] \tag{8.31}$$

where $$G_p(s) = \frac{K_p}{1+sT_p};\ T_p = \frac{2H}{f_o d};\ K_p = \frac{1}{d}$$

T_p is in seconds and K_p in Hz/p.u. MW. The 'open' ALFC loop can now be completed (Fig. 8.13) using equation (8.31). T_p and K_p are the ALFC loop parameters.

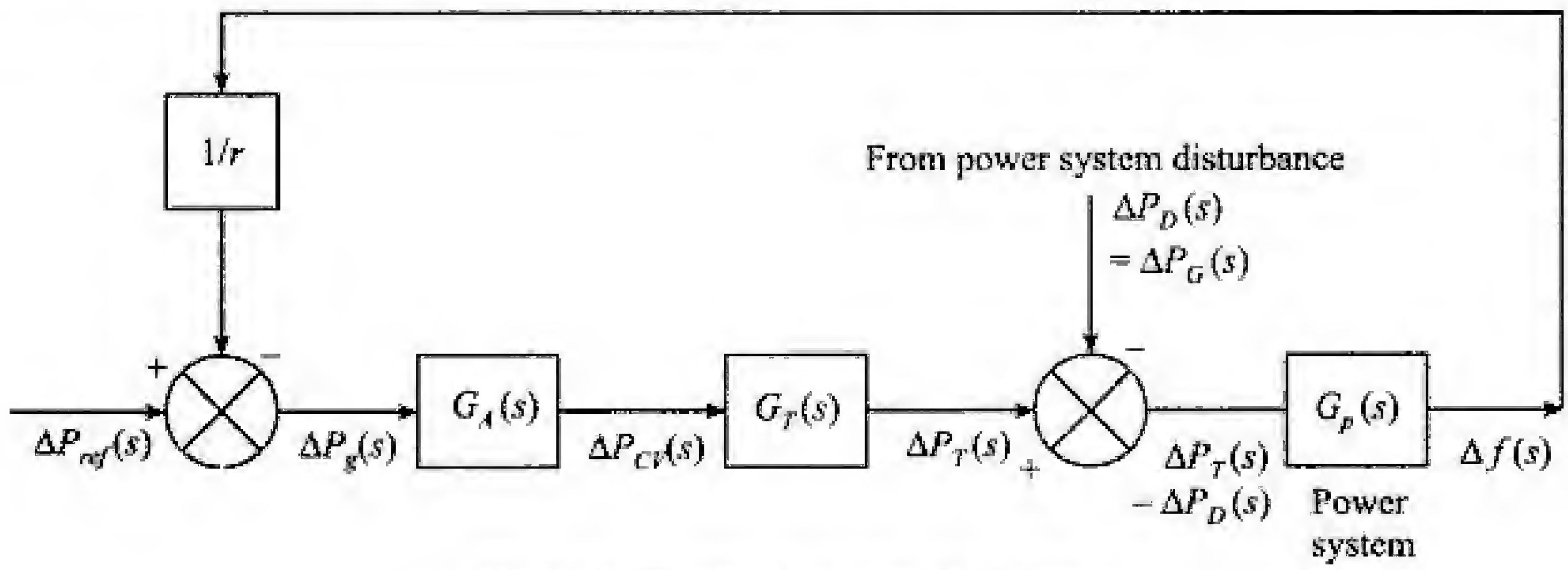


Fig. 8.13 Closing of primary ALFC loop.

8.12 RESPONSES OF PRIMARY ALFC LOOP

8.12.1 Steady State Response

The steady state control equation of the ALFC loop is given by [Fig. 8.13].

$$\left[\left\{\Delta P_{ref_c}(s) - \frac{1}{r}\Delta f(s)\right\}G_A(s)\,G_T(s) - \Delta P_D(s)\right]G_p(s) = \Delta f(s) \tag{8.32}$$

Assuming a constant reference input $(\Delta P_{ref_c}(s) = 0)$, equation (8.32) yields

$$-\frac{1}{r}\Delta f(s)\,G_A(s)\,G_T(s)\,G_p(s) - \Delta P_D(s)\,G_p(s) = \Delta f(s)$$

or, $$\Delta f(s)\left[1 + \frac{1}{r}G_A(s)\,G_T(s)\,G_p(s)\right] = -\Delta P_D(s)\,G_p(s)$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

(*ii*) The magnitude of the transient frequency deviation should be minimised.
(*iii*) Integrated frequency error should be minimum.
(*iv*) ALFC loop should have sufficient degree of stability.

8.13.1 About the Controller

Usually, a *proportional-integral* (PI) controller (also named as network regulator) is used. It yields an output (C) which is the sum of a proportional term ($p \times ACE$) and an integral $\left(\frac{1}{T_r}\int_0^t ACE\, dt\right)$, ACE is the *area control error* explained later; for isolated system $ACE \equiv \Delta f$

$$\therefore \quad C = p \times ACE + \frac{1}{T_r}\int_0^t ACE\, dt$$

$$= -p\Delta f - \frac{1}{T_r}\int_0^t \Delta f\, dt \tag{8.39}$$

[T_r is the time constant of the regulator and ranges between 50 to 200 sec. C is now named as regulation level].

The proportional component ($p \times ACE$) *governs* the dynamic response of the entire loop and the integral component *eliminates* the control deviation in the steady state.

For an increase in load (ΔP_D), the frequency starts decreasing and primary ALFC loop swings into action initiating the speed governors, but it is not able to bring back system frequency to the nominal value. Again, the decrease of frequency provides a positive input to the network PI regulator, that increases the level and secondary ALFC loop acts in the direction of increasing generation. The frequency again comes back to the nominal value.

Thus, the integral component of the network regulator acts in a way so as to keep the system frequency at reference value at steady state condition, while the proportional component is often set to zero.

Then,
$$C\,(= \Delta P_{ref}) = -\frac{1}{T_r}\int_0^t \Delta f\, dt$$

$$= -K_{in}\int_0^t \Delta f\, dt \tag{8.39a}$$

[K_{in} controls the rate of integration and thus the speed response of the loop.]

8.13.2 Modelling of Secondary ALFC Loop

The secondary loop performs slow 'reset' adjustments of the frequency by changing the reference power command (P_{ref}). The dotted portion added to the primary ALFC loop shows this (Fig. 8.14) and this can be best accomplished by low gain integrator loop. (The controller that achieves this property is known as '*Isochronous Controller*' (*or Network Regulator*).)

Let us first amplify and then integrate the frequency error to generate the command of the speed changer.

Thus mathematically, $\Delta P_{ref} = -K_{in}\int \Delta f\, dt$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Thus finally, for the two areas, with $\Delta P_D = L$,

$$-\frac{1}{r_1}\Delta f = L_1 + \Delta P_{12} + d_1\Delta f \tag{8.52}$$

and
$$-\frac{1}{r_2}\Delta f = L_2 + \Delta P_{21} + d_2\Delta f = L_2 - \Delta P_{12} + d_2\Delta f \tag{8.53}$$

Solving for Δf and ΔP_{12} from equations (8.52) and (8.53), we get

$$\Delta f = -\frac{L_1 + L_2}{\left(d_1 + \frac{1}{r_1}\right) + \left(d_2 + \frac{1}{r_2}\right)} \text{ Hz}$$

and
$$\Delta P_{12}\,(= -\Delta P_{21}) = \frac{\left(d_1 + \frac{1}{r_1}\right)L_2 - \left(d_2 + \frac{1}{r_2}\right)L_1}{\left(d_1 + \frac{1}{r_1}\right) + \left(d_2 + \frac{1}{r_2}\right)} \text{ p.u. MW}$$

or,
$$\Delta f = -\frac{L_1 + L_2}{M_1 + M_2} \text{ Hz and } \Delta P_{12} = \frac{M_1L_2 - M_2L_1}{M_1 + M_2} \text{ p.u. MW} \tag{8.54}$$

If both the areas are identical, $r_1 = r_2 = r$, $d_1 = d_2 = d$,

$$\Delta f\,(= \text{static frequency drop}) = -\frac{L_1 + L_2}{2M} \text{ Hz} \qquad [\because\ M_1 = M_2 = M]$$

and
$$\Delta P_{12} = \frac{L_2 - L_1}{2} \text{ p.u. MW}$$

Suppose the load change is in the area-1 only. Then

$$\Delta f = -\frac{L_1}{2M} \text{ and } \Delta P_{12} = -\frac{L_1}{2}, \text{ i.e. } \Delta P_{21} = \frac{L_1}{2}.$$

Thus, we see for a load change in area-1, area-2 'helps' the area-1 by supplying half the load change through the tie line and the frequency drop is also reduced to half.

Example 8.5: We are now concerned with two power areas which may be interconnected by the line for pool operation. Assume no loss in the system.

Area-1 has the following specifications:

(i) Rated capacity = 3000 MW
(ii) Operating power = 2000 MW
(iii) Inertia constant = 5 sec.
(iv) Regulation (r) = 3 Hz/ p.u. MW

Area-2 has the following specifications:

(i) Rated capacity = 9000 MW
(ii) Operating power = 7500 MW
(iii) Inertia constant = 8 sec.
(iv) Regulation (r) = 2.5 Hz/p.u. MW

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

the sum of the change in load power (ΔP_D) and the released energy of the initial loading due to frequency variation (i.e. $d\Delta f$); i.e. mathematically $\Delta P_g = \Delta P_D + d\Delta f$ and $P_g = P_{g_{initial}} + \Delta P_D + d\Delta f$.

Next, we consider the performance of a generating area connected with an *infinite network* through a tie-line. Obviously, the generating area supplies its own loads. Thus, for a step load change ΔP_D, we can write for generating power (P_g)

$$P_g = P_{g_{initial}} + \Delta P_D + d\,(n_{p.u.} - f_{p.u.}) + T_{p.u.} \int_0^t (n_{p.u.} - f_{p.u.})\, dt \tag{8.58}$$

where $d(n_{p.u.} - f_{p.u.})$ is identical to ($d\Delta f$) but expressed in p.u. It may be noted here that the load variation ΔP_D does not appear in equation (8.58) as this load variation is completely absorbed by the infinite network and this has no consequence on the generating area.

Again, it may be observed that an increase in frequency makes ($n_{p.u.} - f_{p.u.}$) negative and hence there is a decrease in power output of the generating area. This makes ($P_{Turbine} - P_{electrical}$) positive and hence the rotor(s) accelerates tending to make ($n_{p.u.} = f_{p.u.}$). ALFC loop response makes the necessary reduction in turbine power output and the *equilibrium* between $P_{Turbine}$ and $P_{electrical}$ is established. Hence a variation of the system frequency has the effect of producing an identical variation in the speed of the unit and a variation of opposite algebraic sign in the power output of the generating area when this area is connected to an infinite network.

In the preceding sections we have seen that in the primary ALFC loop, reduction of r (the regulation) results in lower static frequency drop as well as faster transient response. Moreover, the primary ALFC loop having lower value of r, transient response of the two-area system is also improved. However, it is not possible to reduce r arbitrarily without causing an unsatisfactory transient system performance. Referring to the transient response of the primary ALFC loop, we see from equation (8.56) that the denominator of $\Delta f(s)$ is a third order polynomial in s and cannot be factored *simply*. However, a qualitative analysis is possible on the basis of 'root-locus technique'. It reveals that the system characteristic equation being consisting of three roots, for a given system if r be reduced, two of the three characteristic roots start becoming complex conjugates and more closer to the imaginary axis. The system response becomes oscillatory, with amplitudes of the overshoots and undershoots increasing as the complex pair of roots move closer to the imaginary axis. This restriction thus imposes limitation on choosing very lower values of the regulation r of the primary ALFC loop.

8.20 APPLICATION ASPECT OF SECONDARY ALFC LOOP

In large networks consisting of many generating units, the load variation (ΔP_D) can be of such an amplitude that cannot be compensated by a single generator. In this case, it is required to perform secondary regulation by involving more units and larger generating area.

It is evident that the permanent frequency drop is smaller provided the regulation r of the corresponding generating area (or units) is smaller. However, for small area it may be possible to have one or two units having small regulation; but for a larger network, due to stability problem of the ALFC loop, it may not be possible to have much lower values of regulation. It is then advisable to import power through tie-lines in case of load increments and involve the secondary ALFC loop to compensate for the frequency error.

The affected area first involves the primary ALFC loop of all the interconnected network depending upon the value of r of each of them and thereafter the affected network utilises the spinning reserve. The spinning reserve, which may overlap the primary action of the ALFC loop, is often taken equal to the power of the largest capacity unit of each network concerned so that the connected network can successfully encounter the loss of its biggest unit, after the initial assistance of the primary ALFC loop.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

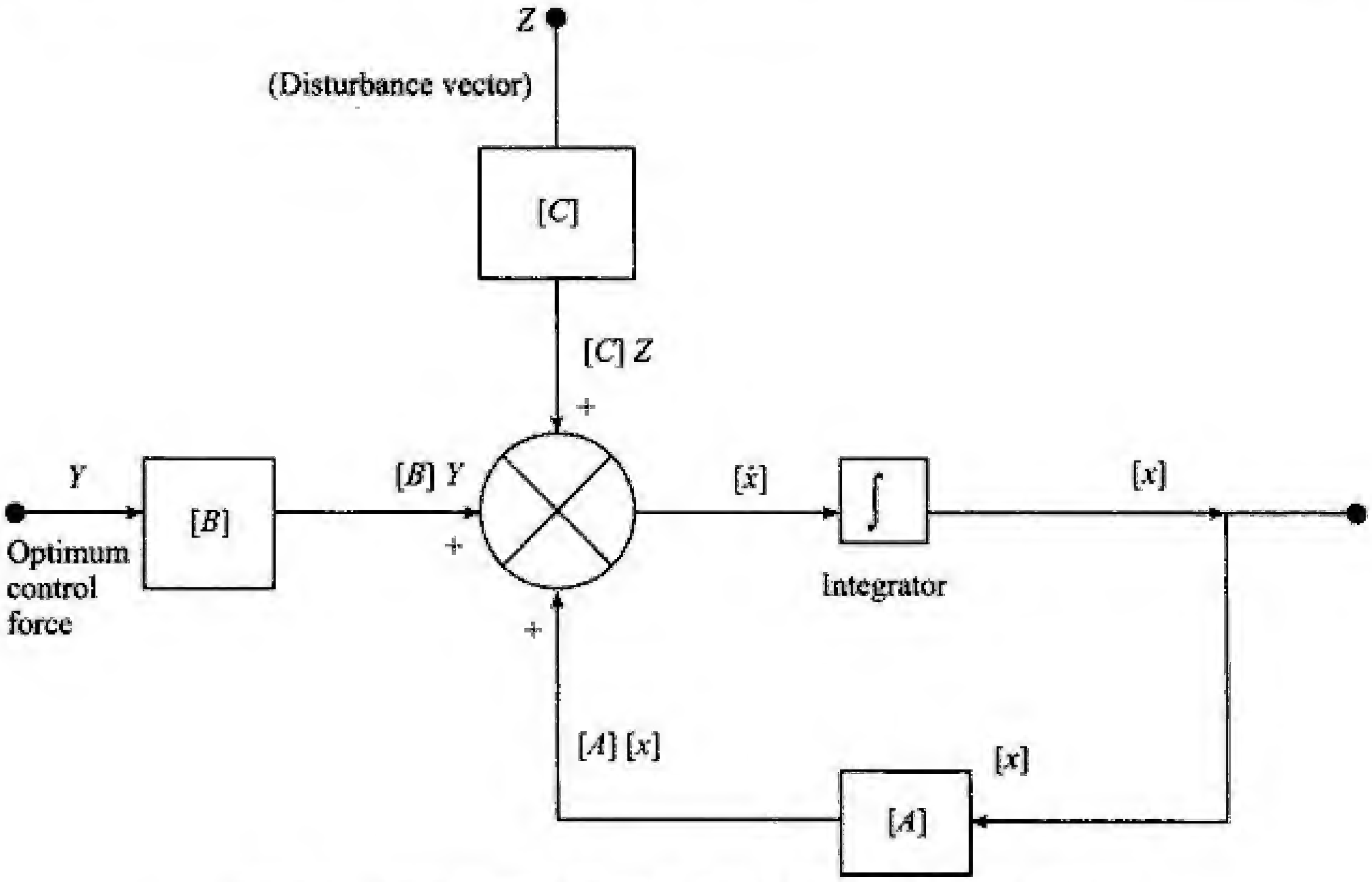


Fig. 8.18 Block diagram representation of the optimal control concept.

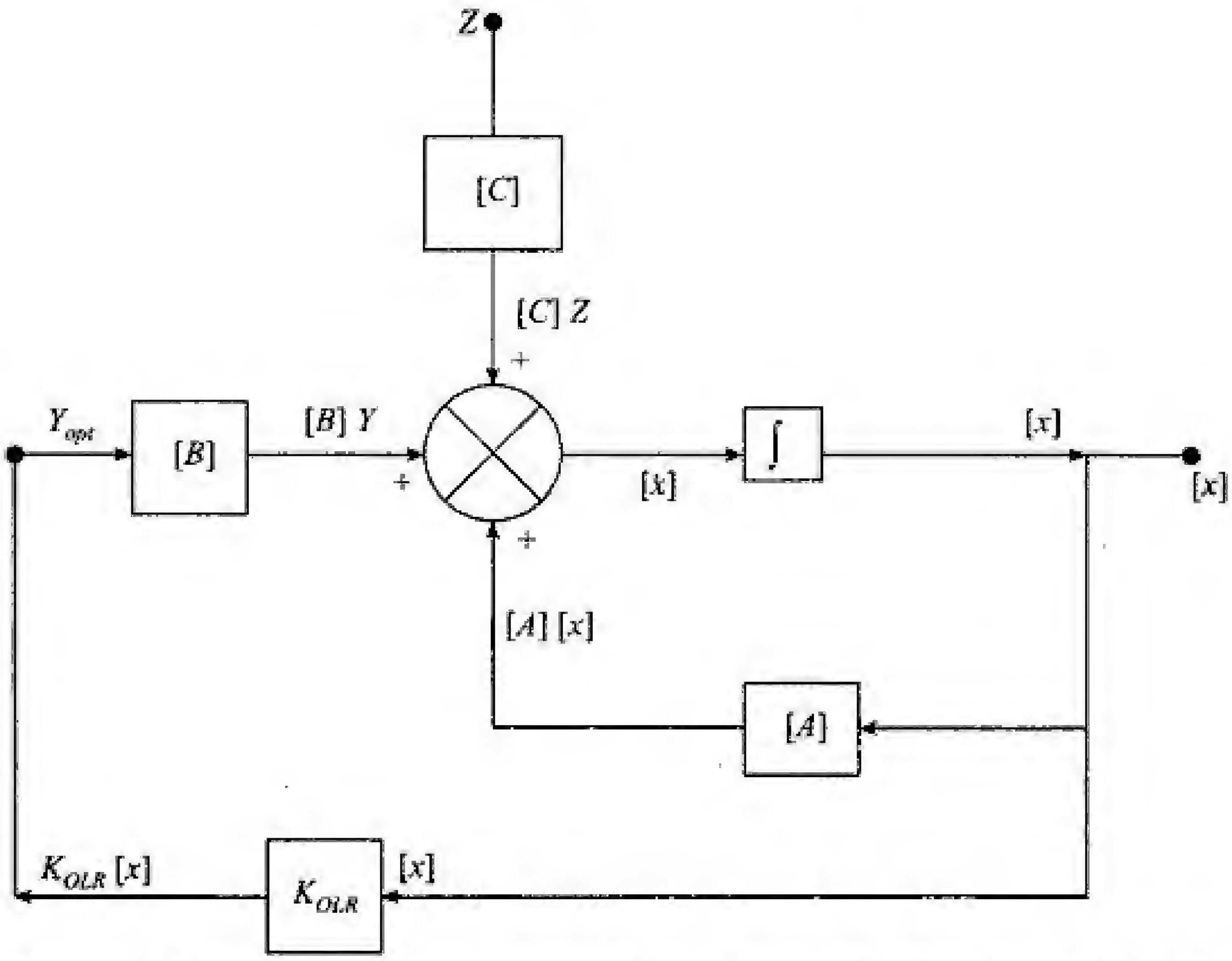


Fig. 8.19 Complete block diagram representation of the optimal control concept in ALFC.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Hence, we can write,

$$\dot{x} = f_1(x_f);\ t = 0^+ < t < t = t_c \tag{9.5}$$

$$\dot{x} = f_2(x_f);\ t > t_c \tag{9.6}$$

Methods have been developed to solve equations (9.5) and (9.6) using computers for the electrical system. The aim is to check whether state x settles down to a new steady state value following the clearing of the fault. *Swing equation* is used to plot *swing curves* to see the stability of the system following faults for different assumed clearing times. Computations to plot swing curves are repeated to find both *critical time* and the system stability aspect.

In the following sections, we will first discuss the aspects of transient stability. In this process, the analytical derivation of the swing equation will be shown along with its application in deriving the conditions for transient stability. At the final stage, the method of finding the transient stability of a multi-machine system will be illustrated. Also, how to derive the conditions of steady state stability from transient stability study will be shown.

9.4 TRANSIENT STABILITY

9.4.1 Representation of Transmission Lines, Loads and Generators in Transient Stability

Let us assume that a transmission network is represented by a n-port network (Fig. 9.1), where the transmission system has n number of buses (excluding the reference or ground bus) with m number of generator buses ($m < n$), while the bus admittance matrix is denoted by $[Y_{Bus}]$. The buses are numbered as follows:

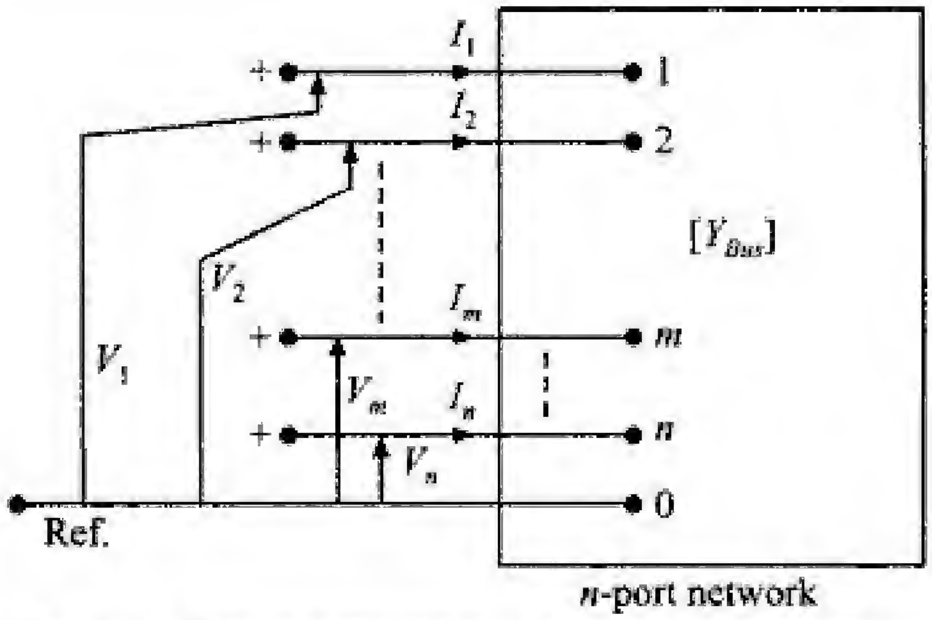


Fig. 9.1 Port representation of a transmission network.
Bus number 2 to $m \rightarrow$ generator (PV) buses
Bus number $m+1$ to $n \rightarrow$ load (PQ) buses
Bus number $1 \rightarrow$ slack bus.

Model of Prefault Network ($t = 0^-$)

This model is available from load flow study. We have the following information:

(i) $P_{gi}, Q_{gi}, P_{di}, Q_{di}$; for $i = 2, 3, \ldots, m$

(ii) P_{di}, Q_{di}; for $i = m + 1, m + 2, \ldots, n$

(iii) P_i and Q_i; for slack bus ($i = 1$)

(iv) $|V_i| \angle \delta_i$; for $i = 2, 3, \ldots, n$.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

then unstable. In an unstable system, δ increases indefinitely with time and the alternator loses synchronisation. For a system to be stable, at the final instant $\frac{d\delta}{dt} = 0$.

We have seen earlier (equation 9.15b) that $M\frac{d^2\delta}{dt^2} = P_a$.

i.e.,
$$2\frac{d\delta}{dt}\cdot\frac{d^2\delta}{dt^2} = 2\frac{d\delta}{dt}\cdot\frac{P_a}{M}$$

Upon integration, it gives

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int_{\delta_0}^{\delta} P_a d\delta$$

$\therefore$
$$\frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int_{\delta_0}^{\delta} P_a d\delta} \tag{9.26}$$

[where δ_0 is the initial power angle before the rotor swing starts following a disturbance]

Application of stability criterion $\left[\frac{d\delta}{dt} = 0\right]$ leads

$$\sqrt{\frac{2}{M}\int_{\delta_0}^{\delta} P_a d\delta} = 0$$

i.e.,
$$\int_{\delta_0}^{\delta} P_a d\delta = 0 \tag{9.27}$$

In words, it means that *the area under the graph of accelerating power P_a versus δ must be zero for some value of δ*. It simply means that positive (or accelerating) area under the graph must be equal to the negative (or decelerating) area. This condition is, therefore, known as *equal area criterion* for stability.

9.6 INTERPRETATION OF EQUAL AREA CRITERION

Let us suppose, a generator is operating at steady state with initial operating angle δ_0 and delivering an electrical power $P_{e_0} (= P_{m_0})$, the $P_e - \delta$ profile being exhibited in Fig. 9.4.

Let us further assume that there be a step change in mechanical power input from P_{m_0} to P_{m_1} at $t = 0$. Due to rotor inertia, the rotor position cannot change instantaneously (i.e., $\delta(0^-) = \delta(0) = \delta(0^+)$). This means that electrical power output remains unchanged and hence $P_e(0^-) = P_{e_0} = P_e(0^+)$. However, due to change (say increase) of P_{m_0} to P_{m_1} we say that $P_{m_1} > P_{e_0}$ and this will lead to consequent acceleration of the rotor ($P_a(0^+)$ is +ve) leading to $d^2\delta/dt^2(0^+)$ positive. With acceleration of the rotor, δ will increase to δ_1 till $P_{e_1} = P_{m_1}$. At this point (when $P_{e_1} = P_{m_1}$), $d^2\delta/dt^2$ is zero though $d\delta/dt$ is still positive and δ continuous to increase due to inertia of the rotor. It (δ) may even overshoot its final steady state operating point. When δ exceeds δ_1, P_{m_1} becomes less than P_e and obviously P_a (accelerating power) becomes negative. The rotor is then decelerating. Assuming absence of any damping, δ will continuously oscillate around δ_1. In practical machines, damping due to mechanical and electrical losses causes δ to stabilize at its final steady state operating point δ_1. It may be noted here that if δ exceeds δ_3, P_m would exceed P_e and rotor would accelerate again. This will lead to further increase in δ and loss of stability.

9.8 APPLICATION OF EQUAL AREA CRITERION TO TRANSIENT STABILITY OF SYNCHRONOUS MOTOR

Let us consider a synchronous motor connected to an infinite busbar. It supplies a load of P_d and is operating at synchronous speed with torque angle δ_0, corresponding to this output power (P_e) on the power-angle diagram (sinusoidal) as shown in Fig. 9.7. The power-angle equation is given by

$$P_e = \frac{|E_m||V|}{X}\sin\delta$$

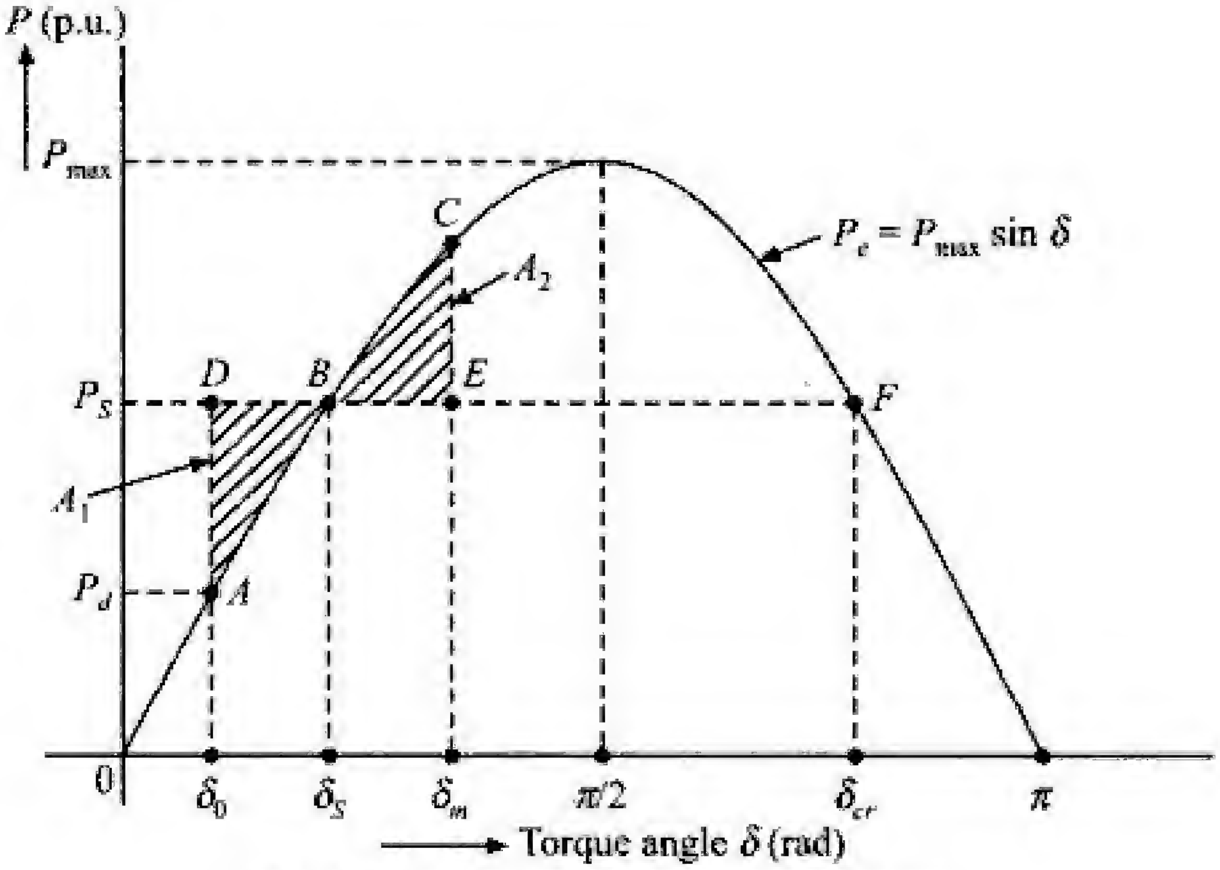


Fig. 9.7 *P-δ curve of a synchronous motor.*

where,

$|V|$ = voltage of the infinite bus,

$|E_m|$ = voltage behind the transient reactance of the motor,

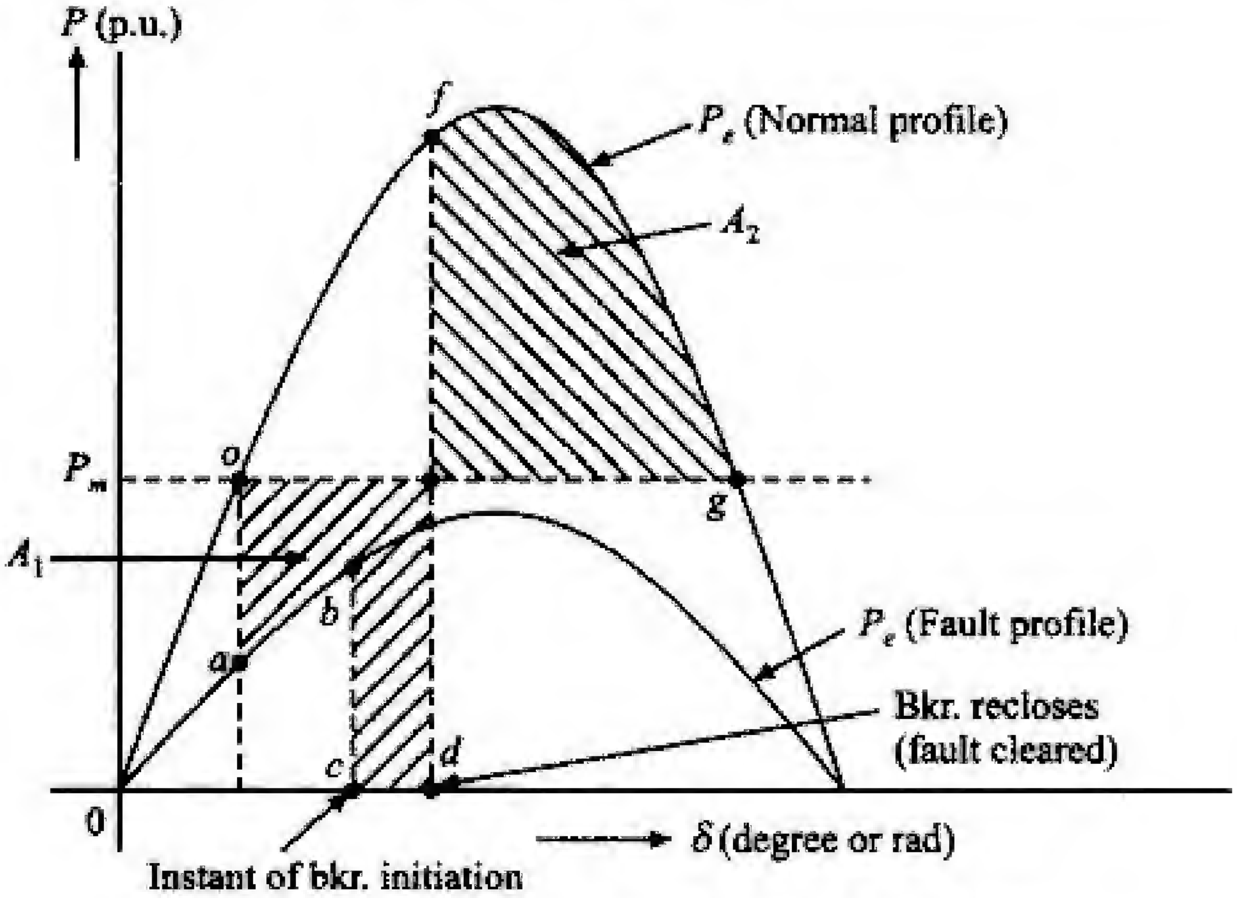
X = transient reactance of the motor including reactance of the transmission lines, transformers, etc., if any, between the infinite bus and the motor terminal.

It is clear that under these conditions the mechanical power output P_d and the electrical power input P_e are equal. If the load on the motor is suddenly increased to P_s, the electrical input power will initially be the same. The sudden excess of load must be available from the kinetic energy stored in the rotating system. This is possible only if the motor slows down. Immediately after the motor starts slowing down, the torque angle δ increases and with the increase in δ, there is an increase in electrical input power. This goes on till P_e and P_s are equal (corresponding to new increased load), at which point the deceleration ceases. The operating point on the power-angle diagram is then B. However, from A to B the motor has been decelerating at a finite rate and at B its speed is below synchronous speed. The result is that δ continues to increase until point C is reached, at which point the speed is again synchronous. Torque angle corresponding to C is δ_m. During the interval, B to C electrical power input P_e has been more than P_s and speed starts increasing till synchronous speed is reached. δ now starts decreasing till decreases point B is reached again with the speed above the synchronous speed. At B, the acceleration ceases and the rotor moves towards A with a finite deceleration; the cycle of oscillations then repeats

Distance protection can give faster clearance than differential protection. In distance protection, we use a combination of (a) definite impedance relay and (b) impedance time relay/mho/reactance relay.

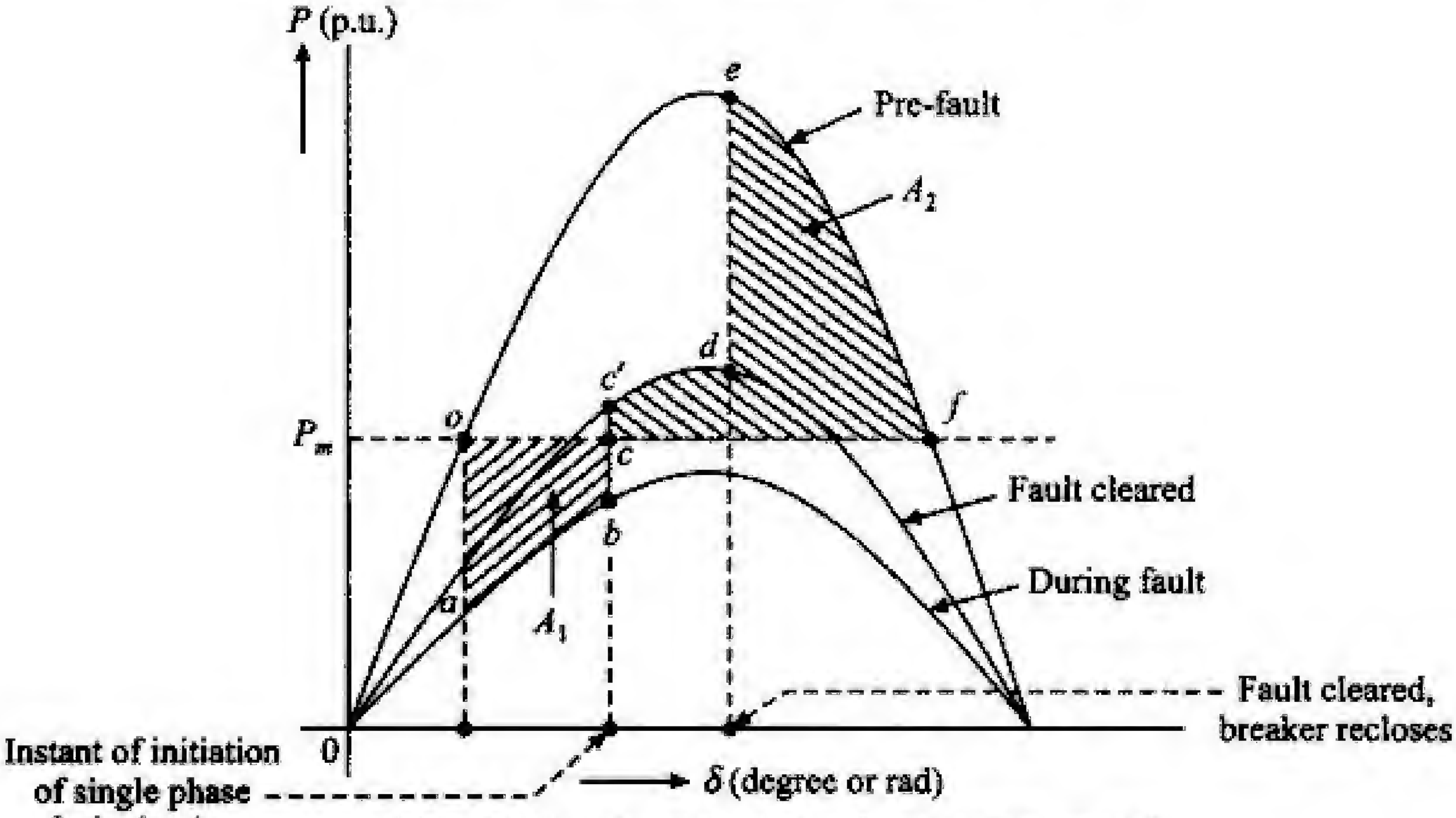
9.9.3 Automatic Reclosing

When a transient fault occurs, the relay trips the circuit breaker. The circuit breaker is allowed to close automatically only after a certain time. This autoreclosing method is more effective in a single circuit line than in a double circuit line (Fig. 9.10(a) and Fig. 9.10(b)).



[A_1 = Accelerating area; A_2 = Decelerating area]

Fig. 9.10(a) Transient stability with autoreclosure.



[A_1 = Accelerating energy; A_2 = Decelerating energy]

Fig. 9.10(b) Single phase autoreclosing.

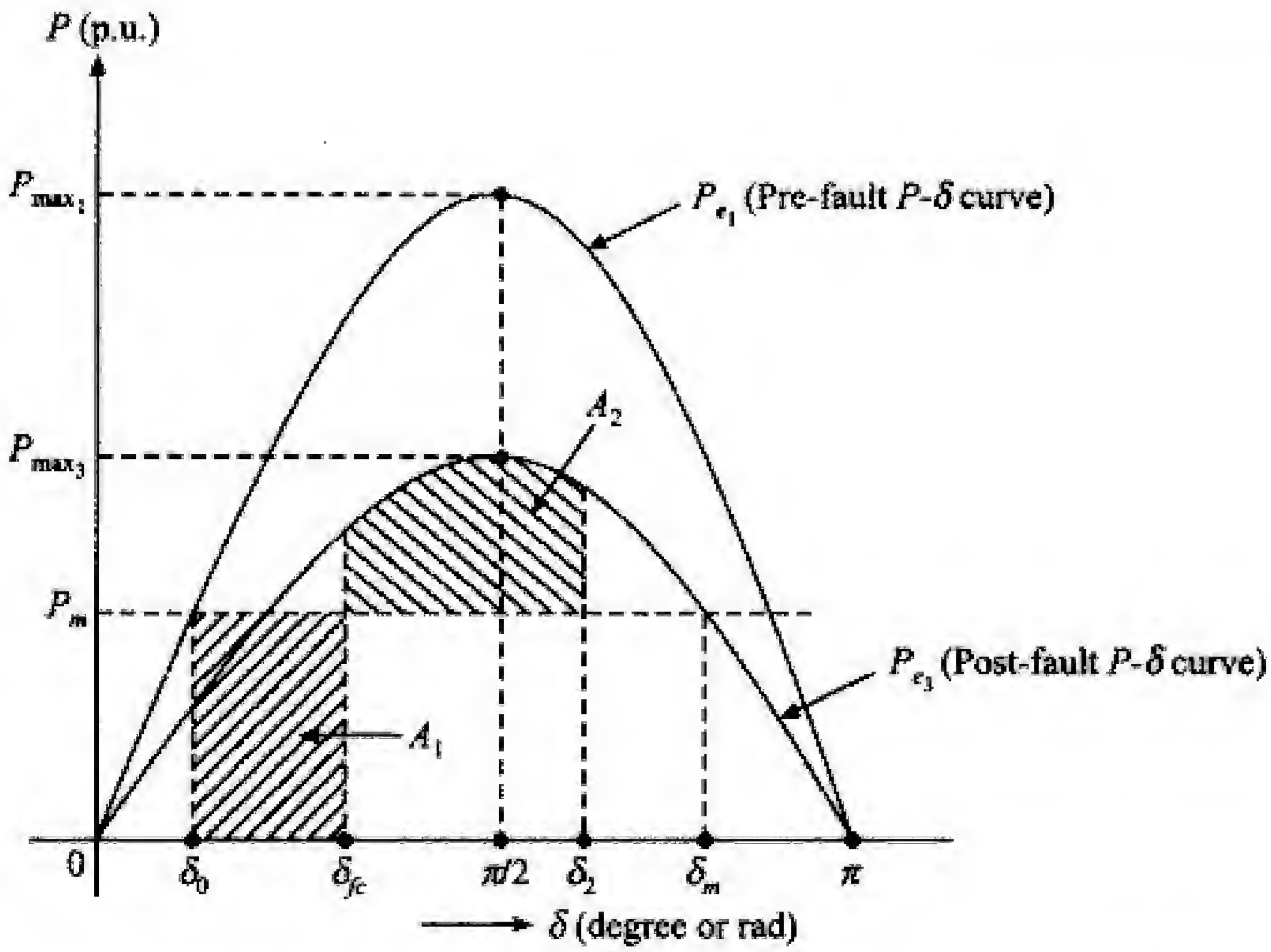
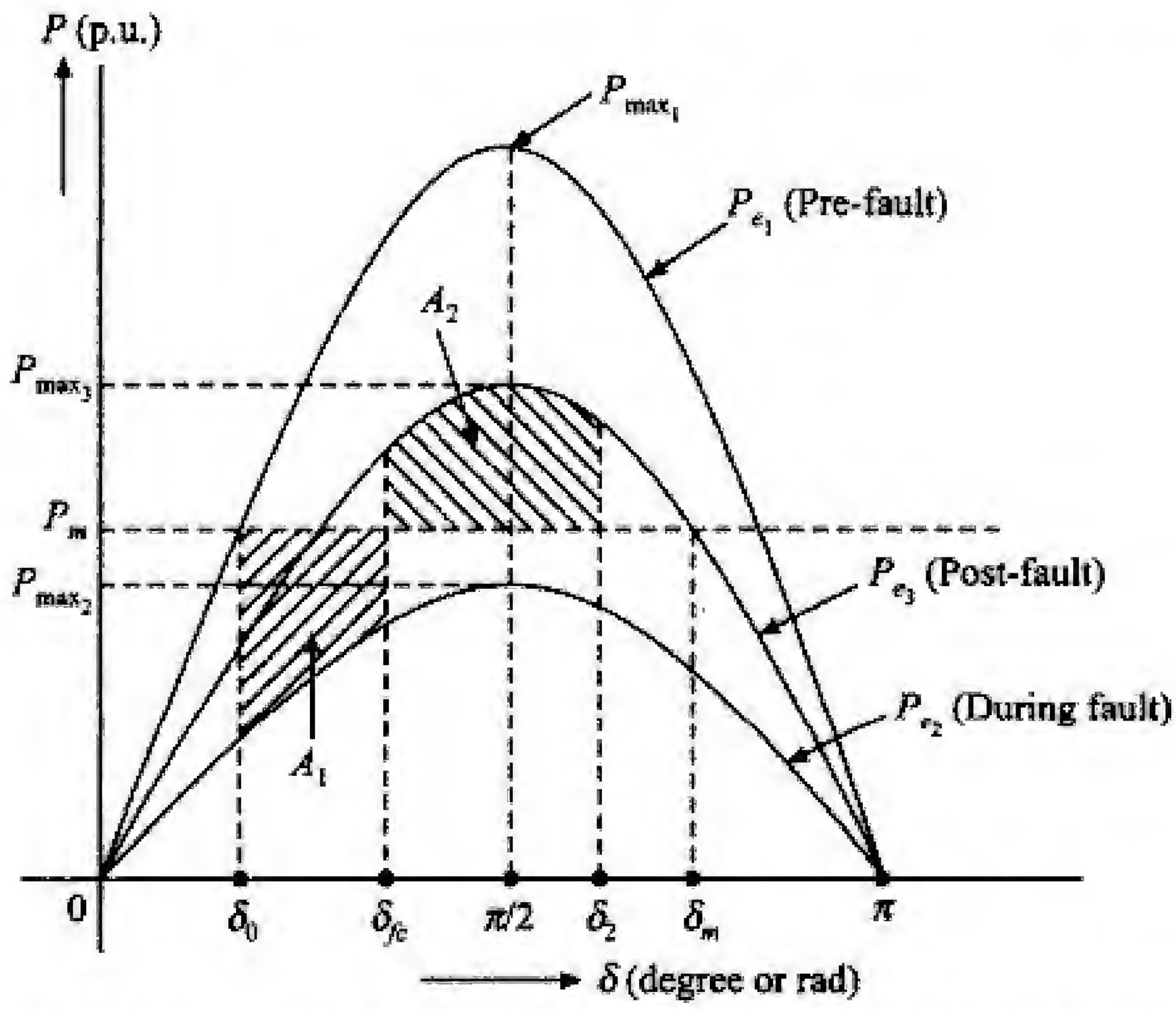


Fig. E9.6(a) During fault, the electrical power output is assumed to be zero, thus P_{e_2} touches x-aixs.



The fault clearing angle δ_{fc} can have maximum value δ_c, the critical clearing angle. Also, δ_2 can have highest value of δ_m, till area A_1 will be equal to A_2

Fig. E9.6(b)

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

and $$\Delta\omega_3^p = \Delta\omega_2 + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_2} \times \Delta t = 1.4463 + 13.8909 \times 0.05 = 2.1408 \text{ rad/sec.}$$

$\therefore$ $$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_3^p} = \Delta\omega_3^p = 2.1408 \text{ rad/sec}$$

and $$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_3^p} = \left.\frac{P_a}{M}\right|_{\delta_3^p} = \frac{\pi \times 50}{8}(1 - 0.6 \sin 0.5816) = 13.1629 \text{ rad/sec}^2$$

The corrected values are thus

$$\delta_3 (\equiv \delta_3^c) = \delta_2 + \left[\frac{\left.\frac{d\delta}{dt}\right|_{\Delta\omega_2} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_3^p}}{2}\right] \times \Delta t$$

$$= 0.5093 + \left[\frac{1.4463 + 2.1408}{2}\right] \times 0.05$$

$$= 0.5990 \text{ rad} = 34.3189°$$

and $$\Delta\omega_3 (\equiv \Delta\omega_3^c) = \Delta\omega_2 + \left[\frac{\left.\frac{d\Delta\omega}{dt}\right|_{\delta_2} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_3^p}}{2}\right] \times \Delta t$$

$$= 1.4463 + \left[\frac{13.8909 + 13.1629}{2}\right] \times 0.05 = 2.1226 \text{ rad/sec}$$

$\therefore$ At $t = 0.15$ sec, $\delta_3 = 0.599$ rad $= 34.3189°$

After 0.05 sec, i.e. at 0.2 sec, the fault is cleared. We will now find the value of δ and $\Delta\omega$ at $t = 0.2$ sec. For this, the fourth iteration is to be performed.

Here, $P_a = 1 - 1.5 \sin \delta$; $\Delta t = 0.05$ sec

Also, $$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_3} = \Delta\omega_3 = 2.1226 \text{ rad/sec}$$

and $$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_3} = \frac{P_a}{M} = \frac{\pi \times 50}{8}(1 - 1.5 \sin 0.599) = 3.0292 \text{ rad/sec}^2$$

The predicted values of δ and $\Delta\omega$ at the fourth iteration are:

$$\delta_4^p = \delta_3 + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_3} \times \Delta t = 0.599 + 2.1226 \times 0.05 = 0.7051 \text{ rad}$$

and $$\Delta\omega_4^p = \Delta\omega_3 + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_3} \times \Delta t = 2.1226 + 3.0292 \times 0.05 = 2.2741 \text{ rad/sec}$$

$\therefore$ $$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_4^p} = \Delta\omega_4^p = 2.2741 \text{ rad/sec}$$

and $$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_4^p} = \left.\frac{P_a}{M}\right|_{\delta_4^p} = \frac{\pi \times 50}{8}(1 - 1.5 \sin 0.7051) = 0.5465 \text{ rad/sec}^2$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

In a matrix form,

$$[Y_{Bus}] = \begin{bmatrix} (1.1646 - j7.1064) & 0 & 0 & (-0.5405 + j3.2432) & (-0.6240 + j3.9002) \\ 0 & -j8.3333 & 0 & j8.3333 & 0 \\ 0 & 0 & -j9.0909 & 0 & j9.0909 \\ (-0.5405 + j3.2432) & j8.3333 & 0 & (2.1981 - j18.6800) & (-0.8734 + j6.5502) \\ (-0.6240 + j3.9002) & 0 & j9.0909 & (-0.8734 + j6.5502) & (2.2247 - j20.0526) \end{bmatrix} \text{ p.u.}$$

Step 5: The short circuit fault is in line 4–5, near bus 4. Therefore, during fault all the elements of $[Y_{Bus}]$ matrix corresponding to bus 4 (i.e., row-4 and column-4) will be zero. Therefore, $[Y_{Bus}]$ matrix during fault is given then by

$$[Y_{Bus}]_{fault} = [Yf] = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} (1.1646 - j7.1064) & 0 & 0 & 0 & (-0.6240 + j3.9002) \\ 0 & -j8.3333 & 0 & 0 & 0 \\ 0 & 0 & -j9.0909 & 0 & j9.0909 \\ 0 & 0 & 0 & 0 & 0 \\ (-0.6240 + j3.9002) & 0 & j9.0909 & 0 & (2.2247 - j20.0526) \end{bmatrix} \end{array} \text{ p.u.}$$

Let us delete row and column corresponding to bus 4

$$\therefore \quad [Yf] = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 5 \end{array} & \begin{bmatrix} (1.1646 - j7.1064) & 0 & 0 & (-0.6240 + j3.9002) \\ 0 & -j8.3333 & 0 & 0 \\ 0 & 0 & -j9.0909 & -j9.0909 \\ (-0.6240 + j3.9002) & 0 & j9.0909 & (2.2247 - j20.0526 \end{bmatrix} \end{array} \text{ p.u.}$$

The injected current at all buses except the internal buses of the generators is zero. Therefore, to obtain bus admittance matrix at internal nodes during fault, we apply the Kron's network reduction technique to eliminate bus 5.

From Chapter 1, we know (equation (1.22)) for Kron's reduction,

$$Y_{ij(new)} = Y_{ij(old)} - \frac{Y_{ik(old)} Y_{kj(old)}}{Y_{kk}}$$

$$\therefore \quad [Yf] = \begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} (1.1646 - j7.1064 & 0 & 0 \\ 0 & -j8.3333 & 0 \\ 0 & 0 & -j9.0909 \end{bmatrix} \end{array}$$

$$- \frac{1}{(2.2247 - j20.0526)} \begin{bmatrix} (-0.6240 + j3.9002) \\ 0 \\ j9.0909 \end{bmatrix} [(-0.6240 + j3.9002) \quad 0 \quad j9.0909]$$

Therefore, $[Y_{Bus}]$ matrix in p.u. during fault is given by

$$\therefore \quad [Yf] = \begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} (1.0058 - j6.3496) & 0 & (-0.0857 + j1.7777) \\ 0 & -j8.3333 & 0 \\ (-0.0857 + j1.7777) & & (0.4517 - j5.0196) \end{bmatrix} \end{array}$$

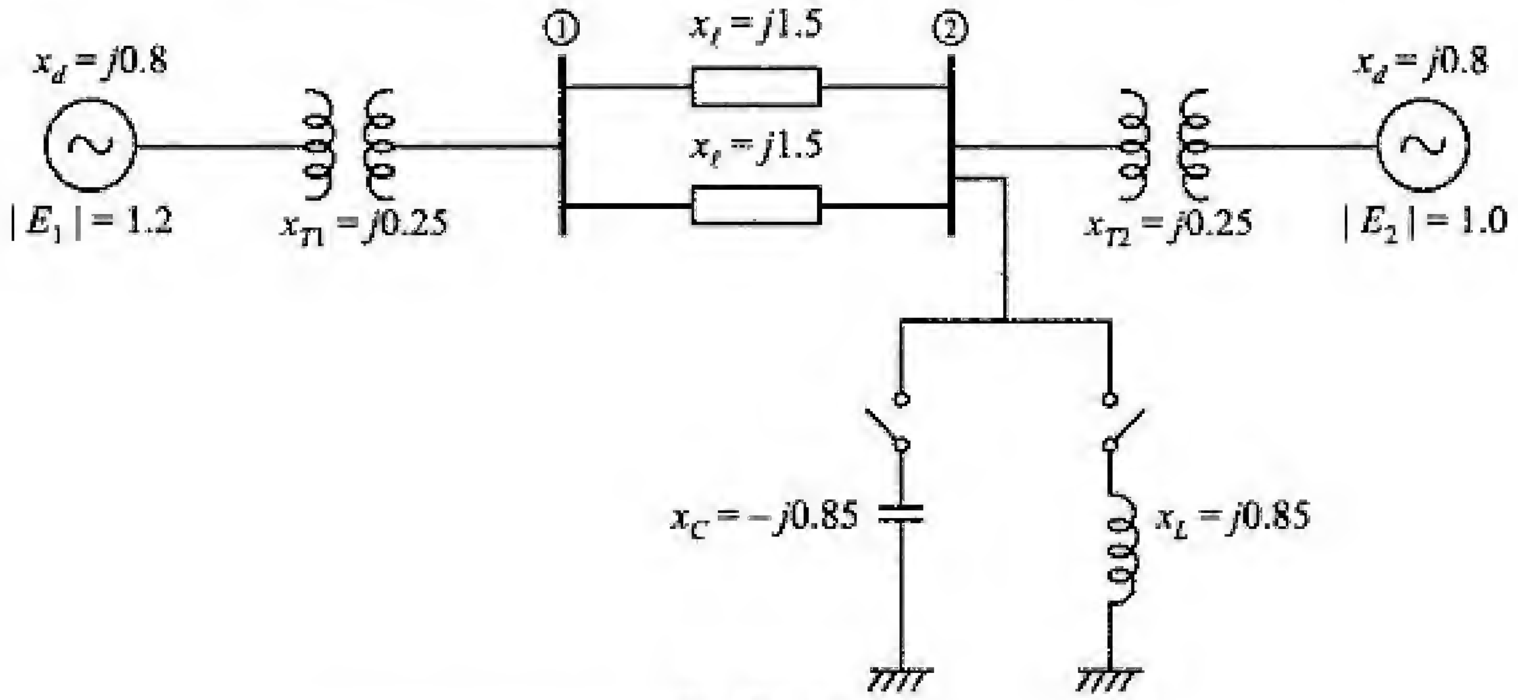


Fig. E9.10

Solution: *Case (a):* With inductor at bus-2

Transfer reactance between the two generating sources is

$$x = x_1 + x_2 + \frac{x_1 x_2}{x_3}$$

where $x_1 = x_d + x_{T1} + \frac{x_\ell}{2} = j0.8 + j0.25 + \frac{j1.5}{2} = 1.8$ p.u.

$x_2 = x_{T2} + x_d = j0.25 + j0.8 = j1.05$ p.u.

$x_3 = x_L = j0.85$ p.u.

Therefore, $x = j1.8 + j1.05 + \frac{j1.8 \times j1.05}{j0.85} = j5.07$ p.u.

$\therefore$ Maximum power transfer under this condition is

$$P_{\max_1} = \left| \frac{E_1 E_2}{x} \right| = \frac{1.2 \times 1.0}{5.07} = 0.237 \text{ p.u.}$$

Case (b): With capacitor at bus 2

Transfer reactance between the two generating sources becomes

$$x = x_1 + x_2 + \frac{x_1 x_2}{x_3}$$

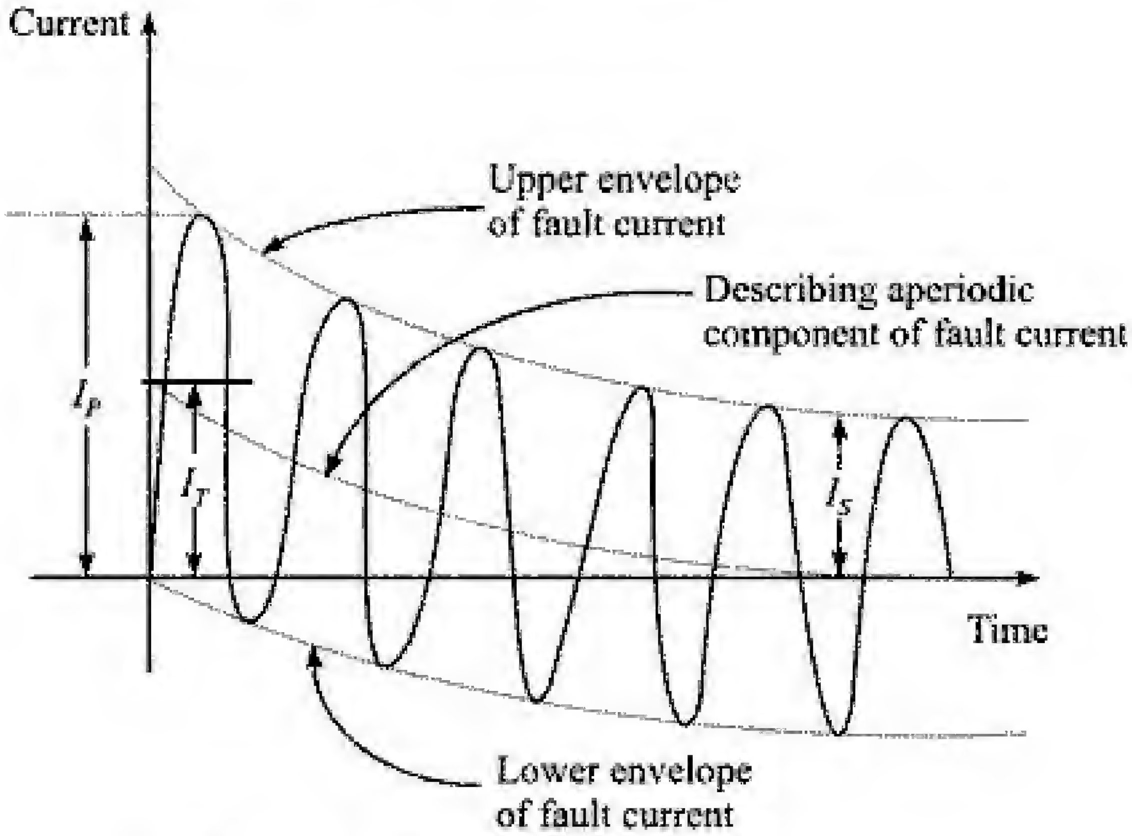
where $x_1 = 1.8$ p.u. and $x_2 = j1.05$ p.u., but $x_3 = x_C = -j0.85$ p.u.

Therefore, $x = j1.8 + j1.05 + \frac{j1.8 \times j1.05}{-j0.85} = j0.626$ p.u.

$\therefore$ Maximum power transfer under this condition is

$$P_{\max_2} = \left| \frac{E_1 E_2}{x} \right| = \frac{1.2 \times 1.0}{0.626} = 1.915 \text{ p.u.}$$

It may be observed that the connection of the inductor at bus-2 reduces the power transfer limit while the capacitor connection improves the power limit. However, inductor connection is needed in order to control the overvoltage at the receiving end when the line is at no load or lightly loaded. Capacitor



I_P = Peak value of short circuit current

I_T = Initial value of aperiodic component of fault current

I_S = Steady state short circuit current (peak value)

$i_p = i_S + i_T$

Fig. 10.1 Variation of fault current with variation in time.

Short circuit faults commonly occurring in practice are *three-phase balanced fault* (L-L-L fault), *single line to ground fault* (L-G fault), *double line fault* (L-L fault) and *double line to ground fault* (L-L-G fault), the most common being the single line to ground fault. However, the three-phase fault in lines, is the severemost fault, though it is basically *symmetrical* in nature; all other faults are *unsymmetrical*.

A conventional fault study usually includes the following assumptions:

(*i*) All the voltage sources are assumed to be of one per unit (p.u.) magnitude with *zero* relative phase displacement and the power flow currents are ignored in the presence of fault currents.

(*ii*) Transformer magnetising current components are neglected and the line may be assumed to have contained only inductances with no shunt susceptance (though in accurate approaches, this assumption is not mandatory). However, in modern digital computer fault study, the assumptions are less rigorous. The generators being represented by their no load voltages behind subtransient reactances, the loads, if at all taken to contribute to the fault, are represented by the equivalent shunt admittance to neutral. Further assumptions are as follows:

The fault strikes the system when the network remains *balanced* electrically. Normal single phase equivalent circuits may be assumed as done in ordinary load flow equations. The pre-fault load currents, system losses and harmonics are neglected.

10.2 SYMMETRICAL COMPONENT ANALYSIS

The analysis of a three phase circuit in which phase voltages and currents are balanced (of equal magnitude in three phases and displaced by 120° from each other) and in which all circuit elements in each phase are balanced and symmetrical, is relatively simple since the treatment of a single phase leads directly to the three phase solution. However, the analysis by Kirchoff's laws is much more difficult, when the circuit is not symmetrical, as a result of unbalanced loads, unbalanced faults or short circuits that are not

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

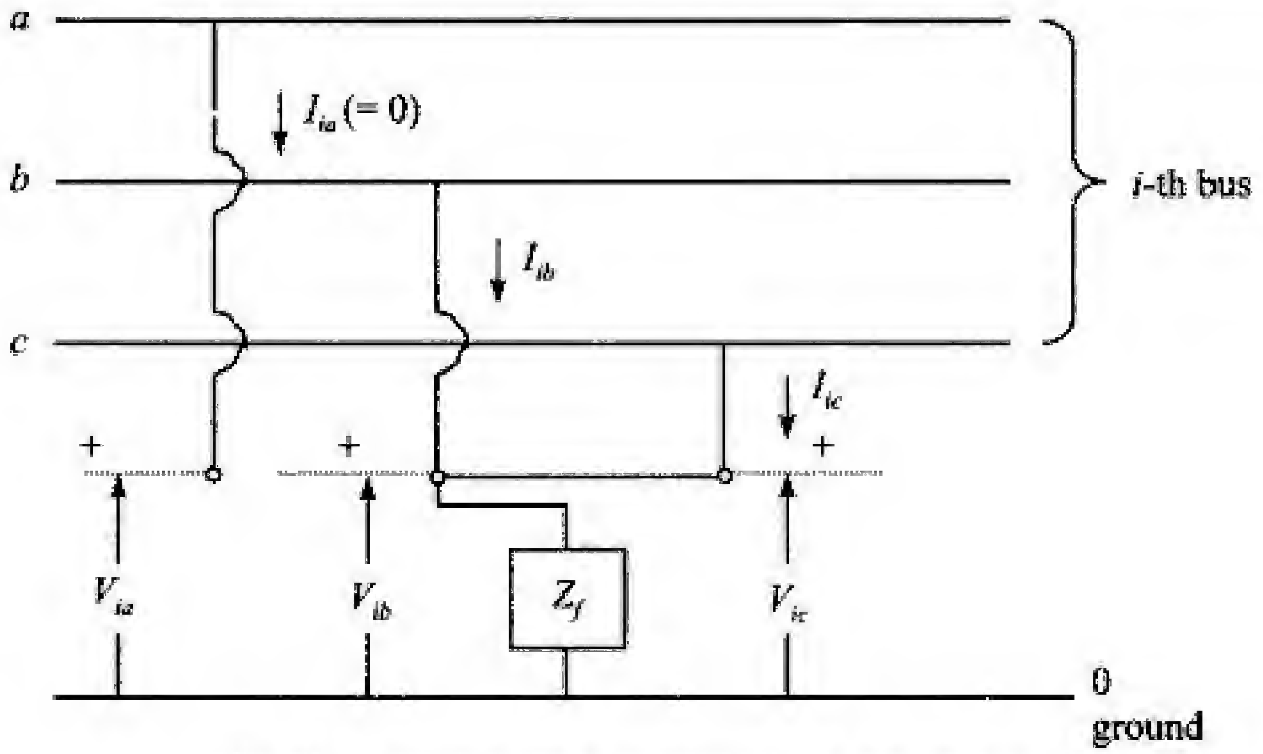


Fig. 10.7 Double line to ground fault at the i-th bus.

Here,
$$I_{ia} = 0 \tag{10.26}$$
$$V_{ib} = V_{ic} \tag{10.27}$$
and
$$V_{ib} = (I_{ib} + I_{ic})\, Z_f \tag{10.28}$$

It follows from equation (10.26),
$$I_i^{(0)} + I_i^{(1)} + I_i^{(2)} = 0 \tag{10.29}$$

From equation (10.27) we write
$$V_i^{(0)} + a^2 V_i^{(1)} + a V_i^{(2)} = V_i^{(0)} + a V_i^{(1)} + a^2 V_i^{(2)}$$

simplification yields $\quad V_i^{(1)} = V_i^{(2)}$

Again, from equation (10.28),
$$V_i^{(0)} + a^2 V_i^{(1)} + a V_i^{(2)} = (I_i^{(0)} + a^2 I_i^{(1)} + a I_i^{(2)} + I_i^{(0)} + a I_i^{(1)} + a^2 I_i^{(2)})\, Z_f$$

simplification yields
$$V_i^{(0)} - V_i^{(1)} = [2 I_i^{(0)} + (a^2 + a)\,(I_i^{(1)} + I_i^{(2)})]\, Z_f$$

or,
$$V_i^{(0)} - V_i^{(1)} = 3 Z_f I_i^{(0)} \tag{10.29a}$$

The sequence network interconnection is shown in Fig. 10.8.

Next we apply the derived concept for a double line to ground fault in i-th bus of a multi-bus power system.

Here also, as we have derived above,
$$V_i^{(1)} = V_i^{(2)}, \quad I_i^{(0)} + I_i^{(1)} + I_i^{(2)} = 0 \tag{10.30}$$

Also,
$$V_i^{(1)} = E - Z_{ii}^{(1)} I_i^{(1)}$$

or,
$$I_i^{(1)} = \frac{E - V_i^{(1)}}{Z_{ii}^{(1)}} \tag{10.31}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Old bus no.	New bus no.
1	Reference
2	1
3	2
4	3

The reconfigured system is shown in Fig. E11.1(a).

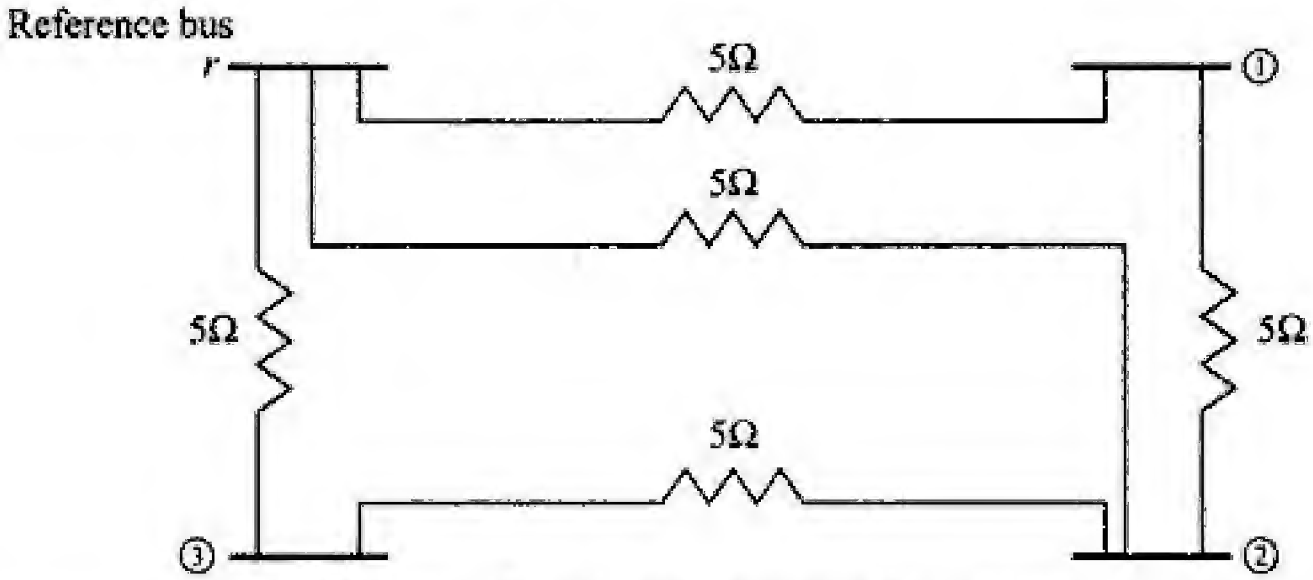


Fig. E11.1(a) Reconfigured system of Fig. E11.1.

Initially order of old $[Z_{Bus}]$ is zero.

Step 1: Consider the element between bus 1 and reference $[Z_b = 5\ \Omega]$. This is type 1 modification.

Added impedance value = 5 Ω; New bus no. = 1

$$\therefore \qquad [Z_{Bus}] = \begin{array}{c|c} & 1 \\ \hline 1 & [5] \end{array}\Omega$$

Step 2: In the next modification, consider the element between bus 2 and reference $[Z_b = 5\ \Omega]$. This is also type-1 modification.

Added impedance value = 5 Ω; New bus no. = 2

$$\therefore \qquad [Z_{Bus}] = \begin{array}{c} \\ 1 \\ 2 \end{array}\!\!\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \end{array}\Omega$$

Step 3: Consider the element between bus 3 and reference $[Z_b = 5\ \Omega]$. This is again a type-1 modification.

Added impedance value = 5 Ω; New bus no. = 3

$$\therefore \qquad [Z_{Bus}] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \end{array}\Omega$$

Step 4: Consider the element between bus 1 and bus 2 $[Z_b = 5\ \Omega]$ in the next modification. This is type-4 modification.

Added impedance value = 5 Ω; Two old bus nos. are 1 and 2.

$$[Z_{Bus}] = [Z_{Bus}]_{old} - \frac{1}{5 + Z_{11} + Z_{22} - 2Z_{12}} \begin{bmatrix} (Z_{11} - Z_{12}) \\ (Z_{21} - Z_{22}) \\ (Z_{31} - Z_{32}) \end{bmatrix} \times (Z_{11} - Z_{21})(Z_{12} - Z_{22})(Z_{13} - Z_{23})$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

We have thus observed that when we form $[Z_{Bus(new)}]$ by multiplying $[C^T]$ with $[Z_{Bus}]$, we got subtraction of the 3rd row elements from each of the other rows while the sign of the elements of the third row are changed. Thus, for computer solution, a generalization can be made assuming that if we select bus j to be the reference node, the new bus impedance matrix can be formed by the following algorithm.

Step 1: To subtract the existing j-th row from each of the other rows in $[Z_{Bus}]$ and reverse the sign of the elements in the j-th row. This gives $[C^T Z_{Bus}]$.

Step 2: To subtract column j of the resultant $[C^T Z_{Bus}]$ matrix from each of its other columns and reverse the sign of the elements in the j-th column. This gives $[Z_{Bus(new)}]$.

11.6 CONTINGENCY ANALYSIS

Contingency analysis is a method by which we can predict steady state bus voltages and line currents in a power system following switching on or off a line in the system. This method does not plead for exact values of voltages and currents and rather assess the approximate values to check whether the system components or buses will be overloaded or will face under/overvoltage following switching on or off the prescribed line. For this purpose, line resistance, presence of off-nominal tap changing transformers, and line charging effects are often neglected and linear models are assumed where we can apply the principle of superposition. Contingency analysis frequently uses $[Z_{Bus}]$ and loads are assumed to be treated as constant current injectors. Removing a line is treated as adding a negative impedance. Hence, a generalized method of developing an algorithm for addition of line is first developed.

11.7 ADDITION AND REMOVAL OF LINES IN POWER SYSTEM

Let

$V_1, V_2, ..., V_N$ = bus voltages in p.u. in the power network,

$I_1, I_2, ..., I_N$ = known current (p.u.) injections at respective buses,

z_X and z_Y = p.u. impedances of lines to be added in the system between buses i-j and k-ℓ respectively,

I_X and I_Y = currents (p.u.) in branches z_X and z_Y respectively,

$[V_1', V_2', ..., V_N']^T$ = bus voltages in p.u. in the same power network after addition of z_X and z_Y in the network.

Here, we can write

$$[V] = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_i \\ V_j \\ V_k \\ V_\ell \\ \vdots \\ V_N \end{bmatrix}; \quad [I] = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_i \\ I_j \\ I_k \\ I_\ell \\ \vdots \\ I_N \end{bmatrix}; \quad [V'] = \begin{bmatrix} V_1' \\ V_2' \\ \vdots \\ V_i' \\ V_j' \\ V_k' \\ V_\ell' \\ \vdots \\ V_N' \end{bmatrix}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Substituting the values of $(V_i - V_j)$ and $(V_k - V_\ell)$ from equation (1.69) in equation (11.73), we get

$$\therefore \quad \begin{bmatrix} I_X \\ I_Y \end{bmatrix} = \frac{1}{1-\phi_{k\ell(i-j)}\phi_{ij(k-\ell)}} \left[\begin{array}{c|c} \dfrac{z_X}{Z_{Th(ij)} - z_X} & \dfrac{z_Y^2}{z_X} \dfrac{\phi_{k\ell(i-j)}}{Z_{Th(k\ell)} - z_Y} \\ \hline \dfrac{z_X^2}{z_Y} \dfrac{\phi_{ij(k-\ell)}}{Z_{Th(ij)} - z_X} & \dfrac{z_Y}{Z_{Th(k\ell)} - z_Y} \end{array} \right] \times \begin{bmatrix} I_{ij} \\ I_{k\ell} \end{bmatrix} \tag{11.74}$$

Now we are to find the current change ΔI_{pq} in a healthy line because of outage of lines between buses i-j and k-ℓ. Mathematically the assumed contingent case of tripping of two lines having p.u. series impedances z_X and z_Y being equivalent to adding impedances $(-z_X)$ and $(-z_Y)$, we can model the system as current injection I_X at bus i and $-I_X$ at bus j while current injection is I_Y at bus k and $-I_Y$ at bus ℓ. The bus voltage changes at bus p and q are obtained as

$$\left.\begin{aligned} \Delta V_p &= (Z_{pj} - Z_{pi})I_X + (Z_{p\ell} - Z_{pk})I_Y \\ \text{and} \qquad \Delta V_q &= (Z_{qj} - Z_{qi})I_X + (Z_{q\ell} - Z_{qk})I_Y \end{aligned}\right\} \tag{11.75}$$

$$\therefore \quad \Delta I_{pq} = \frac{\Delta V_p - \Delta V_q}{z_{pq}} \tag{11.76}$$

[z_{pq} being the series impedance of the line between buses p and q]

Utilising equation (11.75) in equation (11.76), we get

$$\Delta I_{pq} = \frac{[(Z_{pj} - Z_{pi})I_X + (Z_{p\ell} - Z_{pk})I_Y] - [(Z_{qj} - Z_{qi})I_X + (Z_{q\ell} - Z_{qk})I_Y]}{z_{pq}}$$

$$= \left[\begin{array}{c|c} \dfrac{(Z_{pj} - Z_{pi}) - (Z_{qj} - Z_{qi})}{z_{pq}} & \dfrac{(Z_{p\ell} - Z_{pk}) - (Z_{q\ell} - Z_{qk})}{z_{pq}} \end{array} \right] \begin{bmatrix} I_X \\ \hline I_Y \end{bmatrix} \tag{11.76a}$$

[z_{pq} being the series impedance of the line between buses p-q]

Following the concept of line outage distribution factor, we can write

$$\phi_{pq(i-j)} = -\frac{z_X}{z_{pq}} \left[\frac{(Z_{pi} - Z_{pj}) - (Z_{qi} - Z_{qj})}{Z_{Th(ij)} - z_X} \right] \tag{11.77a}$$

$$\phi_{pq(k-\ell)} = -\frac{z_Y}{z_{pq}} \left[\frac{(Z_{pk} - Z_{p\ell}) - (Z_{qk} - Z_{q\ell})}{Z_{Th(k\ell)} - z_Y} \right] \tag{11.77b}$$

Using equations (11.77a) and (11.77b) in equation (11.76a), we get

$$\Delta I_{pq} = \left[\begin{array}{c|c} \dfrac{Z_{Th(ij)} - z_X}{z_X} \phi_{pq(i-j)} & \dfrac{Z_{Th(k\ell)} - z_Y}{z_Y} \phi_{pq(k-\ell)} \end{array} \right] \begin{bmatrix} I_X \\ \hline I_Y \end{bmatrix} \tag{11.78}$$

$$\therefore \quad \Delta I_{pq} = \frac{Z_{Th(ij)} - z_X}{z_X} \phi_{pq(i-j)} I_X + \frac{Z_{Th(k\ell)} - z_Y}{z_Y} \phi_{pq(k-\ell)} I_Y \tag{11.78a}$$

From equation (11.74)

$$I_X = \frac{1}{1-\phi_{k\ell(i-j)}\phi_{ij(k-\ell)}} \frac{z_X}{Z_{Th(ij)} - z_X} I_{ij} + \frac{1}{1-\phi_{k\ell(i-j)}\phi_{ij(k-\ell)}} \frac{z_Y^2}{z_X} \frac{\phi_{k\ell(i-j)}}{Z_{Th(k\ell)} - z_Y} I_{k\ell}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Again, utilising the expression of I_1 from equation (12.5) in (12.2)

$$Q_1 = -\operatorname{Im}\left[\left(|V_1|^2|Y_{11}|\epsilon^{j\phi_{11}} + |V_1V_2Y_{12}|\epsilon^{j(\phi_{12}+\delta_2-\delta_1)}\right)\right]$$

$$= -|V_1|^2|Y_{11}|\sin\phi_{11} - |V_1V_2Y_{12}|\sin(\phi_{12}+\delta_2-\delta_1)$$

$$\therefore \quad Q_1 = -|V_1|^2|Y_{11}|\sin\phi_{11} - |V_1V_2Y_{12}|\sin(\phi_{12}-\delta_{12}) \tag{12.10}$$

while from equations (12.6) and (12.4), the expression of Q_2 has been obtained as

$$Q_2 = -|V_2|^2|Y_{22}|\sin\phi_{22} - |V_1V_2Y_{21}|\sin(\phi_{21}+\delta_{12}) \tag{12.11}$$

In EHV lines (r/x), ratio of conductors is small and in analysis it is a reasonable assumption to neglect the resistance in the line. In such a case, when there is no resistance in the equivalent network connecting two buses,

$$\phi_{11} = \phi_{22} = -90^\circ, \quad \phi_{12} = \phi_{21} = 90^\circ$$

Thus modified expressions in loss-less frame for real and reactive powers at terminals 1 and 2 are then given by

$$\left.\begin{aligned} P_1 &= |V_1V_2Y_{12}|\sin\delta_{12}; \quad Q_1 = |V_1|^2|Y_{11}| - |V_1V_2Y_{12}|\cos\delta_{12} \\ P_2 &= -|V_1V_2Y_{21}|\sin\delta_{12}; \quad Q_2 = |V_2|^2|Y_{22}| - |V_1V_2Y_{21}|\cos\delta_{12} \end{aligned}\right\} \tag{12.12}$$

12.3 VOLTAGE REGULATION IN A TRANSMISSION SYSTEM AND ITS RELATION WITH REACTIVE POWER

Voltage regulation may be defined as the per unit change in the sending end voltage magnitude for a specific variation in the receiving end voltage from no load to full load, and is caused by the drop in voltage due to passage of load current through the supply impedance. Thus the voltage regulation in p.u. for a simple transmission system (Fig. 12.2) is given by

$$\Delta V = \frac{|E| - |V|}{|V|} \tag{12.13}$$

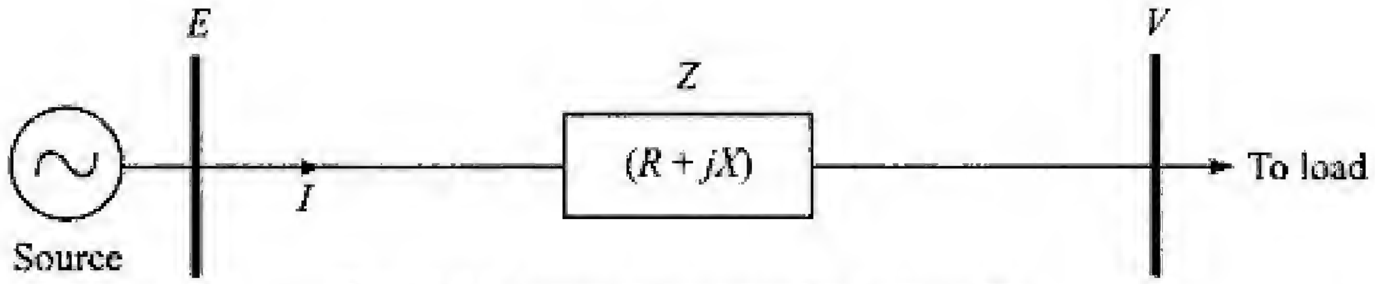


Fig. 12.2 A simple transmission line model.

From the phasor diagram (Fig. 12.3) corresponding to the system shown in Fig. 12.2,

$$\Delta V = E - V = IZ \tag{12.14}$$

where I is the load current through the line having the line impedance Z (= $R + jX$).

Example 12.1: An inductive load draws power of $(2 + j1)$ MVA at a receiving end bus of a radial three phase line. The receiving end bus voltage is 11 KV $(L - L)$ at 50 Hz. and the system reactance is 0.5 Ω/phase. Calculate (i) the receiving end current, (ii) the regulation, (iii) the sending end voltage, and (iv) the short circuit capacity of the system. Assume the system to be lossless.

Solution: Load current (I_L) can be obtained as follows:

$$I_L = \frac{\sqrt{P^2 + Q^2}}{\sqrt{3}V_L} = \frac{\sqrt{2^2 + 1^2} \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 117.363132 \text{ A}$$

and
$$\phi = \tan^{-1}\frac{Q}{P} = \tan^{-1}\frac{1}{2} = 26.565051°$$

∴ $I_L (= |I_L| \angle \phi) = 117.363132 \angle -26.565051°$ A (*Ans. of (i)*).

From equation (12.16), for lossless line the regulation is obtained as

$$\Delta V = \frac{XQ}{V} + j\frac{XP}{V} = \frac{0.5 \times \dfrac{1 \times 10^6}{3}}{\dfrac{(11 \times 10^3)}{\sqrt{3}}} + j\frac{0.5 \times \dfrac{2 \times 10^6}{3}}{\dfrac{(11 \times 10^3)}{\sqrt{3}}}$$

$$= (26.243194 + j\,52.486388) \text{ V} = 58.681566 \angle 63.434950° \text{ V (\textit{Ans. of (ii)})}.$$

The sending end voltage (E) is obtained as

$$E = V + \Delta V = \frac{11 \times 10^3}{\sqrt{3}} + 26.243194 + j\,52.486388$$

$$= 6377.312145 \angle 0.471560° \text{ V/Ph}$$

$$= E(L - L) = \sqrt{3}E = 11045.82865 \text{ V} \cong 11.46 \text{ kV (\textit{Ans. of (iii)})}.$$

The short circuit capacity can be obtained as

$$S_{s/c} = \frac{E_{ph}^2}{\text{system reactance}} = \frac{E^2}{0.5} = 81.340220 \text{ MVA/Ph (\textit{Ans. of (iv)})}.$$

Example 12.2: Assuming the value of short circuit capacity same as obtained in Example 12.1 for the system described in that example, find the ratio of (V/E) using the relation given by equation (12.32) in the text.

Solution: $S_{s/c} = 81.340220$ MVA/Ph, as obtained in Example 12.1.

From relationship of E and V in terms of short circuit capacity (equation (12.32)), we obtain

$$V = E\left(1 - \frac{Q}{S_{s/c}}\right)$$

∴
$$\frac{V}{E} = 1 - \frac{Q}{S_{s/c}} = 1 - \frac{1 \times \dfrac{10^6}{3}}{81.340220 \times 10^6} = 0.995902.$$

[It may be observed that in Example 12.1 we obtained the ratio $(V/E) = (11/\sqrt{3})/6377.312145 = 0.995851$ while in Example 12.2, this ratio is 0.995902. The difference is due to the fact that expression (12.32) gives an approximate value.]

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

From the graph, it is evident that there is a maximum transmissible power, P_{max} ($=P_m$, say) for each load power factor. This is the steady state stability limit at that power factor. For any value of $P(< P_{max})$, there are two possible solutions of V_r. Stable voltage operation is possible at the upper values of $|V_r|$. The lower half of the curves provides voltage unstable zone. Load power factor has a governing effect on the receiving end voltage. At lagging power factor $|V_r|$ tends to reduce while for better power factor loads $|V_r|$ tends to increase. The maximum transmissible power also increases with improvement of power factor.

Instead of assuming a radial line, let us now assume that the line is basically a synchronous tie interconnecting two grids having voltages E_1 and E_2 (Fig. 12.6). Let the midpoint voltage and the current be V_m and I_m, respectively. Equations for the sending end half of the line are

$$E_1 = V_m \cos\frac{\theta}{2} + jZ_0 I_m \sin\frac{\theta}{2} \tag{12.51a}$$

$$I_1 = j\frac{V_m}{Z_0}\sin\frac{\theta}{2} + I_m \cos\frac{\theta}{2} \tag{12.51b}$$

At the midpoint,

$$P_m + jQ_m = V_m I_m^* = P \tag{12.52}$$

It may be noted here that $Q_m = 0$ indicates that no reactive power flows through the midpoint. The transmitted power is P and the real and reactive power at the sending end are

$$P_1 + jQ_1 = E_1 I_1^* \tag{12.53}$$

Substituting for E_1 and I_1 from equation (12.51) and assuming V_m as reference phasor, with $P_m = V_m I_m$, equation (12.53) reduces to

$$(P_1 + jQ_1) = P_m + j\left[Z_0 I_m^2 - \frac{V_m^2}{Z_0}\right]\frac{\sin\theta}{2} \tag{12.54}$$

Since the system is assumed to be lossless, $P_1 = P_m = P_2$. Equation (12.54) then becomes

$$Q_1 = \left[Z_0 I_m^2 - \frac{V_m^2}{Z_0}\right]\frac{\sin\theta}{2}$$

However, $$P_0 = \frac{V_0^2/Z_0}{P_m} = V_m I_m.$$

$$\therefore \quad Q_1 = \left[\frac{V_0^2}{P_0}\frac{P_m^2}{V_m^2} - \frac{V_m^2}{V_0^2/P_0}\right]\frac{\sin\theta}{2} = P_0\left[\frac{P_m^2}{P_0^2}\frac{V_0^2}{V_m^2} - \frac{V_m^2}{V_0^2}\right]\frac{\sin\theta}{2} \tag{12.55}$$

If the terminal voltages are adjusted so that the midpoint voltage $V_m = 1.0$ at all levels of power transfer then from equation (12.55) we have

$$Q_1 = P_0\left[\frac{P_m^2}{P_0^2} - 1\right]\frac{\sin\theta}{2} = P_0\left[\frac{P^2}{P_0^2} - 1\right]\frac{\sin\theta}{2} = -Q_2 \tag{12.56}$$

[assuming $P_m = P$].

Thus we find,

(i) if P (transmitted power) $> P_0$ (SIL), $Q_1 > 0$ (positive) and $Q_2 < 0$ (negative). This means there is a deficit of line charging capacitive power and also, $V_m < E_1, E_2$; to have $V_m = 1.0$, we need to supply capacitive power to the line.

$\therefore$ $$P_s = 528 \text{ MW}, Q_s = 423.212350 \text{ MVAR (inductive)}.$$

$$\text{Here regulation} = \frac{V_s - V_r}{V_r} \times 100\% = \frac{321.157926 - 220}{220} \times 100\% = 45.980876\%$$

For uncompensated lines voltage regulation becomes very high.

Example 12.7: If the load of 220 MVA in Example 12.6 is suddenly disconnected, and the sending end voltage is maintained at the value obtained in the solution of the problem described in Example 12.6, find the rise in the receiving end voltage. Also, determine sending end current and the charging reactive power available at the sending end.

Solution: Since the line described in Example 12.6 is now open circuited with sending end voltage at 321.157926 kV (L–L), we can write

$$V_r = \frac{V_s}{\cos\theta} = \frac{321.157926}{\cos 31.38°} = 376.180928 \text{ kV (L–L)}.$$

$$I_s = j\frac{V_s}{Z_0}\tan\theta = j\frac{\dfrac{321.157926}{\sqrt{3}}}{109.56}\tan 31.38° = j1.032242 \text{ kA}.$$

Q (charging MVAR at sending end) $= \text{Img}(\sqrt{3}V_s I_s^*)$

$$= \text{Img}[\sqrt{3} \times 321.157926 \times (-j1.032242)] \text{ MVAR} = -574.196701 \text{ MVAR}$$

Negative MVAR indicates that here the reactive power is capacitive (which is obvious due to Ferranti effect in transmission line).

$$\text{Here regulation} = \frac{V_s - V_r}{V_r} \times 100\% = \frac{321.157926 - 376.180928}{376.180928} \times 100\% = -14.626739\%$$

So with receiving end open, voltage regulation is negative.

Example 12.8: A cable has surge impedance of 50 Ω and operates at 500 kV (L–L) at 50 Hz. If the electrical line length is 30° equivalent, find the steady state stability limit. Obtain the value of the transmission angle and the reactive power requirements at both ends of the cable when the transmitted power through the cable is 50% of the SIL value. Recalculate these values for transmitted power of 125%. Assume both ends of the line are maintained at 500 kV.

Solution: $$P_0(\text{SIL}) = \frac{V_0^2}{Z_0} = \frac{500^2}{50} = 5000 \text{ MW}$$

$$P_{\max} \text{ (steady state stability limit)} = \frac{P_0}{\sin\theta} = \frac{5000}{\sin 30°} = 10{,}000 \text{ MW}$$

Also, $$\frac{P}{P_0} = \frac{\sin\delta}{\sin\theta}$$

But as per the question, $$\frac{P}{P_0} = 0.5$$

$\therefore$ $$\sin\delta = \frac{P}{P_0} \times \sin\theta = 0.5 \times \sin 30° = 0.25$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

To find the sensitivity of the voltage with tap change, the ratio $\partial|V|/\partial a$ is to be obtained;

$$\frac{\partial|V|}{\partial a} = \frac{R(R+a^2X)(R-a^2X)}{\left(R^2+(a^2X)^2\right)^{3/2}}E \tag{12.108}$$

To have stable voltage state,

$$\frac{\partial|V|}{\partial a} > 0 \tag{12.109}$$

Using equation (12.109) in (12.108) we observe that this is only possible if

$$R > a^2X \tag{12.110}$$

Equation (12.110) shows that *the secondary voltage drops if the tap position 'a' is raised in order to boost up the load bus voltage. At the condition when $a^2X > R$, the voltage stability is lost.* In case of EHV transmission line, X being higher in magnitude, voltage instability may spontaneously occur when an attempt is made to raise the tap position. With enhancement in a, a^2X factor further increases showing that even if the load bus voltage is raised, the system is pushed towards voltage instability. Voltage instability will spontaneously occur under this condition when at any moment the magnitude of a^2X exceeds R.

Thus, it has been analytically established that the tap changer does not serve the purpose of maintaining voltage stability once the system operation enters into a critical state. The following discussion relates the same findings through qualitative discussion. Figure 12.19 represents the V-P curve of a typical transmission system for different tap positions neglecting the effect of induction motors on the load bus, i.e. only the static impedance loads are considered.

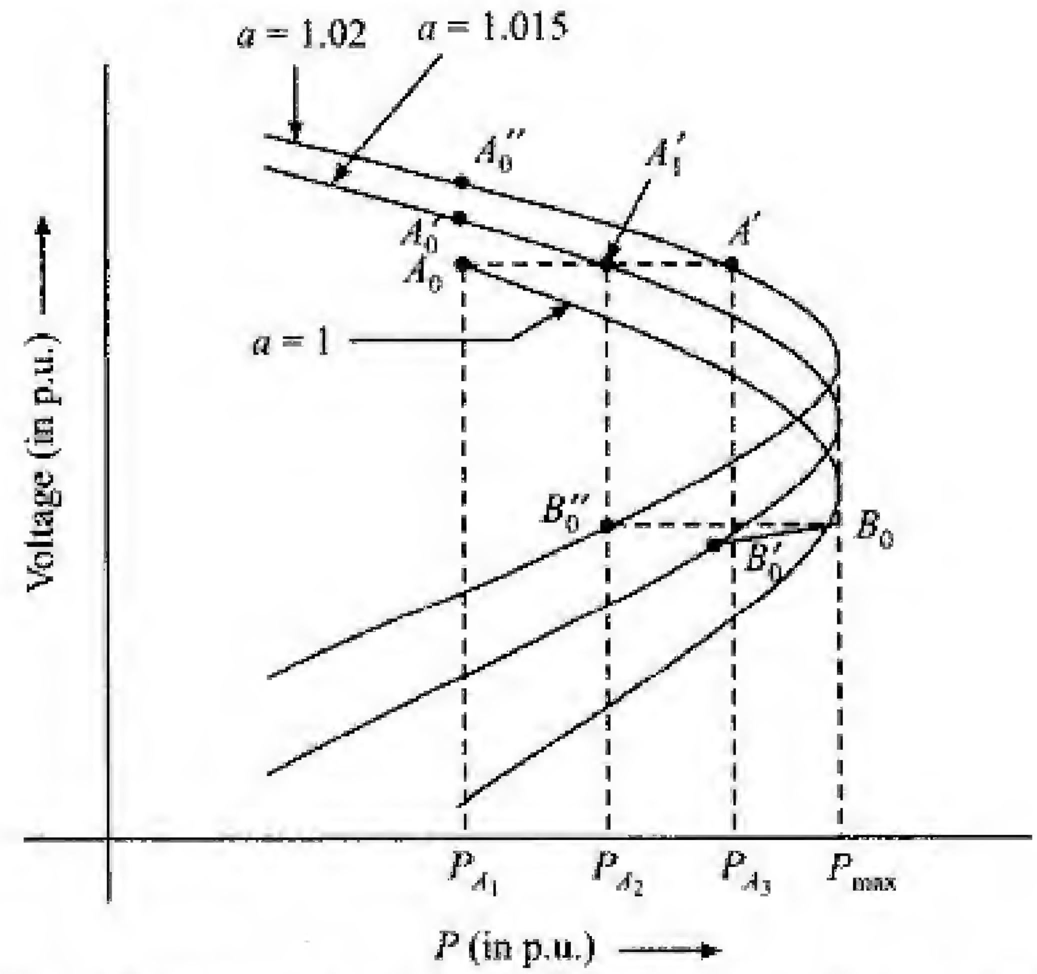


Fig. 12.19 *V-P* characteristic with static impedance load at the receiving end bus.

It may be observed that if the initial system operation be considered at point A_0, by tap changing, the operating point first shifts to A_0'. At point A_0', the voltage is slightly enhanced and the system is

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Operation of synchronous condenser

To explain the operation of the synchronous condenser in steady state, *V-I* characteristic is redrawn superimposed with load lines (Fig. 12.28). Assuming the shifting of load level, the load line-1 shifts to load line-2. Had there been no condenser, the system voltage would have dropped from point 'a' to point 'c', i.e. from V_1 to V_2. However, the presence of the condenser raises the voltage again near to the nominal value as is evident by the intersection of the load line-2 with the transient state characteristic (point 'b'). The machine is to operate in the lead p.f. zone.

Thus, the new operating point is the point 'b' where the condenser is operating in the overexcited stage. However, if the capacity of the condenser at this operating point is beyond the rated capacity of the condenser, the operation at this point is limited for a short interval of time depending on the machine's overload capability. Then the field current will be automatically reduced by adjusting the voltage regulator and a new operating point corresponding to the nearest steady state characteristic (at maximum excitation limit) will be attained at point 'd'.

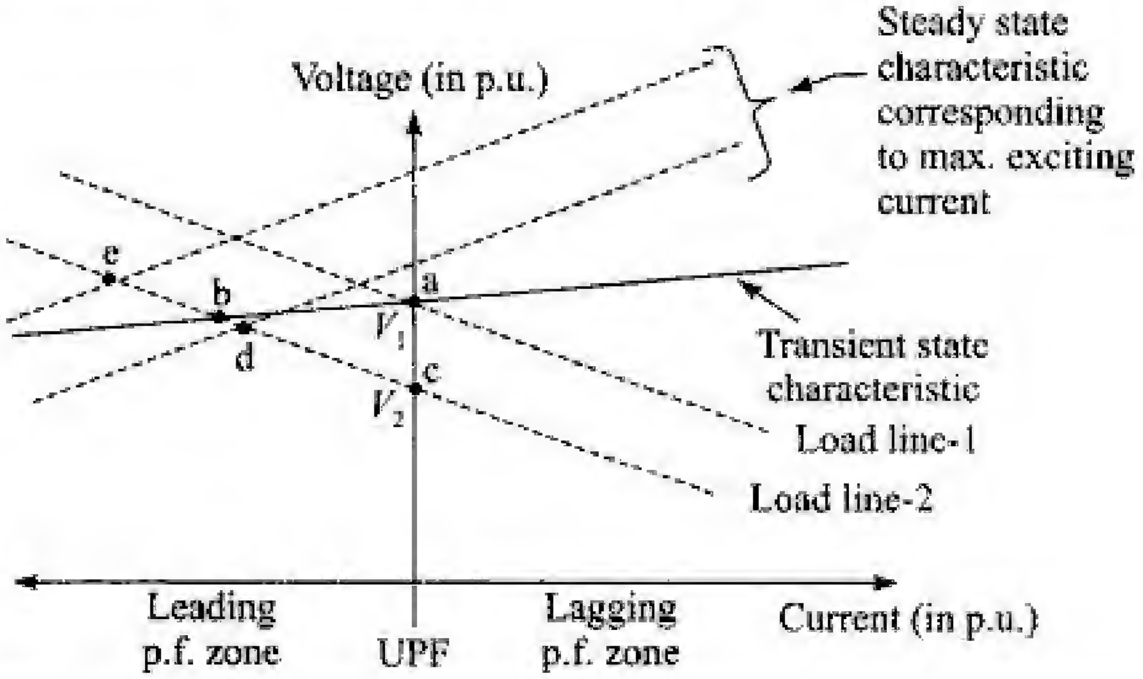


Fig. 12.28 *V-I* characteristic of synchronous condenser with load lines.

On the other hand, if the operation of the condenser corresponding to the transient state of operating point 'b' is within the capability limit, the steady state operation is then settled at point 'e' of the nearest steady state operating characteristic.

Application of synchronous condensers to have limited control of voltage at the receiving end

Here, a constant reactive power source (a synchronous condenser at a fixed excitation limit) with its VAR output Q_{max} is considered to be added at the receiving end in order to increase the power transfer, enhance the stability limits and regulate the receiving end voltage profile, keeping it well within the design limit allowing maximum voltage regulation of 5% (i.e. V_{min} = 0.95 p.u.).

With enhancement of VAR reserves, the operating point voltage and stability margin improve. The capability of this VAR reserve, with the complete utilisation of the desired stability margin (M_d), can be obtained from equation (12.115) and is given by

$$Q_{max} = \frac{V_d^2}{X} - \frac{E_d}{X}\cos\delta_d \tag{12.115}$$

where

$$\delta_d = \sin^{-1}\left(\frac{P_{Rd}X}{V_d E}\right) \tag{12.116}$$

The Q-V curve for bus-6 (with and without SVC) is shown in Fig. E12.2.

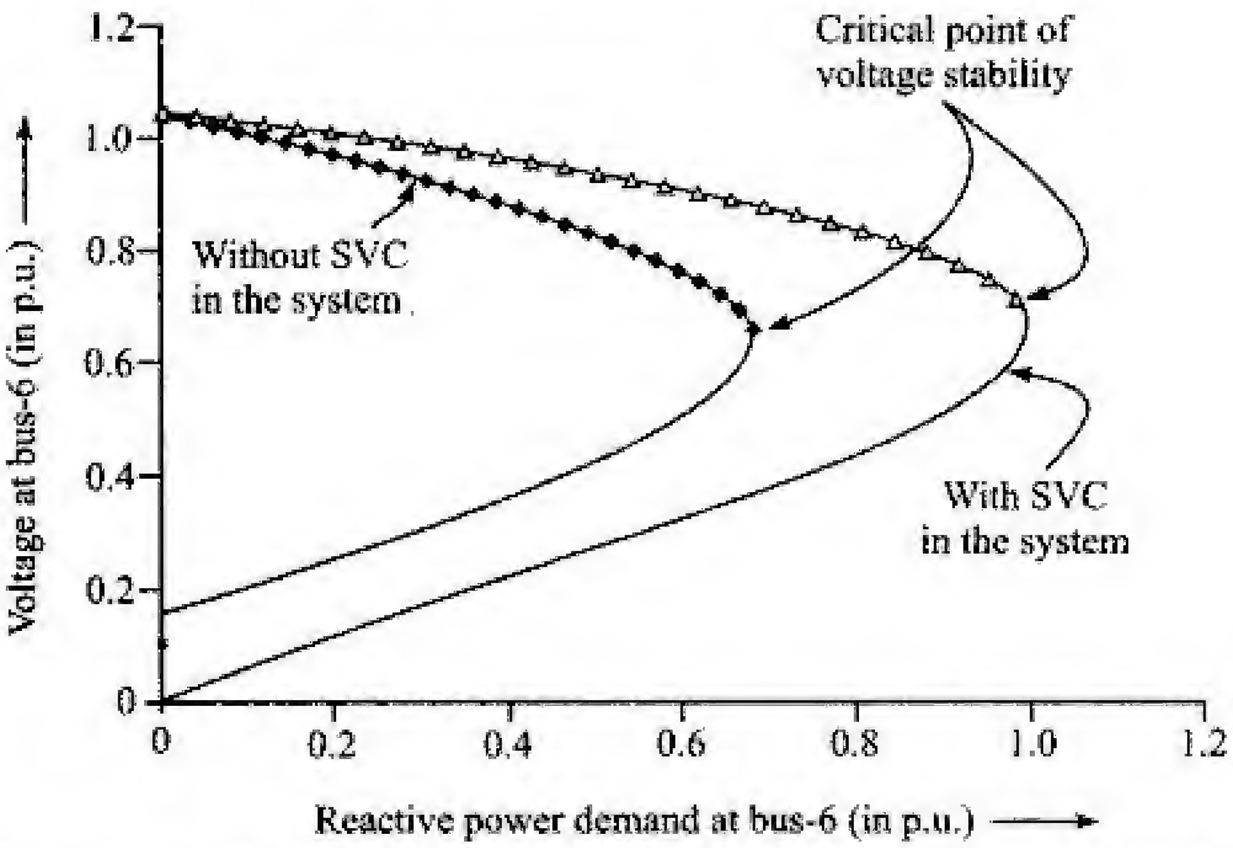


Fig. E12.2 *Q-V* curve for bus-6 with and without SVC at bus-6 (Susceptance model of SVC being assumed).

12.22 DETERMINATION OF VOLTAGE STABILITY USING SENSITIVITY INDICATOR

The basic equations used in Newton-Raphson load flow method are given below:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \dfrac{\partial P}{\partial \delta} & |V|\dfrac{\partial P}{\partial |V|} \\ \dfrac{\partial Q}{\partial \delta} & |V|\dfrac{\partial Q}{\partial |V|} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \dfrac{\Delta |V|}{|V|} \end{bmatrix} \tag{12.123}$$

$$\delta_{new} = \delta_{old} + \Delta\delta \tag{12.124}$$

$$|V|_{new} = |V|_{old}\left(1 + \frac{\Delta |V|}{|V|}\right) \tag{12.125}$$

Here the Jacobian is:

$$[J] = \begin{bmatrix} \dfrac{\partial P}{\partial \delta} & |V|\dfrac{\partial P}{\partial |V|} \\ \dfrac{\partial Q}{\partial \delta} & |V|\dfrac{\partial Q}{\partial |V|} \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \tag{12.126}$$

To obtain real and reactive power sensitivity, the basic load flow equation becomes

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \dfrac{\partial P}{\partial \delta} & \dfrac{\partial P}{\partial |V|} \\ \dfrac{\partial Q}{\partial \delta} & \dfrac{\partial Q}{\partial |V|} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta |V| \end{bmatrix} \tag{12.127}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

(*ii*) The data obtained to plot *VSI* with SVC in the system: **VSI2.DAT**

Bus no.	Pload (p.u.)	Qload (p.u.)	VSI	Element
6	.250000	.000000	.315835	(5, 5)
6	.250000	.050000	.319431	(5, 5)
6	.250000	.100000	.323654	(5, 5)
6	.250000	.150000	.328110	(5, 5)
6	.250000	.200000	.332828	(5, 5)
6	.250000	.250000	.337837	(5, 5)
6	.250000	.300000	.343175	(5, 5)
6	.250000	.350000	.348886	(5, 5)
6	.250000	.400000	.355022	(5, 5)
6	.250000	.450000	.361649	(5, 5)
6	.250000	.500000	.368845	(5, 5)
6	.250000	.550000	.376709	(5, 5)
6	.250000	.600000	.385367	(5, 5)
6	.250000	.650000	.394979	(5, 5)
6	.250000	.700000	.409694	(5, 5)
6	.250000	.750000	.423932	(5, 5)
6	.250000	.800000	.441447	(5, 5)
6	.250000	.850000	.464251	(5, 5)
6	.250000	.900000	.496902	(5, 5)

(*iii*) The data obtained to plot *VSI* when line 5-6 trips and there is no SVC in the system: **VSI2.DAT**

Bus no.	Pload (p.u.)	Qload (p.u.)	VSI	Element
6	.250000	.000000	.434444	(5, 5)
6	.250000	.050000	.444419	(5, 5)
6	.250000	.100000	.455381	(5, 5)
6	.250000	.150000	.467532	(5, 5)
6	.250000	.200000	.481132	(5, 5)
6	.250000	.250000	.496531	(5, 5)
6	.250000	.300000	.514204	(5, 5)
6	.250000	.350000	.534816	(5, 5)
6	.250000	.400000	.559323	(5, 5)
6	.250000	.450000	.602708	(5, 5)
6	.250000	.500000	.657264	(5, 5)
6	.250000	.550000	.845259	(5, 5)

The profile of *VSI* with the variation in the reactive power status of bus-6 is shown in Fig. E12.5.

```
» JR [dV/dQ calculated direct from diagonal element of inverse of [JR].]
JR =
      6.0000    0.3858   [Positive dV/dQ, stable]
      4.0000    0.3285
      5.0000    0.3117
      2.0000    0.2321
      3.0000    0.1778

» DA [the eigenvalues of [JR]]
DA =
      0.8840         0         0     [Positive eigenvalue of [JR], stable]
           0    6.5857         0
           0         0   14.1669
» Db [maximum eigenvalues of inverse of [JR] (starting from maximum)]
Db =
      1.1312         0         0   [Positive eigenvalue of inverse of [JR], stable]
           0    0.1518         0
           0         0    0.0706
```

For qload(6)=0.4 p.u.

FILE NAME: SOUTL2.DAT

```
Number of weak Modes required: 3 [i.e., 3 modes have been considered here.]
» DS [the bus participation factor]
DS =
      6.0000    0.2667    6.0000    0.4301    2.0000    0.5860
      5.0000    0.2367    4.0000    0.3152    6.0000    0.2499
      4.0000    0.2305    2.0000    0.1332    3.0000    0.1027
      2.0000    0.1442    5.0000    0.0910         0         0
      3.0000    0.1219         0         0         0         0
» dvdq [dV/dQ from eigenvalues (i.e. no. of modes considered) [equation (12.144)]]
dvdq =
      6.0000    0.4368   [Positive dV/dQ, stable]
      4.0000    0.3516
      5.0000    0.3072
      2.0000    0.0670
      3.0000    0.0079
» JR [dV/dQ calculated direct from diagonal element of inverse of [JR].]
JR =
      6.0000    0.4393   [Positive dV/dQ, stable]
      4.0000    0.3746
      5.0000    0.3568
      2.0000    0.2595
      3.0000    0.2003
```

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Chapter 13

Power System Compensation Using Passive and Facts Controllers

13.1 INTRODUCTION

With the ongoing expansion and growth of the electric utility industry, including deregulation in many countries, numerous changes are continuously being introduced to once a predictable business. Although electricity is a highly engineered product, it is increasingly being considered and handled as a commodity. Thus, transmission systems are being pushed closer to their stability and thermal limits while the focus on the quality of power delivered is greater than ever.

In the evolving utility environment, financial and market forces continue and will continue, to demand a more optimal and profitable operation of the power system with respect to generation, transmission and distribution. Now, more than ever, advanced technologies are paramount for reliable and secure operation of power systems. To achieve both operational reliability and financial profitability, it has become clear that more efficient utilization and control of the existing transmission system infrastructure is required.

The chief objective of the power system operation being the supply of electric power from source end to load end stably and economically, it is imperative that the frequency and voltage at any point (node) of the network remains constant while the voltage waveform at any bus is free form harmonics, and the phases are balanced. It is preferred in practice that the power factor is either unity or very close to unity and bus voltage, even if it varies, remains within specified limits and the load performs its function in an optimal way. The maintenance of constant frequency in a power system needs an exact balance between the generated power and the load plus loss (i.e. power generated = power consumed at load + power loss). Also, voltage plays an important role in maintaining stable operation of the system. As shown in Chapter 12 following the discussion relating to the concept of voltage control and stability, it is very much evident that the receiving end voltage in a power system network is extremely sensitive to any change in reactive power status at the receiving end bus. In order to achieve optimum performance of the power system, it is required to control the reactive power flow in the network and this is possible by applying reactive power management at the different nodes in the power system. However, for reactive power management, the focus may be either towards the performance of load or line performance. Traditional solutions for upgrading the electrical transmission system infrastructure have been primarily in the form of new transmission lines, substations and associated equipment. However, as experiences

The expression for voltage regulation in simple two bus system has been derived in Chapter 12 [equations (12.16) and (12.17)] and is reproduced here.

$$\Delta V = \left(\frac{RP_\ell + XQ_\ell}{V}\right) + j\left(\frac{XP_\ell - RQ_\ell}{V}\right) = \Delta V' + \Delta V'' \tag{13.9}$$

(Q and P in equation (12.16) are replaced by Q_ℓ and P_ℓ).

We have termed $\Delta V'$ as the inphase drop while $\Delta V''$ the quadrature drop. Thus, voltage regulation is a function of both of these drops and subsequently depends on magnitude and phase of the bus voltage (V) and the load current. In other words, the voltage change depends upon both the real and reactive power demands of the load, precisely more on reactive power demand of the load.

By installing a reactive compensator in parallel to the load, it is possible to make $|E| = |V|$ (i.e. voltage regulation becomes zero). This is a purely reactive compensator and Q_ℓ in the equation of voltage regulation being replaced by Q, where $Q = Q_\ell + Q_c$. For heavy inductive loading Q_ℓ is positive and the compensation is negative, (i.e. $-Q_c$) the compensator being a pure capacitance. On the other hand, in order to compensate for the Ferranti effect where $|\Delta V|$ $(= |E| - |V|)$ is negative, we need to have inductive compensator and Q_c is positive (Q_c is the reactive-power capability of the compensator).

Once the compensator is installed at the load bus, we can write (following the phasor diagram shown in Fig. 12.3)

$$|E|^2 = \left[V + \frac{RP_\ell + XQ}{V}\right]^2 + \left[\frac{XP_\ell - RQ}{V}\right]^2 \tag{13.9a}$$

Q_ℓ being replaced by Q, where $Q = Q_\ell + Q_c$.

Rearranging equation (13.9), we get

$$AQ^2 + BQ + C = 0 \tag{13.9b}$$

where $A = R^2+X^2$; $B = 2V^2X$; $C = (V^2 + RP_\ell)^2 + X^2P_\ell^2 - E^2V^2$

Solution of equation (13.9b) yields

$$Q = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = Q_\ell + Q_c \tag{13.10}$$

Here Q is obtained and Q_ℓ is given, Q_c can also be obtained from equation (13.10); the reactive load demand being compensated, the voltage regulation obtained from equation (13.9) is given by

$$\Delta V = \left(\frac{RP_\ell + jXP_\ell}{V}\right) = (R + jX)\frac{P_\ell}{V} \tag{13.11}$$

Expression (13.11) indicates that even with a purely reactive compensator (with $Q_\ell = 0$), there will be a voltage regulation being governed by line impedance and real power demand.

Algorithm to determine capacity of a reactive compensator to improve voltage regulation

Step 1: Store data of load bus voltage (V), line impedance (z) [$= r + jx$] and load demand $S_\ell = P_\ell + jQ_\ell$.

Step 2: Calculate the constants A, B and C in equation (13.9b).

Step 3: Obtain Q using equation (13.10).

Step 4: Find Q_c using equation [$Q_c = Q - Q_\ell$].

Step 5: Calculate ΔV using equation (13.11).

Step 6: Obtain Q using equation (13.10).

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

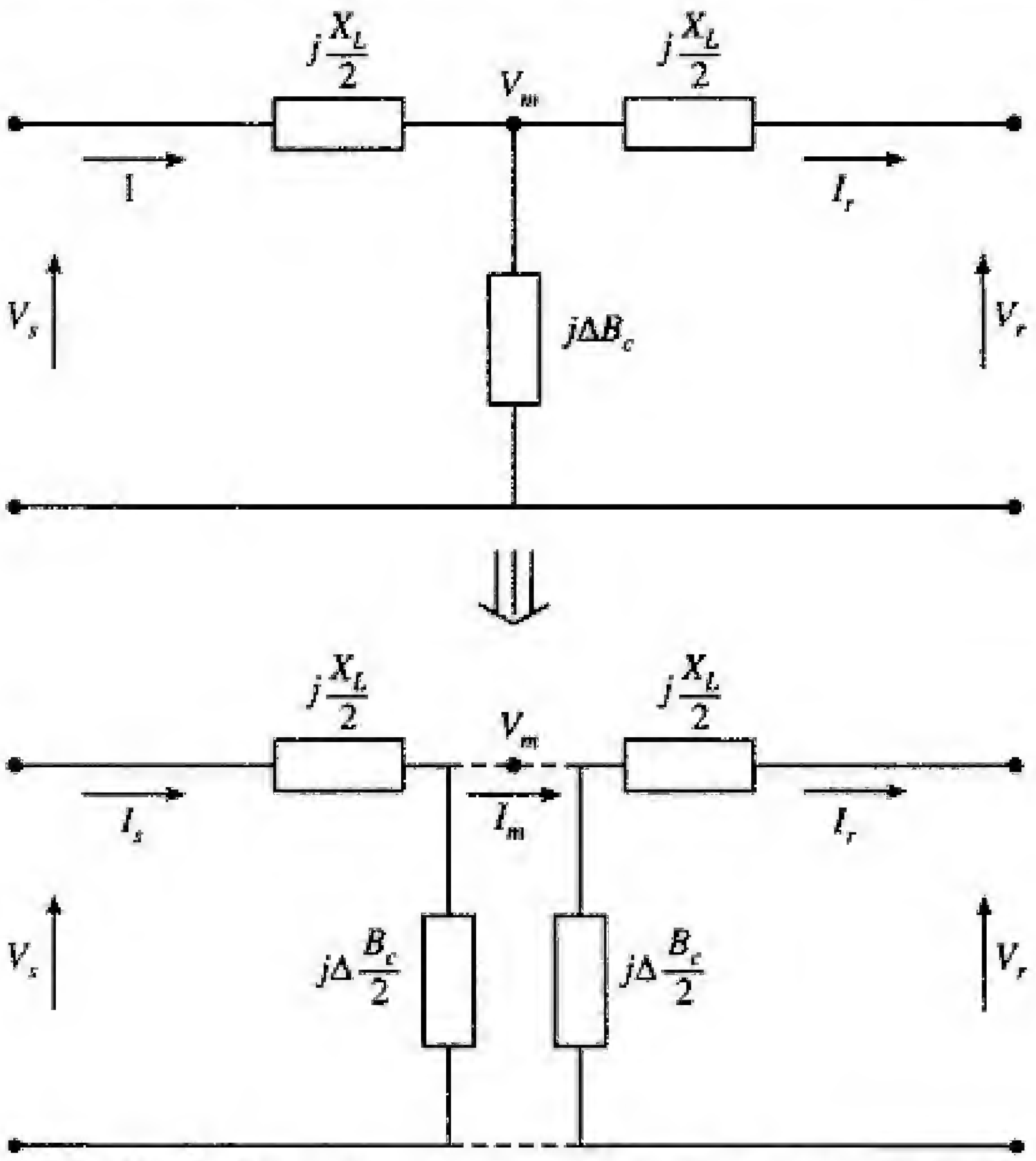


Fig. 13.16 Equivalent circuit of mid-point shunt compensation.

Thus, the receiving end current is obtained as

$$I_r = I_m - V_m\left(j\frac{\Delta B_c}{2}\right) \tag{13.42}$$

The mid-point voltage V_m can be expressed as

$$V_m = V_r + jI_r\frac{X_L}{2} = V_r + j\frac{X_L}{2}\left(I_m - jV_m\frac{\Delta B_c}{2}\right)$$

$$= V_r + j\frac{X_L}{2}I_m + \frac{V_m \Delta B_c}{2}\left(\frac{X_L}{2}\right) \tag{13.43}$$

From equation (13.43), we can write

$$\Delta V_m = V_m\frac{\Delta B_c}{2}\left(\frac{X_L}{2}\right) \tag{13.44a}$$

$$= \frac{\Delta I_c}{2}\cdot\frac{X_L}{2} \text{ [Using equation (13.41)]} = \frac{\Delta I_c X_L}{4} \tag{13.44b}$$

Substituting the value of ΔV_m from equation (13.44a) in (13.40), we get

$$\Delta P = \frac{V_s V_m}{2}\sin\frac{\delta}{2}\Delta B_c \tag{13.45}$$

However, the reactive power charged by the mid-point capacitor is given as

$$\Delta Q_{Sh} = V_m^2 \Delta B_c \tag{13.46}$$

The power mismatch equation can be written now as

$$\begin{bmatrix} \Delta P_j \\ \Delta Q_{sc} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial P_j}{\partial \delta_j} & \dfrac{\partial P_j}{\partial |V_{sc}|} \\ \dfrac{\partial Q_{sc}}{\partial \delta_j} & \dfrac{\partial Q_{sc}}{\partial |V_{sc}|} \end{bmatrix} \begin{bmatrix} \Delta \delta_j \\ \Delta |V_{sc}| \end{bmatrix} \tag{13.71}$$

At the end of iteration p, the variable voltage $|V_{sc}|$ can be corrected as

$$|V_{sc}|^{(p+1)} = |V_{sc}|^{(p)} + \Delta |V_{sc}|^{(p)} \tag{13.72}$$

13.6.3 Series-Series Controllers

Any standard series controller (FACTS device) may be suitably connected with another type of series FACTS controller to form a series-series controller (Fig. 13.25). As a typical example, we may think of a thyristor controlled series capacitor (TCSC) in series with a thyristor switched series capacitor (TCSC). It is reasonable to arrange this series connection such that one module could be smooth thyristor control while the other could be thyristor switched control. Though not very common in use, the series-series controller (Fig. 13.25) may be applied for control of power in double circuit ac lines.

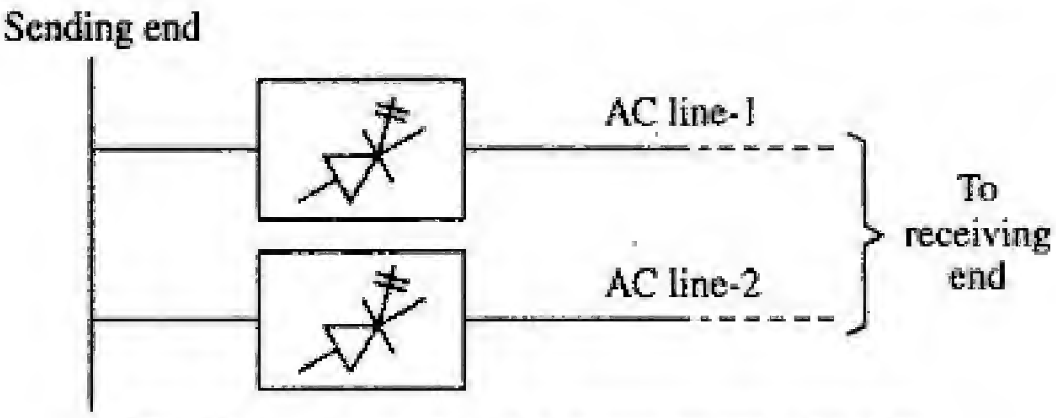


Fig. 13.25 Schematic of a series-series controller.

13.6.4 Combined Shunt-Series Connected Controllers

Static phase shifter (SPS)

A static phase shifter (SPS) or a *thyristor controlled phase shifting transformer* (TC PST) is basically a phase shifting transformer adjusted by thyristor switches to provide smooth variation in phase angle.

Figure 13.26 shows such an arrangement where phase shifting is arranged by adding a quadrature voltage vector in series with the phase voltage. This quadrature voltage vector is derived from the other two phases via shunt connected transformers. The perpendicular series voltage is made variable using thyristor based voltage controller. SPS is connected in all the three phases. Using proper control circuitry, it is possible to even reverse the voltage so that phase shift in either direction is possible.

Let us denote the sending end as suffix s while the receiving end as suffix r. We assume a lossless system. The equivalent circuit of the SPS is shown in Fig. 13.27.

Let us assume I_s and I_r be the sending end and receiving end currents with V_s and V_r, the respective voltages, X_t the series transformer reactance. We can express the tap changing variables n_V and n_1 as

$$n_V = 1 \angle \phi = \cos \phi + j \sin \phi \tag{13.73a}$$

$$n_1 = 1 \angle -\phi = \cos \phi + j \sin \phi \tag{13.73b}$$

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

In order to calculate the effect of damping on the system, the detailed system representation has to be considerably studied. The total damping in the power system is typically very small and is made up of both positive and negative components. In larger networks, the only two components that add to positive damping are the prime mover and electrical loads.

In earlier researches, the damper windings with high resistive components were employed to suppress oscillations as it was revealed in the researches that high line resistance and armature winding resistance give rise to higher oscillations, while higher resistance in damper windings is useful to provide damping to machine oscillations. Later, *Power System Stabilizers* (PSS) were developed to suppress these low frequency electromechanical oscillations effectively.

14.4 MECHANISM OF TIE-LINE OSCILLATIONS

Let us assume that a few generators operate in parallel in a closely connected system and oscillations are getting damped due to the presence of the damper windings. If oscillations did occur, there was little variation in the system voltage. For example, two generators sharing equal load in oscillations between them would produce practically no voltage variation and what is produced would be practically at the twice oscillation frequency. Thus, generator voltage regulators need not be simulated in this case. Moreover, the close coupling between the generators reduces the effective generator AVR gain for the oscillation mode. If the generator voltage regulator gain is increased, there would be much improvement on transient stability but not considerable increase in negative damping. However, with high regulator gain, negative damping torque increases and there may be some possibility of loss of stability due to this negative damping torque.

Under these circumstances, let us assume that the existing system is connected to another generator by a tie-line. We further assume that this tie-line is strong enough to sustain heavy power flow without appreciable drop in voltage and will not have thermal or stability limitations. Because of the presence of high external impedance seen by either system, not only positive damping by the generator damping winding is lost, but the generator terminal voltages become responsive to angular swings. This causes the generator AVR to act, producing negative damping as an unwanted side effect. This sensitivity to voltage to angle increases as a strong function of a tie-line loading. Tie-line oscillations are likely to occur, especially at heavy line loading. These tie-line oscillations are not desired, especially as a restriction on the allowable power transfer, as relatively large oscillations are taken as a precursor to instability. As the interconnection proceeds, more oscillations are likely to appear.

14.5 NATURE OF OSCILLATIONS AND ITS STUDY PROCEDURE

We can point out the nature of oscillations in the power system having the following properties.

1. Oscillations are due to natural modes of the system and therefore cannot be completely eliminated.
2. With increase in complexity of the power system, the frequency and damping of oscillations may increase and new ones may be added.
3. AVR control is the primary source of introducing negative damping torque in the power system. With increase in the number of controls, negative damping may further increase.
4. Interarea oscillations are associated with weak transmission lines and larger line loadings.
5. Interarea oscillations may involve more than one utility.
6. Damping of the system is to be enhanced to control these tie-line oscillations.

The small signal analysis (i.e. modal analysis or eigenvalue analysis) based on linear techniques is ideally suitable for investigating problems associated with oscillations. Here, the characteristics of a

From Fig. 14.2, we obtain

$$\Delta\delta = \frac{\omega_0}{s}\left[\frac{1}{2Hs}(-T_c\Delta\delta - k_d\Delta\omega + \Delta T_m)\right]$$

$$= \frac{\omega_0}{s}\left[\frac{1}{2Hs}\left(-T_c\Delta\delta - k_d s\frac{\Delta\delta}{\omega_0} + \Delta T_m\right)\right]$$

i.e.

$$s^2(\Delta\delta) + \frac{k_d}{2H}s(\Delta\delta) + \frac{T_c}{2H}\omega_0(\Delta\delta) = \frac{\omega_0}{2H}\Delta T_m \tag{14.15}$$

The characteristic equation is then given by

$$s^2 + \frac{k_d}{2H}s + \frac{T_c}{2H}\omega_0 = 0 \tag{14.16}$$

[General form: $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$]

The undamped natural frequency is then given as

$$\omega_n = \sqrt{T_c\frac{\omega_0}{2H}}\ \text{rad/sec} \tag{14.17}$$

Damping ratio (ξ) is given by

$$\xi = \frac{1}{2}\frac{k_d}{2H\omega_n} = \frac{1}{2}\frac{k_d}{\sqrt{T_c 2H\omega_0}} \tag{14.18}$$

It may be observed that with increase in T_c, the synchronizing torque coefficient, the natural frequency ω_0 increases while the damping ratio (ξ) decreases. If k_d is increased, the damping ratio (ξ) increases. On the other hand, if H is increased, ω_n decreases along with reduction in the damping ratio.

Example 14.1: A generator supplies power in steady state to an infinite bus (Fig. E14.1). Due to some contingencies, line-2 gets tripped. Find the following:

(i) Damped frequency of oscillation, (ii) Damping ratio,
(iii) Undamped natural frequency, and (iv) Eigenvalues,

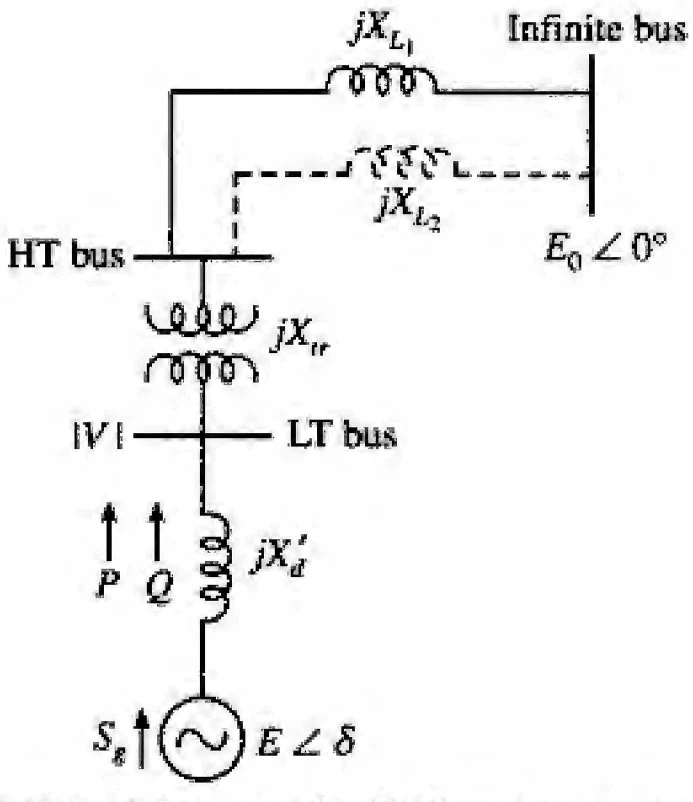


Fig. E14.1 Schematic of a SMIB system of Ex. 14.1.

From equation (14.40), we get

$$k_4 = E_0 \frac{(X_d - X_d')(X_q + X_\ell)\sin\delta_0}{(X_q + X_\ell)(X_d' + X_\ell)}$$

$$\therefore \qquad k_4 = E_0 \frac{X_d - X_d'}{X_\ell + X_d'}\sin\delta_0 \tag{14.45d}$$

From equation (14.43), we get

$$k_5 = \left[\frac{V_{d0}X_q}{V}\frac{E_0\cos\delta_0(X_d' + X_\ell)}{(X_q + X_\ell)(X_d' + X_\ell)} - \frac{V_{q0}}{V}\frac{X_d' E_0(X_q + X_\ell)\sin\delta_0}{(X_q + X_\ell)(X_d' + X_\ell)}\right]$$

$$\therefore \qquad k_5 = \frac{X_q}{X_q + X_\ell}\frac{V_{d0}}{V}E_0\cos\delta_0 - \frac{X_d'}{X_d' + X_\ell}\frac{V_{q0}}{V}E_0\sin\delta_0 \tag{14.45e}$$

From equation (14.44), we get

$$k_6 = -\frac{V_{q0}}{V}\frac{X_d'(X_q + X_\ell)}{(X_q + X_\ell)(X_d' + X_\ell)} + \frac{V_{q0}}{V}$$

or,

$$k_6 = -\frac{V_{q0}}{V}\left(1 - \frac{X_d'}{X_d' + X_\ell}\right)$$

$$\therefore \qquad k_6 = \frac{X_\ell}{X_\ell + X_d'}\frac{V_{q0}}{V} \tag{14.45f}$$

The parameters (k_1-k_6) all change with the operating condition, except k_3 (which is ratio of impedance). These equations represent the linearised small perturbation relations of a single generator connected with an infinite bus through an external impedance. Suffix 0 stands for initial values.

14.8 COMPUTATIONAL STEPS TO FIND k_1 TO k_6 PARAMETERS

In this section, a generalised method of determining k_1-k_6 parameters will be presented for a SMIB system from the machine data and system operating data. The steps are as follows:

Step 1: To read: X_d', X_q, ω_0, k_d, k_A, T_A, T_{d0}' and H. Also, to read R_ℓ, X_ℓ, $V \angle \theta$, $E_0 \angle 0°$. All the variables have p.u. value except T_A, T_{d0}' and H which are expressed in secs. θ is expressed in degree. Infinite bus voltage is treated as reference phasor. [k_A represents amplifier gain in exciter circuit of the machine while T_A is its time constant.]

Step 2: To calculate I_g, use equation

$$I_g \angle \phi \,[=(I_d + I_q)\,e^{j(\delta-\pi/2)}] = \frac{V\angle\theta - E_0\angle 0°}{R_\ell + jX_\ell}$$

[Frequently, for the external line we assume R_ℓ to be negligibly low; this makes $R_\ell = 0$ and $I_g \angle \phi = \dfrac{V\angle\theta - E_0\angle 0°}{jX_\ell}$.]

Step 3: To calculate δ, the angle of E', the following equation may be used;

$$E' = V\angle\theta + jX_q I_g \angle\phi = |E'|\angle\delta$$

[In expressions of k_1-k_6, this angle δ is the same as δ_0.]

The eigenvalues of the state matrix [A] are

$$\lambda_1, \lambda_2 = -2.795768 \pm j6.508242; \lambda_{3,4} = 0.129102 \pm j5.263960.$$

Since they have positive real parts, the small signal stability of the system is lost in this case. Hence the small signal stability of the given power system is lost with increase of amplifier gain (k_A) from 50 to 200.

Execution of the computer program as per the algorithm given in section 14.12 for calculation of *k*-coefficients for Example 14.3 gives the following result:

R_EX14P3.DAT

```
For ka=  50.000000

k1=    0.874066  k2=    0.822567 k3=    0.333333
k4=    1.498794  k5=   -0.022797 k6=    0.412980

[A] matrix
  -0.333333    -0.166533     0.000000    0.111111
   0.000000     0.000000   314.000000    0.000000
  -0.082257    -0.087407     0.000000    0.000000
-103.244975     5.699289     0.000000   -5.000000

Eigenvalues of [A] matrix are

 -2.620565 + 2.521554i
 -2.620565 - 2.521554i
 -0.046102 + 5.183920i
 -0.046102 - 5.183920i

Small signal stability is maintained.

For ka= 200.000000

k1=    0.874066  k2=    0.822567 k3=    0.333333
k4=    1.498794  k5=   -0.023521 k6=    0.426090

[A] matrix

  -0.333333    -0.166533     0.000000    0.111111
   0.000000     0.000000   314.000000    0.000000
  -0.082257    -0.087407     0.000000    0.000000
-426.090460    23.520879     0.000000   -5.000000

Eigenvalues of [A] matrix are

 -2.795769 + 6.508242i
 -2.795769 - 6.508242i
```

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Solution:

Test system 1:

The state matrix $[A]$ is

$$[A]=\begin{bmatrix} -1.333333 & -0.933333 & 0 & 0.333333 \\ 0 & 0 & 314 & 0 \\ -0.352 & -0.352 & 0 & 0 \\ -70 & -10 & 0 & -4 \end{bmatrix}$$

The eigenvalues of the state matrix $[A]$ are

$$\lambda_{1,2} = -2.035352 \pm j4.123291;\ \lambda_{3,4} = -0.631315 \pm j10.607173$$

Since they have negative real parts, the small signal stability of the system is maintained.
From equation (14.85), we can write for $[A_{Mod}]$ for inclusion of PSS,

$$\therefore \qquad [A_{Mod}]=\left[\begin{array}{c:c:c:c:c} -1.333333 & -0.933333 & 0 & 0.333333 & 0 \\ \hdashline 0 & 0 & 314 & 0 & 0 \\ \hdashline -0.352 & -0.352 & 0 & 0 & 0 \\ \hdashline -70 & -10 & 0 & -4 & \dfrac{50}{0.25} \\ \hdashline \dfrac{-3.52\times 1}{0.1}\left(\dfrac{1.1}{2\times 5}\right) & \dfrac{-3.52\times 1}{0.1}\left(\dfrac{1.1}{2\times 5}\right) & \dfrac{1.1}{0.1} & 0 & -\dfrac{1}{0.1} \end{array}\right]$$

$$=\left[\begin{array}{c:c:c:c:c} -1.333333 & -0.933333 & 0 & 0.333333 & 0 \\ \hdashline 0 & 0 & 314 & 0 & 0 \\ \hdashline -0.352 & -0.352 & 0 & 0 & 0 \\ \hdashline -70 & -10 & 0 & -4 & 200 \\ \hdashline -3.872 & -3.872 & 11 & 0 & -10 \end{array}\right]$$

Eigenvalue analysis reveals that for $[A_{Mod}]$

$$\lambda_{1,2} = -1.825668 \pm j3.611861$$
$$\lambda_{3,4} = -0.195111 \pm j11.360022$$
$$\lambda_5 = -11.291775$$

Since the modified state matrix has an extra negative real root, it is concluded that there is improvement in small signal stability.

Test system 2:

$$[A]=\begin{bmatrix} -0.813008 & -0.506667 & 0 & 0.333333 \\ 0 & 0 & 314 & 0 \\ -0.12 & -0.082 & 0 & 0 \\ -104 & 20 & 0 & -4 \end{bmatrix}$$

The eigenvalues of the state matrix $[A]$ are

$$\lambda_1, \lambda_2 = -2.963578 \pm j5.956531;\ \lambda_{3,4} = 0.557074 \pm j5.069225.$$

Since they have positive real parts, the small signal stability of the system is not maintained.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

You have either reached a page that is unavailable for viewing or reached your viewing limit for this book.

Therefore, equation (15.8) is a single equation and is a function of $\hat{x}$. To minimize $f\,[= f(\hat{x})]$, the following relation must be satisfied

$$\frac{\partial f}{\partial \hat{x}} = 0$$

or,
$$-z^T H - H^T z + 2H^T H\hat{x} = 0$$

$\therefore$
$$H^T H\hat{x} - H^T z = 0 \quad [\because\ H^T z = z^T H] \tag{15.9}$$

Equation (15.9) is called *normal equation* and the solution of this equation will provide least square estimate of $\hat{x}$ as follows:

$$\hat{x} = (H^T H)^{-1} H^T z \tag{15.10}$$

Example 15.1: Estimate two random variables by least square estimation method for a given measurement vector z as follows:

$$z = \begin{bmatrix} 0.5 \\ 0.45 \\ 0.51 \end{bmatrix} \text{ and } H = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Solution: Given,
$$H = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \therefore \quad H^T = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$\therefore$
$$H^T H = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

or,
$$(H^T H)^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} = \begin{bmatrix} 0.6667 & -0.3333 \\ -0.3333 & 0.6667 \end{bmatrix}$$

$\therefore$
$$(H^T H)^{-1} H^T = \begin{bmatrix} 0.6667 & -0.3333 \\ -0.3333 & 0.6667 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0.3334 & 0.6667 & -0.3333 \\ 0.3334 & -0.3333 & 0.6667 \end{bmatrix}$$

Let us assume, $z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$ where $\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.45 \\ 0.51 \end{bmatrix}$

From equation (15.10), we have

$$\hat{x} = (H^T H)^{-1} H^T z = \begin{bmatrix} 0.3334 & 0.6667 & -0.3333 \\ 0.3334 & -0.3333 & 0.6667 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$= \sum_{\substack{k=1 \\ k \neq i}}^{n} | V_i V_k Y_{ik} | \sin(\phi_{ik} + \delta_k - \delta_i) \qquad \text{for } i = j \qquad (15.40a)$$

for $i = 1, 2, 3, ..., N$, for $j = 2, 3, ..., N$

$$H_{2_{ij}} = \frac{\partial P_i}{\partial |V_j|} = | V_i Y_{ij} | \cos(\phi_{ij} + \delta_j - \delta_i) \qquad \text{for } i \neq j$$

$$= 2V_i Y_{ii} \cos \phi_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{N} | V_k Y_{ik} | \cos(\phi_{ik} + \delta_k - \delta_i) \qquad \text{for } i = j \qquad (15.40b)$$

for $i = 1, 2, 3, ..., N$, for $j = 1, 2, 3, ..., N$

$$H_{3_{i(j-1)}} = \frac{\partial Q_i}{\partial \delta_j} = -| V_i V_j Y_{ij} | \cos(\phi_{ij} + \delta_j - \delta_i) \qquad \text{for } i \neq j$$

$$= \sum_{\substack{k=1 \\ k \neq i}}^{n} | V_i V_k Y_{ik} | \cos(\phi_{ik} + \delta_k - \delta_i) \qquad \text{for } i = j \qquad (15.40c)$$

for $i = 1, 2, 3, ..., N$, for $j = 2, 3, ..., N$

$$H_{4_{ij}} = \frac{\partial Q_i}{\partial |V_j|} = -| V_i Y_{ij} | \sin(\phi_{ij} + \delta_j - \delta_i) \qquad \text{for } i \neq j$$

$$= -2V_i Y_{ii} \sin \phi_{ii} - \sum_{\substack{k=1 \\ k \neq i}}^{N} | V_k Y_{ik} | \sin(\phi_{ik} + \delta_k - \delta_i) \qquad \text{for } i = j \qquad (15.40d)$$

for $i = 1, 2, 3, ..., N$, for $j = 1, 2, 3, ..., N$

Equations (15.40a–15.40d) may be utilized to compute the Jacobian matrix for any specified value of the state vector x. Here the measurement vector has a non-linear relationship with the state vector x. Therefore, the iterative method described in Section 15.3 may be employed to estimate the state vector x using equation (15.31a) or (15.31b).

If we apply the concept of decoupling as described in Section 4.11, the submatrices H_2 and H_3 will be zero and equation (15.28) can be rewritten in this case as given by

$$\begin{bmatrix} \Delta z_p \\ \Delta z_q \end{bmatrix} = \begin{bmatrix} H_1 & 0 \\ 0 & H_4 \end{bmatrix} \begin{bmatrix} \Delta x_\delta \\ \Delta x_v \end{bmatrix} + \begin{bmatrix} r_p \\ r_q \end{bmatrix} \qquad (15.41)$$

where,

z_p = measurement vector y for active power injection,
z_q = measurement vector y for reactive power injection,
x_δ = state vector for δ,
x_v = state vector for $| V |$,
r_p = vector for error in z_p,
r_q = vector for error in z_q,
Δz_p and Δz_q = change in z_p and z_q respectively,
Δx_δ and Δx_v = change in x_δ and x_v respectively.

Equation (15.41) can be written separately as

$$\Delta z_p = H_1 \Delta x_\delta + r_p \qquad (15.42a)$$

and

$$\Delta z_q = H_4 \Delta x_v + r_q \qquad (15.42b)$$

where $\hat{x}\left(\frac{\sigma}{\sigma}\right)$ = filtered estimate of $x(\sigma)$,

$\hat{x}\left(\frac{\sigma}{\sigma-1}\right)$ = single step prediction of $x(\sigma)$,

$L_x(\sigma)$ = filter gain vector of same dimension as $x(\sigma)$,

$R(\sigma)$ = covariance of $e(\sigma)$,

$\xi_x\left(\frac{\sigma}{\sigma}\right)$ = filtering error covariance,

$\xi_x\left(\frac{\sigma}{\sigma-1}\right)$ = prediction error covariance,

and $Q(\sigma)$ = covariance of $u(\sigma)$.

The prediction $\hat{x}\left(\frac{\sigma+1}{\sigma}\right)$ can easily be obtained from equation (15.80a) at the one step lead time $t = \sigma + 1$. Now, load forecast may be obtained from equation (15.80b) in one step ahead as follows:

$$d_s(\sigma) = h^T \hat{x}\left(\frac{\sigma+1}{\sigma}\right) \tag{15.83}$$

Similarly, we can obtain a multi-step ahead (say, p step ahead) prediction of the load from the multi-step ahead prediction of the vector $x(\sigma)$, given by (from equation (15.80a))

$$x(\sigma+p) = F^p\, x\left(\frac{\sigma}{\sigma}\right) \tag{15.84}$$

For this estimation, it is very much necessary that the noise statistics and some other information must be available. The value of $R(\sigma)$ is often estimated from a knowledge of the accuracy of the meters used. Generally, the value of the covariance $Q(\sigma)$ is unknown initially and required to be obtained by some other means. An adaptive version of the Kalman filtering algorithm is used to estimate the noise statistics along with the state vector $x(\sigma)$. Now, if we assume that both $R(\sigma)$ and $Q(\sigma)$ are known quantities, and the initial estimate $\hat{x}\left(\frac{0}{0}\right)$ and the covariance $\xi_x\left(\frac{0}{0}\right)$ are known, then using these as a *priori* information, it is possible to utilize equations (15.82a)–(15.82e) recursively to process the data for $d_s(1), d_s(2), d_s(3), \ldots, d_s(\sigma)$ to generate the filtered estimate $\hat{x}\left(\frac{\sigma}{\sigma}\right)$. Then, it is very easy to forecast the load at any desired lead time using equation (15.84).

Short-term Forecasting Using the Periodic Load Model

As described in Section 15.13.4, the periodic load can be modelled using equation (15.69a) or (15.69b) considering the dominant harmonics. Now, equation (15.69b) can be used as the output equation in the Kalman algorithm. If we assume that all the coefficients are constant, the required dynamic model for the state vector $x(\sigma)$ can be given by

$$x(\sigma+1) = x(\sigma) \tag{15.85}$$

Second Edition

Power System Analysis

Operation and Control

Abhijit Chakrabarti • Sunita Halder

This well-received book, now in its Second Edition, continues to discuss a number of engineering and economic aspects relating to power system operation and control. It gives an in-depth analysis of the subject, bringing together the many dimensions of power system operation and control. It also ably illustrates the application of advanced methods in solving power system problems. In this edition, five new chapters are included, some existing chapters are revised, and more solved examples are given, to enlarge the goal of the book.

What is New to This Edition

→ Provides separate chapters on Power System Stability, Contingency Analysis and Power System Security, Power System Compensation, Small Signal Stability, and State Estimation and Load Forecasting.

→ Includes appendices on load flow calculation using bus impedance matrix and the decoupling of real and reactive power in terms of load angle and voltage.

The book is designed both for postgraduate students in power systems/energy systems engineering and for senior undergraduate students of electrical engineering. It will also be useful to the industry professionals and researchers.

THE AUTHORS

Abhijit Chakrabarti is Professor, Department of Electrical Engineering, Bengal Engineering and Science University, Shibpur, Howrah. A Fellow of the Institution of Engineers (India), he has published/presented 85 research papers in international and national journals and conferences and authored/co-authored ten books on electrical engineering. He has nearly eight years of industrial experience and around twenty years of teaching and research experience. Professor Chakrabarti is a recipient of Pandit Madan Mohan Malviya award, and the power medal, merit and best paper awards (twice) from Central Electricity Authority, India. His areas of interest include power system, electric circuits and power electronics and drives.

Sunita Halder is with the Department of Electrical Engineering, Jadavpur University, Kolkata. She has published/presented several research papers in international and national journals and conferences. Her areas of interest include power system and electric circuits.

ISBN:978-81-203-3585-1

9 788120 335851

Rs. 525.00

www.phindia.com